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A Simple Approach to Project Extreme Old Age Mortality Rates and Value Mortality-Related Financial Instruments

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Abstract

This article shows how mortality models that involve age effects can be fitted to ages beyond the sample range using projections of age effects as replacements for age effects that might not be in the sample. This 'projected age effect' approach allows insurers to use age-effect mortality models to obtain valuations of financial instruments such as annuities that depend on projections of extreme old age q rates. Illustrative results suggest that the proposed approach provides a good approximation to both q rates and term annuity prices. The practical import of this approach is to allow insurers to apply a much wider range of mortality models to such problems than would otherwise be possible.

Key Words: Age-Period-Cohort mortality model, age effect, projection, extreme old age, term annuity

JEL codes: G220, G230, J110

1. Introduction

Insurers frequently encounter situations in which they want to project mortality rates out to extreme old age, such as ages over 90. A standard example is where an insurer wishes to price an annuity.

However, most mortality rate (or q rate) models cannot project extreme old age q rates. The problem is that most mortality models – think of models of the Lee-Carter family – have an age-related state variable or age effect, and for such models the maximum age in the sample age range constrains the ages for which one can obtain q rates. If the maximum age in our sample age range is 90, say, then we cannot use these models to obtain q rates for ages over 90. A further problem is that deaths rate data tend to deteriorate for extreme old ages, so our data for such ages might be unreliable.[1] A case in point is where we might have data going to age 90, but might have reason to regard death rates as unreliable for ages over 85.

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[1] The implications of unreliable data are explored further in Cairns *et al.*, 2016.

The only exceptions are models of the CBD family (Cairns *et al*., 2006, 2009). The original CBD model (later reparameterised as M5 using the classification of Cairns *et al*., 2009) had two period effects and no age or cohort effects, but a later extension, M6, added a cohort effect and a further extension, M7, added a third period effect to M6. Thus, the original CBD/M5 model has only period effects, and models M6 and M7 have only period effects and cohort effects, but none of these models has any age effect. Consequently, these models can be used to project q rates to any ages without being constrained by the range of ages in the sample data used to calibrate them.

One solution available to insurers who want extreme old age q projections is therefore to use one of the CBD family of models, but then they are restricted to a choice of 3 models.

So what do we do if we wish to project extreme old ages using a model that has an age effect? This article suggests a simple answer: we can project the age effects and then use the projected age effects to project the q rates we want. The idea is that we project the age effects from those obtained from within the available sample age range and treat the projected age effects as if they were sample age effects. We can then fit any model that includes an age effect, regardless of any maximum age constraints imposed by our available sample age range, and can use the fitted model to project future q rates and price financial instruments that depend on these q rates.

This article is organised as follows. Section 1 examines age effects and their projections and sets out the proposed 'age effect projection' approach. Section 2 examines how well this approach performs when used for projections of cohort q rates and section 3 examines its performance when used for pricing term annuities. Both sections show that the proposed approach performs well in the illustrative settings considered. Section 4 concludes.

2. Age Effects and their Projections

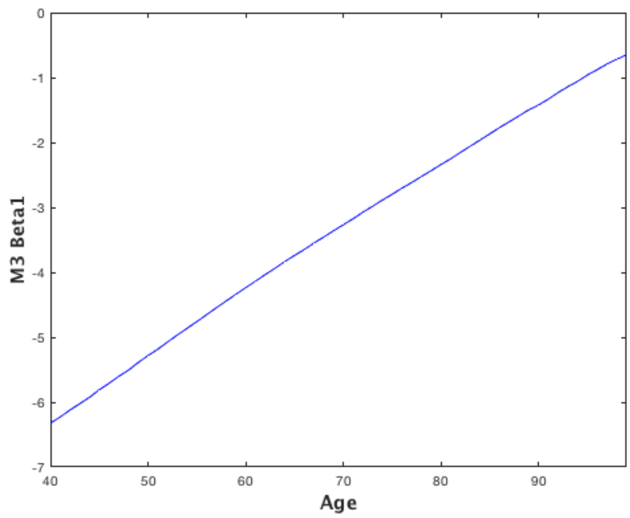
Consider the Age-Period-Cohort model (Jacobsen *et alia*, 2002; Osmond, 1985), sometimes known as M3, which is an extension of the Lee-Carter model that allows for a cohort effect. M3 postulates that $m(t,x)$ satisfies:

(1) $$log\, m(t,x) = \beta_1(x) + \frac{1}{n_a}\kappa_2(t) + \frac{1}{n_a}\gamma(c)$$

where $\beta_1(x)$ is an age effect predicated on age x, $\kappa_2(t)$ is a period effect predicated on period t, $\gamma(c)$ is a cohort effect predicated on cohort or year of birth $c = t - x$, and n_a is the number of ages in the sample data age range. This model satisfies various conditions set out, e.g., in Cairns *et alia* (2009, p. 8).

Figure 1 shows a plot of $\beta_1(x)$ estimated for ages 40:99 on England & Wales male deaths and exposures data for 2015.

Figure 1: Plot of M3 $\beta_1(x)$

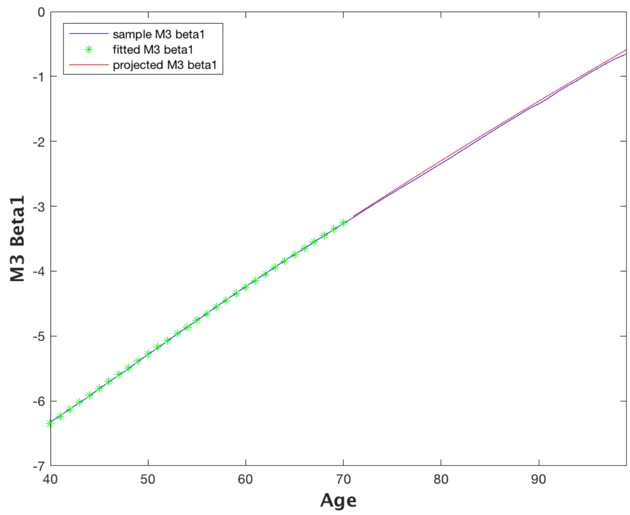


Note: Based on England & Wales data for 1961: 2015.

It is immediately apparent that the plot has a very gentle concave shape.

This point suggests that the $\beta_1(x)$ curve should be projectable, i.e., that we ought to be able to get a good projection of later age $\beta_1(x)$ from earlier age $\beta_1(x)$.[2]

Figure 2: Plot of Fitted vs. Projected M3 $\beta_1(x)$



Note: Based on England & Wales data for 1961: 2015.

[2] A word of caution, however. One needs to be careful about the sample ages range when projecting the age effects. If the sample age range goes down to late teens or early twenties, then the 'accident hump' over those ages can produce projections for the old-age age effects that can be implausible. At the other extreme, if the sample age range includes extreme old age (e.g., beyond 100) then sample-based age effects for those ages be subject to considerable sample variation. As a rule of thumb, we would advise not to use sample ages over 90.

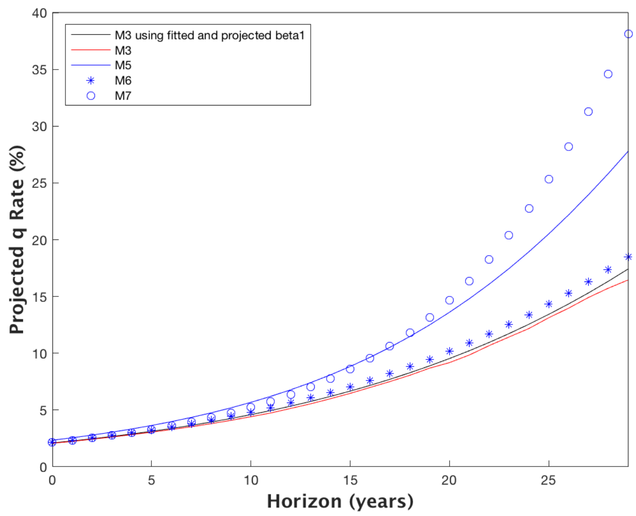
Figure 2 shows plots of fitted vs. projected $\beta_1(x)$ for model M3. The blue line gives the fitted values over ages 40:70 and the red line gives the projected values for ages 71:99 based on quadratic projections from the fits over ages 40:70. The green line underlying these other lines reproduces the sample values given in Figure 1.

We see that the fitted blue values on the left-hand side of the figure are extremely close to the sample green values on the left-hand side, and that the projected red values on the right-hand side are very close to the sample green values on the right-hand side. Thus, the projections are highly accurate when plotted against the actual sample values.

3. Projections of Future Cohort *q* Rates

Figure 3 provides some projected of future cohort *q* rates. The black projection uses M3 with projections of age effects instead of sample age effects. The red projection uses M3 with sample age effects, and the remaining ones are those produced by models M5, M6 and M6 and do not use any age effect.

Figure 3: Projections of Future Cohort *q* Rates



Note: Based on England & Wales data for 1961: 2015.

We see that the black and red lines are very close together, which confirms that using projected age effects gives similar results to using sample age effects. The other lines help give a sense of the model risk across the different models. The error in using projected age effects is therefore small both in absolute terms and when judged against the scale of the model risk.

4. Term Annuity Values

Table 1 shows a set of Term Annuity Values (TAVs) for various start and end ages. For example, for age range 70:99, the annuity is taken out when the annuitant turns 70 and the last payment is made when he turns 99, and so forth. The first line shows the TAVs for M3 when the age effects are replaced by projected age effects. The remaining lines show the TAVs for M3, M5, M6 and M7 respectively:

Table 1: Illustrative Term Annuity Values

Model	Ages Range			
	70:99	**80:99**	**85:99**	**90:99**
M3 (using projected alpha)	12:49	7.18	4.73	2.72
M3	12.73	7.08	4.62	2.68
M5	11.34	6.00	3.91	2.29
M6	12.09	6.42	4.05	2.24
M7	11.55	6.04	3.80	2.09

Notes: Annuity is taken out on birthday given by first age in ages range, and pays out £1 on each subsequent birthday until last birthday in ages range. Annuity has 10% loading factor and is based on a flat 1.5% interest rate term structure. M3 estimations are based on England & Wales males deaths and exposures data for 1961: 2015 and ages 40:99. Other models are based on same data set but sample ages 50:99. The beta1 projection for ages 70:99 is based on a projection from fitted ages 40:69 and so forth.

We see that the M3 TAVs obtained using projected age effects are very close to the M3 TAVs obtained using sample age effects. This result confirms that the projections give accurate TAVs: the typical error rate is between 1% and 2%. We also see that the error in the M3 TAV valuation due to the use of projected instead of sample age effects is also small relative to the range of variation i.e., model risk, that we see across the different mortality models.

5. Conclusions

This article shows how mortality models that involve age effects can be fitted to ages beyond the sample range using projections of age effects as replacements for age effects that are not in the sample. The 'projected age effect' approach proposed allows insurers to use these models to obtain valuations of financial instruments such as annuities that depend on projections of extreme old age q rates. Illustrative results suggest that the proposed approach provides a good approximation to both q rates and term annuity prices. The practical import of this approach is, therefore, to allow insurers to apply a much wider range of mortality models to such problems than would otherwise be possible.

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