Pensions Institute

DISCUSSION PAPER PI-1901

The Impact of Deferral and Adverse Selection on the Actuarial Fairness and Cost Neutrality of the UK State Pension.

John Dagpunar.

February 2019

ISSN 1367-580X

The Pensions Institute
Cass Business School
City University London
106 Bunhill Row
London EC1Y 8TZ
UNITED KINGDOM

http://www.pensions-institute.org/

The Impact of Deferral and Adverse selection on the Actuarial Fairness and Cost Neutrality of the UK State Pension[1]

John Dagpunar, Mathematical Sciences, University of Southampton, SO17 1BJ, UK

Address for correspondence: j.dagpunar@soton.ac.uk

Abstract: *Persons who have achieved UK state pension age (SPA) may defer their pension and instead receive an extra pension on termination of deferral. We define a scheme to be actuarially fair to a category of deferrer with agreed discount rate, when the expected net present value of pre-tax lifetime receipts is independent of the deferral period. After a review of the literature on deferral and early take-up of state pensions in the UK and other countries, this paper argues that the current UK scheme based upon a uniform accrual rate cannot be actuarially fair. Instead, we propose a scheme where the accrual rate is dependent upon deferral period, gender, SPA, deferrer's discount rate, degree of pension uprating, and partnership status of the deferrer. Fair accrual rate curves are plotted for various scenarios and compared with the current uniform rates of 10.4% and 5.8% per annum that apply to those who attain SPA before 6 April and after 5 April 2016 respectively. A scheme that is actuarially fair will not be cost neutral to the Exchequer unless the discount rate is the same for both parties. In addition to this asymmetry, adverse selection will impact upon both actuarial fairness and cost to the Exchequer. Expressions are derived for the cost penalty to the Exchequer for attempting to achieve actuarial fairness both with and without an acknowledgement of adverse selection. Similarly, when the objective is to achieve cost neutrality for the Exchequer, expressions for the cost to the deferrer are obtained. Some numerical examples are given for various scenarios. The methodology should be applicable to public pensions in other countries, in order to inform fair policies for both early and postponed take-up of pensions.*

Keywords: *UK State pension, deferral, actuarial fairness, cost neutrality, adverse selection, discount rate*

JEL: *H55, D63, D12, C44, C41*

I. Introduction and background to state pension deferral and actuarial fairness

Individuals entitled to a UK state pension can defer take-up beyond state pension age (SPA). A person who defers for a period x is entitled on termination of deferral to take his/her un-deferred periodic pension plus an *extra pension* (increment) equal to a proportion βx of the un-deferred pension, where β is an accrual rate. For those who achieve SPA before 6 April 2016 and therefore take the state retirement pension (RP), $\beta = 0.104$ per year deferred, while for others who take the new state pension (SP), $\beta = 0.058$ per annum.

In the former (RP) case, on termination of deferral a person will choose to take either the periodic extra pension, or a lump sum equal to the total pension foregone during deferral, with in the latter case for those who have deferred for at least 12 months, interest added at a rate of 2% per annum above bank base rate. For SP persons the lump sum option is not available. For both RP and SP persons, death during deferral results in neither a periodic pension nor lump sum although the

[1] This paper provides an update to PI discussion paper PI-1802

deceased's estate receives up to three months of pension foregone. A crucial difference between RP and SP pensioners is that in the former case a spouse or civil partner of the deceased will, for death during deferral, choose to inherit either the periodic extra pension or lump sum with interest, and for death after deferral will inherit the extra pension if that was the choice made at the end of deferral. Such inheritance will have to wait until the partner reaches SPA (irrespective of whether or not that occurs before 6 April 2016), and then only if at that point in time he/she has neither remarried nor taken on a new civil partnership. If the deceased had deferred for less than a year then the surviving partner would have to inherit the periodic extra pension. For many RP deferrers the *basic pension* component will account for much if not all of the un-deferred pension. The inheriting partner can inherit 100% of the extra state pension or lump sum that the deferrer has earned on the basic pension. The corresponding proportions for other components of the un-deferred (RP) pension are 50% for Graduated Retirement, 50% for State Second Pension, between 50% and 100% for SERPS, and 50% of inherited extra pension from a recognised legal partner.

In addition to regular reviews of the appropriateness of current state pension ages, the details of the deferral scheme have also come under review. In a report by the Government Actuary it is stated (paragraph 1.3) that '*the concept of actuarially* fair[2] *is subjective*' but it is assumed to mean '… *at state pension age, the benefits available have broadly the same value whether the person chooses to defer or not*... ' and later (paragraph 2.5) *'… the benefits available have broadly the same value in terms of cost to the Exchequer … whether the person chooses to defer or not. '* As the report acknowledges, rates which are truly actuarially fair will vary according to gender, state pension age, length of deferral, rate of pension uprating, to which list one might also add the discount rate to be applied to future payments, and in the case of RP persons their marriage/civil partnership status. In this paper we will define a scheme to be actuarially fair to a category of deferrers if the expected net present value (NPV) of lifetime payments received by a deferrer is the same regardless of the number of years deferred.

It is interesting to note that it was not always the case that actuarial fairness was an objective. Sir William Beveridge, economist and social reformer, says in his famous report to Government, Beveridge (1942, p60), that "The rate of increase above the basic rate that is proposed for each year of postponement (paragraph 246), while it should be sufficient strongly to encourage postponement, is below the full actuarial value of postponement." Thus a 14/-[3] weekly pension payable to a male, at SPA of 65 years would be increased by 1/- for each year's deferment. In Beveridge (1942), Appendix A, paragraph 20, the actuarially correct amounts are given as 14/-, 15/9, 17/10, 20/3, 23/2, 26/8 for pensions payable at retirement ages of 65, 66, 67, 68, 69, and 70 respectively. These are considerably below the proposed payments of 14/- at age 65, 15/- at 66, through to 19/- at age 70. That represents significant unfairness to deferrers and savings for the Exchequer; indeed this is interpreted as a deliberate intention when it is stated that "The saving due to postponement is thus partly returned to the pensioner in the form of a larger pension and partly a profit to the Social Insurance Fund."

[2] Some writers use 'actuarially fair', others 'actuarial neutrality'. 'Actuarially fair' has also been used to describe a different objective of equating Net Present Value of lifetime contributions to lifetime benefits.

[3] The currency was pre-decimal with 12 pence to the shilling and 20 shillings to the pound. Thirteen shillings and four pence for example would be written as 13/4.

It is clear that it is impossible to deliver an actuarially fair scheme to deferrers when the extra pension is awarded according to a linear function βx, as some deferral periods will be more advantageous than others to the deferrer. However, one can achieve fairness by making the marginal accrual rate dependent upon the number of years deferred, in contrast to the current uniform rate. It is of course true that one would not want to change the accrual rate schema retrospectively as that might well compromise historical benefits accrued to date. Therefore, for persons currently deferring when a government proposes a change in accrual rates, the rates experienced could be set so that the expected NPV remains at its value at the time of such change, irrespective of how much longer he/she defers. One might ask: what is the point of actuarial fairness if it has a neutral effect upon pre-tax expected net present value? A response might be that it offers the deferrer choice, an opportunity for tax smoothing, and perhaps encourages such persons to be economically productive without raising their marginal income tax rate. Arguably, these have benefits for both the individual and Society, although the various studies do not indicate a clear correlation.

An obvious question is how much does state pension deferral currently cost the Exchequer? This is unknown as DWP collects no information on the distribution of deferral period. It is also pertinent to ask how many individuals defer. The information available is limited. In the six months to March 2010, approximately 9% of the 371,500 individuals claiming had deferred, Thurley (2017). Of these 35% had opted for the extra pension only, 52 % for the lump sum only and 13% for the option (available at that time) of a mixture of extra pension and lump sum. In March 2010, DWP estimated[4] that 11.31 million (6.82m females and 4.49m males) and 1.17 million (0.93m females and 0.24m males) individuals were in receipt of state pension without and with extra pension (also known as increments) respectively. Regarding the male/female split for those receiving extra pension, the proportion (71%) of females is surprisingly large. This might be explained by greater life expectancy and lower SPA for females, and perhaps a large number of females inheriting extra pension from their deceased husbands who were RP pensioners. Another statistic comes from the Pensions Commission (2004). It is stated that "DWP estimates that 20,000 people in Great Britain currently defer their claim to their state pension each year and on average they defer for around 2 years". That figure of 20,000 seems abnormally low. Referring again to the DWP data set[4] we observe that the total number taking extra pension in March 2004 was 1.25 million. That would imply that the mean claim duration for those who take extra pension, under an extremely conservative assumption that every deferral resulted in extra pension (rather than lump sum), is approximately 1,250,000/20,000 or 62.5 years! It would therefore appear unwise, in the absence of any published methodology to place any reliance on either the estimate of 20,000 or the 2 year figure. Finally, we give the most up to date figures[4]. In March 2018, DWP estimated that 11.72 million (6.26m females and 5.46m males) and 1.05 million (0.78m females and 0.27m males) individuals were in receipt of state pension without and with extra pension respectively and the total bill for pensions in 2017-2018 was £93.8 billion[5] . The numbers for females are less than the corresponding 2010 figures, in part due to the rise in SPA for females during the intervening priod. In summary, while the available data does give a sense of the proportion deferring with extra pension

[4] State Pension caseload statistics (5% sample) to March 2018. Available at https://www.gov.uk/government/statistics/dwp-benefits-statistical-summaries-2018

[5] DWP Annual Report and Accounts 2017-18 for year ending 30 March 2018, HC 1108, London: Stationery Office.

it gives no clue as to the distribution of deferral period; this is something which in future should not be difficult for DWP to collect. Quite possibly, it is also a statistic that may be quite dynamic in nature, not least because of the 2016 major change in accrual rate.

The purpose of the present paper is to develop a model in continuous time which leads directly to actuarially fair rates by equating to zero the marginal benefit of extending the deferral period. It is applied to the UK situation. We also determine the cost to the Exchequer of deferral of stated period, expressed as a percentage of the cost under no deferral. Following a summary of the literature in section II we develop in section III a model for deferrers without spouse or civil partner. Section IV shows specimen values of marginal accrual rates when the pension uprating rate is identical to the deferrer discount rate. Section V gives results when they are not equal. Section VI shows how the derived rates will have to be different for those who are already deferring at the point at which actuarial fairness is introduced. The rates would be different because a deferrer's position cannot be allowed to worsen after the introduction of actuarial fairness. Section VII addresses the case of RP deferrers with partner and inheritance options. In section VIII we discuss the issue of adverse selection, where the partially analogous annuity situation, has, in various studies, shown that annuitants tend to have longer life expectancy than the general population. In section IX we examine the trade-off between actuarial fairness for deferrers and cost neutrality for the Exchequer. Section X highlights some cost penalties and deferrer benefits as a result of adverse selection and the application of different discount rates to the Exchequer and to the deferrer. Section XI summarises and concludes and offers some policy recommendations.

II. Review of Literature

Useful background to the UK state pension system can be found in Bozio, Crawford, and Tetlow (2010) and Thurley (2016). With changing demography including increased life expectancy, the system needs to be kept under regular review. The sustainability has been examined by Blake and Mayhew (2006) while Moizer, Farrar, and Hyde (2017) specifically look at the long term effect on the National Insurance Fund. They use a Systems Dynamics model over a 40-year horizon.

Several authors have looked at the deferral problem in the context of the UK system. Rules for deferral are described in Thurley (2010, 2017). Coleman et al. (2008) conducted surveys to examine the public perception of the deferral scheme. They conclude that better off individuals are more likely to defer. Farrar, Moizer, and Hyde (2012) model the RP situation in continuous time to inform the extra pension/lump sum decision for various deferral periods, tax rates, and residual life. A conclusion they reach is that the scheme does demonstrate 'clear financial benefits…, particularly in relation to the decision to receive an enhanced pension stream…'. That in itself implies that the RP scheme is, in a qualitative sense, actuarially unfair to non-deferrers. Dagpunar (2015) explicitly includes survivor risk by using the distribution of residual life at SPA to derive a simple rule for stopping deferral based upon life expectancy (i.e. mean residual life) at any age and the number of years deferred up until that age. Stubbs and Adetunji (2016) construct a discrete time model to determine the payback period (i.e. residual life at SPA) for deferral to be worthwhile under the extra pension option. Kanabar and Simmons (2016) construct a discrete time model for the extra pension/lump sum decision. A critique of their analysis together with an estimate of the implicit rate of return in the 5.8% case is given by Dagpunar (2018). Dagpunar (2017) extends previous work by deriving a stopping rule for a RP deferrer with marriage or civil partner, again with explicit consideration of the survival risk.

Mirer (1998) looks at the US case where there are accruals for deferral and decrements for early take-up. Coile et al. (2002) develop two models for the US case, one maximizing expected net present value of pension payments, and the other maximizing expected utility. They use simulation in a discrete time setting for various scenarios of gender, discount rate, mortality rates**.** Duggan and Soares (2002), again working in discrete time with the US Social Security system, calculate actuarial adjustment factors for each year's deferral and then compute the difference in benefits compared with the use of the statutory adjustment factors. They mention how authors have differing views on the setting of an appropriate discount factor. A parallel piece of work in Canada is detailed by the Office of the Superintendent of Financial Institutions Canada (2017) which examines the appropriateness of the statutory uniform accrual rate of 8.4 % p.a. for delays from age 65-70, and decrements of 7.2 % for early retirement from 60-65. A conclusion of the report was that these legislated amounts were appropriate. Queisser and Whitehouse (2006) develop a discrete time formula for adjustment factors to be used for actuarial neutrality (a term they, along with many authors, use for actuarial fairness as understood here) and they derive some results based upon OECD average mortality rates. Medijainen (2011) uses these adjustment factors to derive rates for the Estonian system. He deduced that the 10.8% accrual rate for late deferrals, with no upper limit on deferral period, is too generous to the deferrer.

Rasdal (2013) is concerned with reasons for deferral in Norway and the probabilities thereof as a function of an individual's or couple's attributes. There is the question as to deferrer's perception of his/her life expectancy. She highlights the difference between a couple's subjective and objective life expectancy given their attributes, the subjectivity of the discount factor, and the selection/moral hazard issue implicit in the implicit annuity take-up. Shoven and Slavov (2014) return to the US case. There, full retirement age is 66 for births in 1943-54. They show delay is actuarially advantageous for many. The method used is simulation. Pension can be claimed as early as 62, receiving 75% of the primary insurance amount (PIA). For delay beyond 66, there is a uniform accrual rate of 8% p.a. Spousal benefits can be claimed as early as 62 but only after the primary earner has claimed. Calculations are performed for different ethnic groups and health categories. Healthy individuals were assigned 75% of the population mortality rates, the less healthy 200%. The complicated rules for couples justify the use of simulation rather than a purely analytical model. They concluded that primary earners in two earner couples should always delay to 70. Heiland and Yin (2014) look at early and late take-up of US Social Security benefits, computing accrual rates that are approximately actuarially fair over a range of deferral periods, and extend this to the case of married or civil-partnered deferrers. This does not have the additional complexity of the inherited lump sum option that is part of UK RP system. Meneu et al. (2016) look at intergenerational unfairness (contributions vis-à-vis benefits). Oksanen (2005) concentrates on adjusting parameters of the pension system to achieve both actuarial fairness (contributions versus benefits) and actuarial neutrality with respect to deferral and pension payments. He gives a numerical illustration to defined benefit schemes in the UK with some emphasis on inter-generational fairness. Rose (2015) concludes that delaying take-up in the US Social Security system can give substantial gains to married couples due to an inheritance provision. He also deals with the penalty for early retirement. As regards appropriate discount rate, he suggests that since social security is an inflation-linked obligation, the appropriate discount rate is that available on 20-year Treasury Inflation Protected

Securities (TIPS). As others have also observed he concludes that the present regime is very favourable towards deferral, particularly for the primary insured in a married couple. He calculates rate of return and concludes this is better than on Treasury index-linked instruments.

Previous work addressing the UK situation has tended to refer to actuarial fairness in passing. Any modelling of it has tended to be empirical rather than mathematical. The main contribution of this paper is to present a more mathematical treatment for both single and partnered (RP only) deferrers; this is achieved by relaxing the current convention of a uniform accrual rate across all deferral periods. The model is used to derive accrual rates for a range of UK scenarios. We also cost the effect of various degrees of adverse selection and of differing discount rates between deferrer and Exchequer. The rules for deferral do allow for two non-contiguous periods of deferral. We do not consider this, nor do we consider actuarial fairness with respect to after tax income. Both these are more relevant to a deferrer wishing to optimize decisions given highly individualised personal circumstance than to the setting of an actuarially fair scheme for a reasonably homogenous group. Models for such personalised decision making can be developed using stochastic dynamic programming.

III. A model for unattached (single) deferrer, both RP and SP

We propose a modification whereby the uniform accrual rate β is replaced by a time dependent one $\beta(x)$ where x is the time deferred to date. Let $\mathrm{B}(x)$ denote the cumulative accrual multiple for a deferral of x years. Then $\mathrm{B}(x) = \int_0^x \beta(u)du$. Let a denote state pension age, $\alpha(t)$ the rate at which pensions are uprated and $\mu_1(t)$ a nominal discount rate for a deferrer, both at age $a+t$. Then we define a discount rate net of pension uprating (DRNPU) at age $a+t$ to be $\lambda_1(t) = \mu_1(t) - \alpha(t)$. In the case of RP we assume that all components are uprated at the same rate as the basic pension, and in the case of both RP and SP that the extra pension is uprated at the same rate as the un-deferred pension. As the duration of a pension payments can be quite long it would be dangerous to speculate on future uprating policies and so this assumption of equality (which appears to be approximately the case at the moment) across all pension components is felt to be a reasonable one. We might model the evolution of $\lambda_1(t)$ as an Ornstein-Uhlenbeck process as in the Vasicek (1977) model for interest rates. A drawback of that model is sometimes said to be that it can lead to negative interest rates, whereas here we do specifically require a model that does not sign-restrict $\lambda_1(t)$. Accordingly,

$$d\lambda_1 = c(d - \lambda_1)dt + \sigma dW(t) \tag{1}$$

where $\{W(t): t \geq 0\}$ is a Wiener process and σ is the volatility of the process. d is the long term mean of λ_1 and $c(>0)$ is the speed of reversion to that mean. Let $S(t)$ denote the probability of surviving to at least age t; $m(t)$ and $r(t)$ to denote respectively the mean residual life (life expectancy) and age specific mortality rate at age t. Now suppose that an individual plans to stop deferral at age $a+x+dx$ having already deferred to age $a+x$. The present value (PV), referred to age $a+x$, of pension sacrificed in that time increment dx is $[1+\mathrm{B}(x)]dx$, the monetary value expressed in units of the un-deferred pension rate that would have been payable at age $a+x$.

With probability $[1-r(a+x)dx]$, the increase in conditional expected NPV payments over the residual life is $d\mathrm{B}(x)\mathrm{E}\left[\int_x^\infty \frac{S(a+v)\exp\{-\int_x^v \lambda_1(u|\lambda_1(x))du\}}{S(a+x)}dv\right]$, and with probability $r(a+x)dx$ it is zero. Now let $V(x)dx$ denote the conditional increase in expected NPV (for the deferrer) for delaying termination of deferral by this further increment dx. Then bringing the above together

$$V(x)dx = -[1+\mathrm{B}(x)]dx + d\mathrm{B}(x)\int_x^\infty \frac{S(a+v)\mathrm{E}[\exp\{-\int_x^v \lambda_1(u|\lambda_1(x))du\}]}{S(a+x)}dv \tag{2}$$

We note that the integral in the last term of (2) can be interpreted as the premium payable on a 100% money's worth[6] (from a deferrer's point of view) full life annuity purchased at age $a+x$, where the monetary unit is the rate of payment of the un-deferred pension payable at that age, and where the payments are escalated to track pension uprating.

Given our definition of actuarial fairness for the deferrer, this is achieved by setting $V(x)=0$ [7] that is

$$-[1+\mathrm{B}(x)]dx + d\mathrm{B}(x)\int_x^\infty \frac{S(a+v)\mathrm{E}[\exp\{-\int_x^v \lambda_1(u|\lambda_1(x))du\}]}{S(a+x)}dv = 0 \tag{3}$$

The actuarially fair accrual rate is

$$\beta(x) = \frac{d\mathrm{B}}{dx} = \frac{1+\mathrm{B}(x)}{\int_x^\infty \frac{S(a+v)\mathrm{E}[\exp\{-\int_x^v \lambda_1(u|\lambda_1(x))du\}]}{S(a+x)}dv} \tag{4}$$

In principle, one would control $\beta(x)$ dynamically, as $\lambda_1(x)$ evolves. In practice, appropriately timed discrete updates are more expedient. Mamon (2004), for example, gives a closed form expression for the expectation term as

$$\mathrm{E}[\exp\{-\int_x^v \lambda_1(u|\lambda_1(x))du\}] = \exp\{-\lambda_1(x)A(x,v) + D(x,v)\} \tag{5}$$

where

$$A(x,v) = \frac{1-e^{-c(v-x)}}{c} \tag{6}$$

and

$$D(x,v) = \left(d - \frac{\sigma^2}{2c^2}\right)\{A(x,v)-(v-x)\} - \frac{\sigma^2 A(x,v)^2}{4c} \tag{7}$$

The model could be calibrated using historical pension uprating and discount rates. Alternatively, the annuity interpretation might be helpful in avoiding the need to explicitly model the future evolution

[6] An insurance company would be unlikely to offer an annuity that offers the annuitant 100% money's worth. It would lose money because there would be no loading for administrative costs and because the nominal discount rate for the insurance company is likely to be smaller than a deferrer's as it does not bear the additional individual survival risk.
[7] This ensures that the act of extending deferral by an amount dx gives zero change to the deferrer's expected NPV, meaning that we neither add to nor subtract from the expected benefits or dis-benefits that the deferrer has accrued in $(a, a+x)$.

of real discount rates; however that would require suitable scaling to take account of the less than 100% money's worth feature, as mentioned previously.

Such matters are considered to be beyond the scope of the present paper and so here we take a simplified model where at age $a+x$ it is assumed that $\lambda_1(u|\lambda_1(x))=\lambda_1(x)$ for all $u \geq x$. It follows that

$$V(x)=-[1+\mathrm{B}(x)]dx+d\mathrm{B}(x)\int_x^{\infty}\tfrac{S(a+v)\exp\{-\lambda_1(x)(v-x)\}}{S(a+x)}dv \quad ^{8} \tag{8}$$

Setting $V(x)=0$

$$-[1+\mathrm{B}(x)]dx+d\mathrm{B}(x)\int_x^{\infty}\tfrac{S(a+v)\exp\{-\lambda_1(x)(v-x)\}}{S(a+x)}dv=0 \tag{9}$$

and the actuarially fair accrual rate is

$$\beta(x)=\frac{d\mathrm{B}}{dx}=\frac{1+\mathrm{B}(x)}{\int_x^{\infty}\tfrac{S(a+v)\exp\{-\lambda_1(x)(v-x)\}}{S(a+x)}dv} \tag{10}$$

Now suppose that $\mathrm{B}(x_0)$ encapsulates accruals in $(a,a+x_0)$ that are not necessarily actuarially fair. One scenario is that a person has deferred in this interval during which time he/she has been subject to the existing statutory uniform accrual rate or alternatively λ_1 has not been updated to reflect the real economic context. However, we now require actuarial fairness for any $x > x_0$. In that case the solution to (9) subject to $\mathrm{B}(x_0)=\mathrm{B}_0$ say, together with a simplifying assumption that $\lambda_1(x|\lambda_1(x_0))=\lambda_1(x_0)$ for all $x \geq x_0$, is

$$\mathrm{B}(x)=\frac{[1+\mathrm{B}_0]\int_{x_0}^{\infty}S(a+u)e^{-\lambda_1(x_0)u}du}{\int_x^{\infty}S(a+u)e^{-\lambda_1(x_0)u}du}-1 \qquad (x>x_0) \quad ^{9} \tag{11}$$

For the case where the nominal discount rate equals the pension uprating rate, $\lambda_1(x)=0$ and so for all $x > x_0$ results (9-11) become

$$-[1+\mathrm{B}(x)]+\frac{d\mathrm{B}}{dx}m(a+x)=0 \tag{12}$$

$$\beta(x)=\frac{1+\mathrm{B}(x)}{m(a+x)} \tag{13}$$

and

---

[8] It follows that a deferrer's expected NPV of benefits will improve by continuing deferral only if $\beta(x) > \frac{1+\mathrm{B}(x)}{\int_x^{\infty}\frac{S(a+v)e^{-\lambda_1(x)(v-x)}}{S(a+x)}dv}$. In particular, if $\mathrm{B}(x)=\beta x$ and $\lambda(x)=0$ the condition becomes $m(a+x)>x+\beta^{-1}$

[9] $1+\mathrm{B}(x)$ is essentially the adjustment factor used by actuaries who might analyse in discrete time.

$$\mathrm{B}(x) = \frac{[1+\mathrm{B}_0]m(a+x_0)S(a+x_0)}{m(a+x)S(a+x)} - 1 \tag{14}$$

IV. Instantaneous accrual rates when uprating rate is same as discount rate

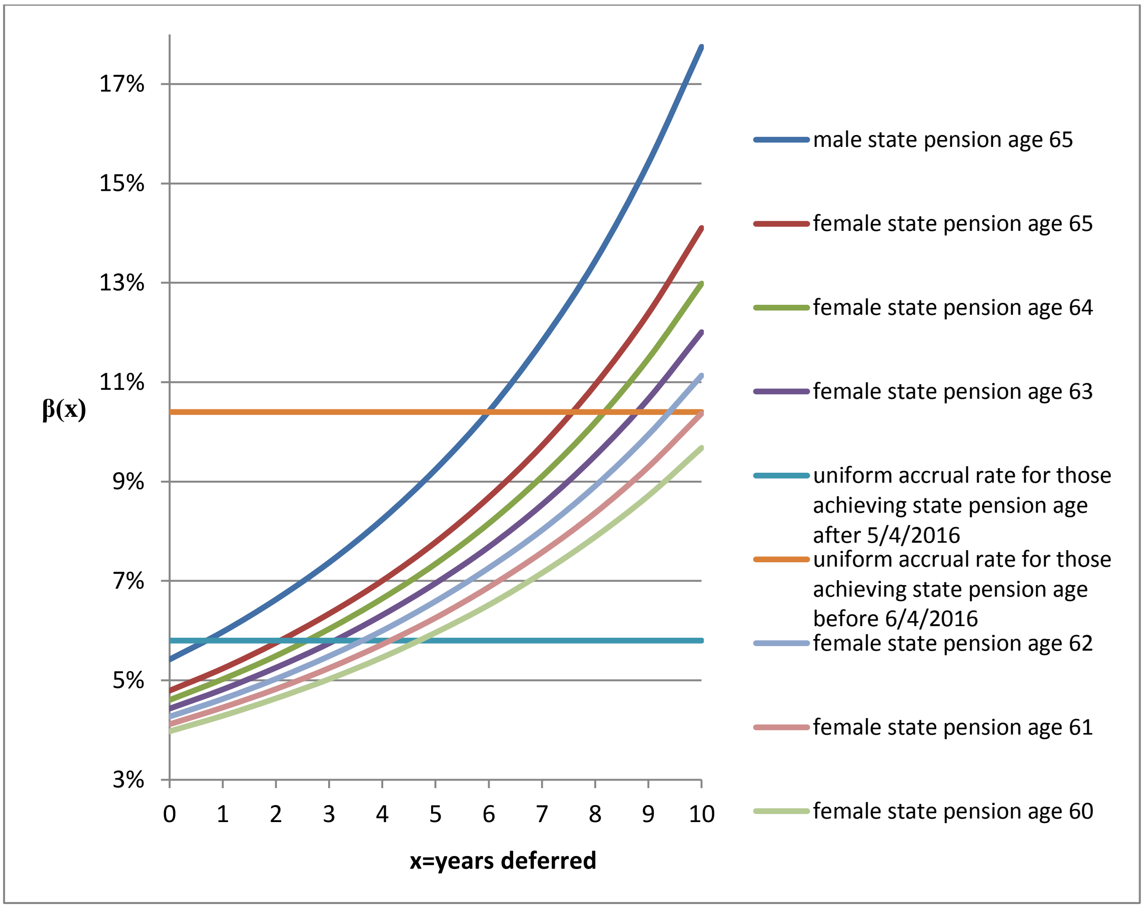
There is a question as to an appropriate nominal discount rate for the deferrer. One might choose a risk-free rate given by the term structure on UK gilts, but arguably that is too low since deferral is not risk free due to the individual survival risk. Each deferrer has an individual attitude towards risk, and so actuarial fairness across any category of deferrers (defined by gender, state pension age, partner status) is technically unachievable. One possibility is to choose a discount rate for the deferrer equal to the inflation rate based upon the consumer price index (CPI). In that case the expected NPV is a measure of purchasing power and in the context of pension income that in itself might be considered as contributing towards deferrer fairness. Further, if the pension uprating rate broadly tracks inflation then the discount rate $\lambda_1(x) = 0$ for all x. That additionally means that for any given individual the purchasing power of his/her pension is the same from year to year, which might be seen as a desirable objective.

In Figure 1 we have used Office of National Statistics (ONS) life tables[10] to calculate and display the actuarially fair accrual rates for males of state pension age 65 and females of state pension age 60 through to 65, for the case where a government introduces actuarial fairness before a person reaches SPA, that is where $x_0 = 0$ and where $\lambda_1(x) = \lambda_1 = 0$, that is where the discount rate tracks the pension uprating rate. For comparison, Figure 1 also displays the current uniform accrual rates of 10.4 % (RP) and 5.8% (SP).

[10] https://www.ons.gov.uk/file?uri=/peoplepopulationandcommunity/birthsdeathsandmarriages/lifeexpectancies/datasets/nationallifetablesunitedkingdomreferencetables/current/nltuk1315reg.xls

FIGURE 1

Instantaneous accrual rates for actuarial fairness



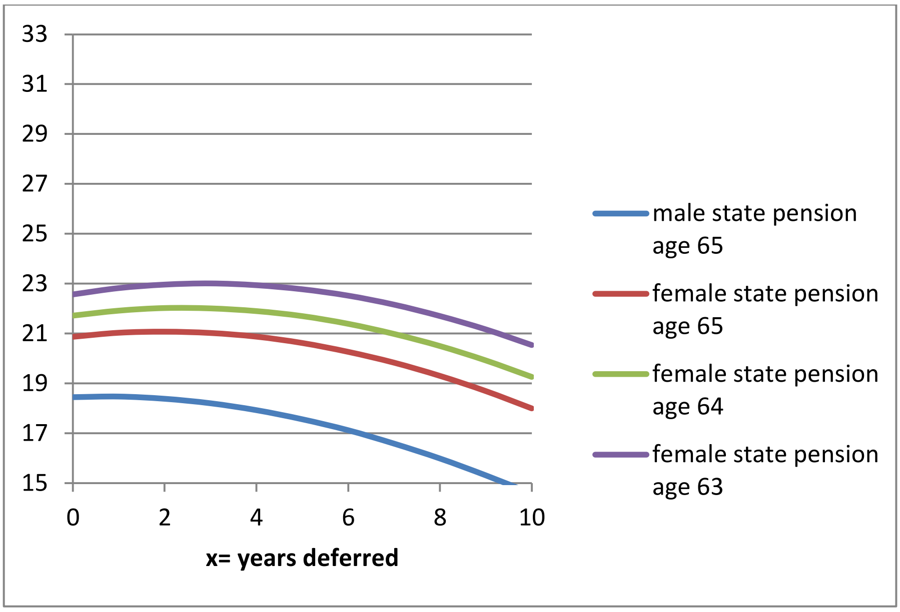
As others have observed, the most obvious feature is that the RP 10.4% is delivering considerable benefits to the deferrer at some cost to those who cannot or choose not to do so. For SP males the current uniform 5.8% is delivering approximate actuarial fairness for deferral periods less than about 1.5 years (the point at which the area under the actuarially fair curve equates to the area under the uniform 5.8% line) but with deferral periods in excess of that it is delivering somewhat inferior benefits compared with not deferring. The situation for females is slightly better due to longer life expectancy. The departure from actuarial fairness can be quantified by plotting the expected equivalent pension years $EPY(x)$ against deferral period where

$$EPY(x) = \frac{S(a+x)}{S(a)}[1+\beta x]m(a+x) \tag{15}$$

Figures 2 and 3 show this for $\beta = 0.058$ and $\beta = 0.104$ respectively. The advantage of deferral is given by $EPY(x) - EPY(0)$. Note that for women achieving SPA after 5 April 2016 (SP), their SPA must be at least 63 years. Figure 2 shows that for SP females any deferral less than about 4 years achieves rough actuarial fairness. For males, the same is true for deferrals under 1.5 years. Longer deferrals are disadvantageous to deferrers and therefore unfair

FIGURE 2

Expected equivalent pension years with uniform accrual rate of 5.8% p.a



.

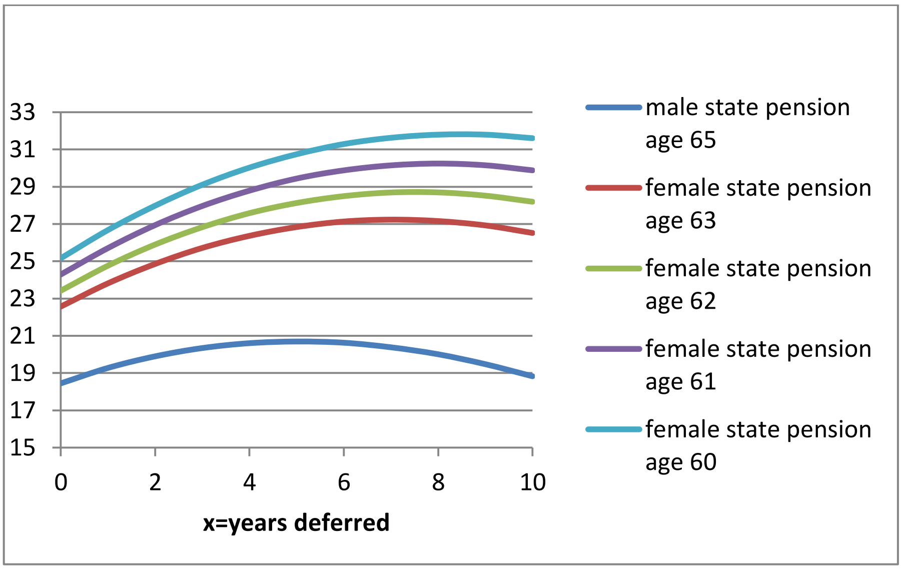
The situation is entirely different for RP persons as shown in Figure 3. For males, any deferral less than 10 years is advantageous to the deferrer and at cost (discussed further in section IX) to the Exchequer, with an optimum period of 5 years giving a modest expected increase equivalent to just over 2 years of un-deferred pension. For RP females, deferral offers even greater rewards due to longer life expectancy and lower SPA, in stark contrast to the effects of the rapid increase in SPA that SP females are experiencing. For example, even for the last cohort of RP females (those having the least to benefit from deferral of all RP female deferrers on account of their greater SPA of 63 years, compared with the previous 60 year SPA) we find that $EPY(7) - EPY(0) = 4.6$ years. It is clear that the current 10.4 % is overly generous to deferrers and actuarially unfair to non-deferrers and at significant cost (per deferrer) to the Exchequer. What is not clear is the total cost per year to the Exchequer of continuing to advantage RP deferrers. Unfortunately, there is no reliable data on the numbers and distribution by deferral period of RP deferrers. Nevertheless, the analysis does show that the current 10.4% accrual rate fails the most basic tests of not imposing an extra burden on the Exchequer and of ensuring that the pensioner is (on average) in neither a better nor worse position than not deferring. The redistributive effects might be considered regressive, arguably benefiting those who find themselves in a financial position to defer, at the expense of those who do not.[11] As Duggan and Soares (2002) say 'benefit

[11] The same might be said of tax relief and employer National Insurance concessions on pension contributions for those subject to higher and additional rate income tax. HM Treasury states 'In 2015/16 income tax and employer National Insurance Contributions relief cost around £50 billion, with around two thirds going to higher rate tax payers' – letter dated July 2017 from Stephen Barclay MP, Economic

adjustments that deviate systematically from actuarial equivalence create some (perhaps unintended) redistribution of benefits'.

FIGURE 3

Expected equivalent pension years with uniform accrual rate of 10.4% p.a.



The foregoing analysis assumes no adverse selection where one may conjecture that people who choose to defer have a somewhat higher life expectancy. We deal with this in section VIII. It also ignores the three-month pension paid to an estate after death during deferral; the impact of the latter is thought to be small compared with that due to uncertainty about future pension uprating and discount rates.

V. Numerical results for different discount rates net of pension uprating (DRNPU)

The previous sections assumed that the discount rate net of pension uprating (DRNPU) is zero. Here we relax that assumption and show how the actuarially fair rates to the deferrer will respond to different DRNPU. We show in Table 1 the situation for a single male of SPA 65, using results (10) and (11) where $\lambda_1(x) = \lambda_1$ for all x and where $x_0 = 0$.

Secretary to the Treasury. Briefing paper number CBP-07505, 12 October 2018, 'Reform of pension tax relief', House of Commons Library, author D. Thurley gives HMRC estimate for tax relief on pensions for 2016/17 as £38.6 billion and £16.2 billion for NI relief. An estimated 65% of tax relief for 2018 is attributable to higher and additional rate tax payers.

TABLE 1

Single male with SPA of 65 years: Specimen percentage values of the actuarially fair instantaneous accrual rate $\beta(x)$ for various values of λ_1, the amount by which the deferrer discount rate exceeds the pension uprating rate.

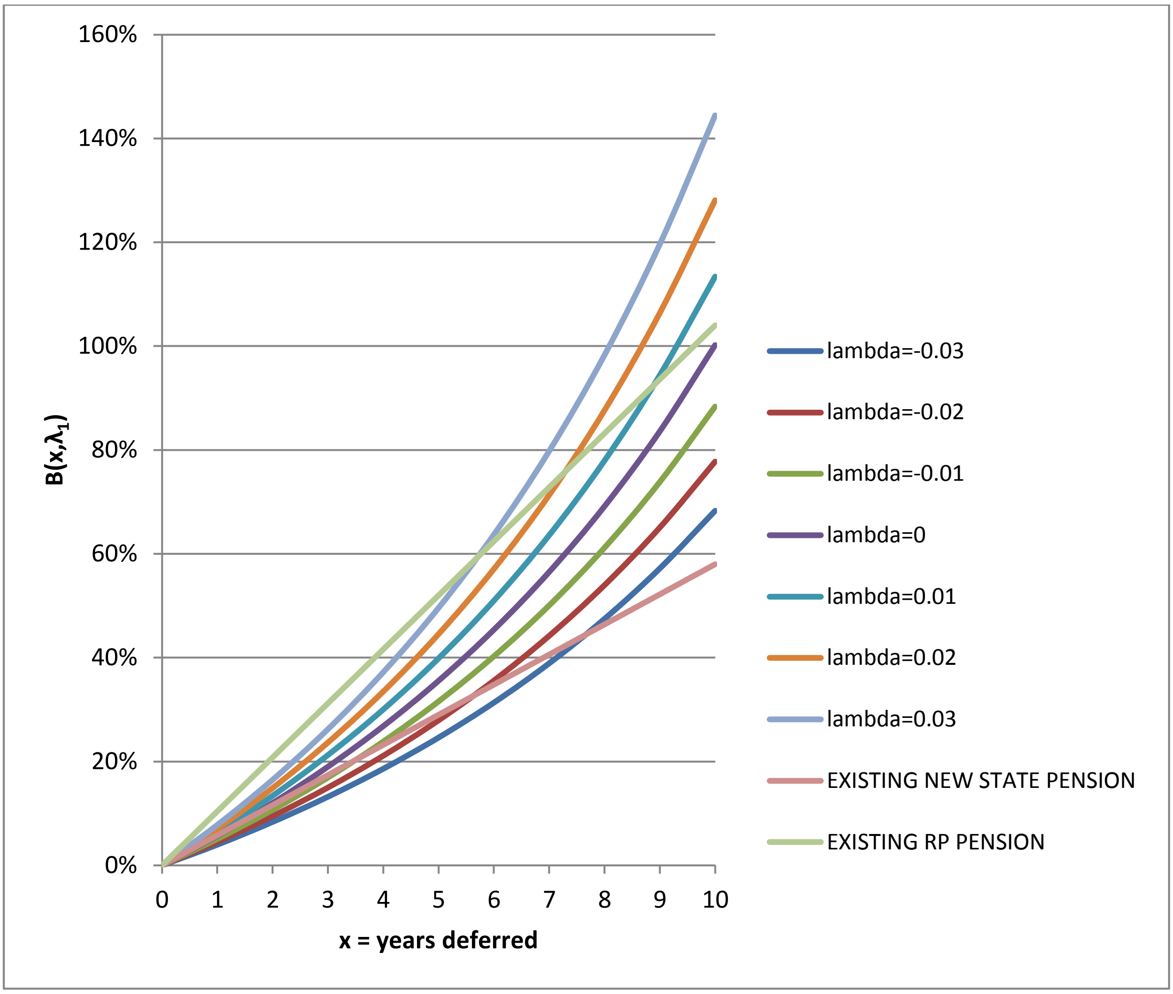
$x \quad \backslash \lambda_1$	-0.03	-0.02	-0.01	0	0.01	0.02	0.03
0	3.8%	4.3%	4.8%	5.4%	6.0%	6.7%	7.4%
1	4.2%	4.7%	5.3%	6.0%	6.7%	7.4%	8.2%
2	4.6%	5.2%	5.9%	6.6%	7.4%	8.3%	9.2%
3	5.1%	5.8%	6.6%	7.4%	8.3%	9.3%	10.3%
4	5.7%	6.5%	7.3%	8.3%	9.3%	10.4%	11.6%
5	6.4%	7.2%	8.2%	9.3%	10.4%	11.8%	13.2%
6	7.1%	8.1%	9.2%	10.4%	11.8%	13.3%	15.0%
7	8.0%	9.2%	10.4%	11.8%	13.4%	15.2%	17.2%
8	9.1%	10.4%	11.8%	13.5%	15.4%	17.5%	19.8%
9	10.4%	11.9%	13.5%	15.5%	17.7%	20.1%	23.0%
10	11.9%	13.6%	15.6%	17.8%	20.4%	23.4%	26.8%
11	13.7%	15.7%	18.0%	20.7%	23.8%	27.4%	31.5%

Positive values of λ_1 lead to the PV of pension payments decreasing as time passes while negative values mean they increase. Consequently $\beta(x)$ is increasing in λ_1 for fixed x. At the time of writing (February 2018) the triple lock value is the CPI rate of 3.0% p.a. Suppose the discount rate is chosen to be 2% p.a. Then $\lambda_1 = \mu_1 - \alpha = 0.02 - 0.03 = -0.01$. If this were to be maintained throughout deferral then reference to the table shows that the rate during the first year should be approximately $0.5(4.8+5.3) = 5.05\%$, during the second year $0.5(5.3+5.9) = 5.6\%$ and so forth.

Figure 4 shows the cumulative accrual to date $\mathrm{B}(x)$ and it is apparent that the choice of λ can have an appreciable effect. It is also evident that since the majority of (x,λ) scenarios lie below the existing linear RP case but above the SP one, that in general RP is delivering to a 65 year old male more than a fair deal, and if anything SP is delivering less.

FIGURE 4

Single male with SPA of 65 - Cumulative accrual for actuarial fairness where λ_1 represents amount by which discount rate exceeds pension uprating rate, and a comparison with existing linear accruals

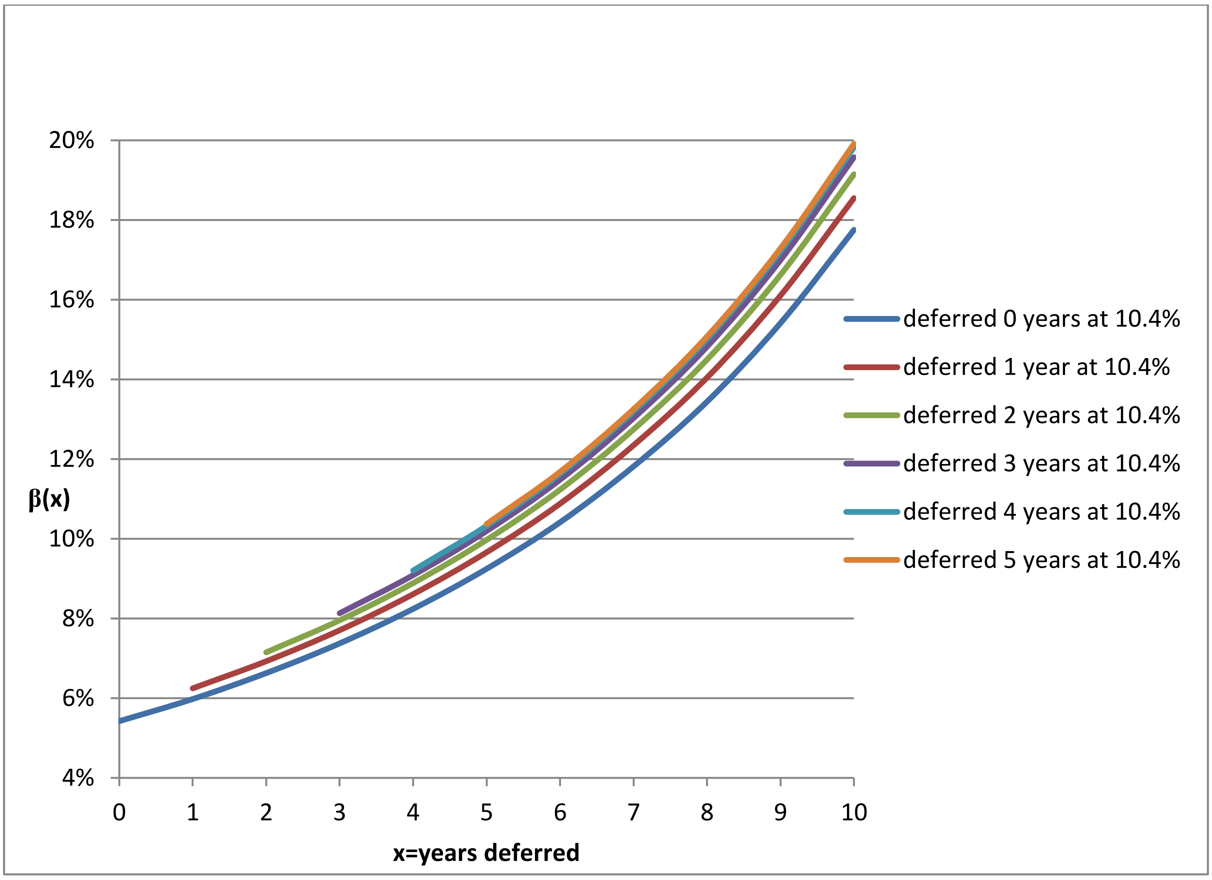


VI. For those currently under deferral

In Figure 5 we show $\beta(x)$ for a RP male with SPA of 65 years when $\lambda_1 = 0$ for $x_0 = 0, 1, 2, 3, 4, 5$. For example, the curve for $x_0 = 2$ is used if a man reached state pension age before 6 April 2016 and had already deferred for two years when a government introduces actuarial fairness. During these two years the accrual rate would have been 10.4% p.a. and then drops to 7.15%, rising to 7.95% after 3 years, 9.08% after 4 years, 9.97% after 5 years, and so on. Had the government introduced actuarial fairness before he reached state pension age then he would receive the equivalent of $m(65) = 18.45$ years of un-deferred pension no matter how long he deferred. As it happens, after deferring 1 year under an accrual rate of 10.4% he would receive the equivalent of 19.28 years, and after 2 years, 19.90 years (see Figure 3). At that point the new lower rates apply and he receives 19.90 years no matter how much longer he defers. He does enjoy a better outcome than someone who is subject to actuarially fair rates throughout deferral, but that is unavoidable given the principle of not worsening the position of someone who already has a history of deferral on introduction of an actuarially fair scheme.

FIGURE 5

Actuarially fair instantaneous rates for male with SPA of 65, given previous accrual rate was 10.4% p.a.



VII. For those having a partner

We consider a RP deferrer (A) with SPA of a and his/her partner (B) having age b at the time at which A reaches age a. It is only partners of RP pensioners who may be able to inherit an extra periodic pension or lump sum. We assume, as in section IV that the DRNPU is zero, that is $\lambda_1(x) = \lambda_1 = 0$ for all x. To illustrate the approach we will assume that b is at least equal to B's state pension age. This means that if A dies first then B will immediately be eligible for inheriting benefits as described in section I, unless A had previously taken the lump sum. It is acknowledged that this will not cover all inheritance cases, but it can provide an upper bound on the joint NPV for all partnerships and therefore a lower bound on actuarially fair rates. To cover all joint scenarios would necessitate an empirical rather than analytical approach which follows; the latter provides some useful insight into partnership cases. For the same reason we also assume that the interest rate used to calculate an inherited lump sum equals the pension uprating rate. Currently these two rates are not very different. With this assumption and zero DRNPU, that means that a lump sum with added interest equates to the pension rate at SPA multiplied by the deferral period.

Let $m_A(x)$ and $m_B(x)$ denote their respective mean residual lives at ages x and let $M(x_A, x_B)$ denote the expected time to the second death of A and B given that they have respective ages x_A and x_B. Let $S_A(x)$ and $S_B(x)$ be the respective survivor functions for A and B and $r_A(x)$ and $r_B(x)$ their respective mortality rates. As before, we express all monetary values in units of the un-deferred periodic pension per annum that A would receive at age a. If A has just stopped deferring after x years where both deferrer and survivor are still alive, we assume that a rational A will take the extra pension in preference to the lump sum[12]. Regardless of who is the last survivor, A will receive an expected NPV of $m_A(a+x)$. The expected residual life of the partnership is $M(a+x,b+x)$ and therefore the expected value of the extra pension that it will earn is $\mathrm{B}(x)M(a+x,b+x)$. Accordingly, the expected NPV of pension payments over the residual lifetime of the partnership as

$$\mathrm{B}(x)M(a+x,b+x)+m_A(a+x) \quad (16)$$

[13]

It is possible that either A or B dies before the end of the planned deferral period x is reached. If A predeceases B at age $a+u$ (where $u<x$), then B can choose to inherit either a lump sum of u or a periodic extra pension of $\mathrm{B}(u)$ per year. If B predeceases A at age $b+u$ (where $u<x$), then A chooses either to take a lump sum of u plus the periodic un-deferred pension, or take both extra pension of $\mathrm{B}(u)$ p.a. and periodic un-deferred pension, or to continue to defer. Now suppose that A plans to stop deferral at age $a+x+dx$ rather than at age $a+x$. The pension sacrificed in that time increment dx is $[1+\mathrm{B}(x)]dx$. With probability $[1-r_A(a+x)dx][1-r_B(b+x)dx]$ the increase in conditional expected NPV over the residual life of the partnership is $M(a+x,b+x)d\mathrm{B}(x)$. With probability $r_A(a+x)dx[1-r_B(b+x)dx]$ the conditional expected additional PV for B (compared with termination at $a+x$) is $\max[x-\mathrm{B}(x)m_B(b+x),0]$[14]. With probability $r_B(b+x)dx[1-r_A(a+x)dx]$ the conditional expected increase in PV for A is $\max[x-\mathrm{B}(x)m_A(a+x),0]$. Note that we have eliminated the possibility of A continuing to defer on B's death[15]. This is a consequence of the fact that

---

[12] Under Actuarial Fairness, surviving the deferral period has implicit value favouring extra pension over lump sum, unless either A or B has suffered a sufficiently large downgrade from the population life expectancy.

[13] $M(a+x,b+x)$ is computed using

$$\frac{dM(a+x,b+x)}{dx}=r_A(a+x)\{M(a+x,b+x)-m_B(b+x)\}+r_B(b+x)\{M(a+x,b+x)-m_A(a+x)\}-1$$

It is assumed that the respective times until death of A and B are independently distributed.

[14] To see this, note that if A terminates deferral at $a+x$ then A will take the extra pension which, if A dies in the next increment dx, leads to a future expected benefit to B of $\mathrm{B}(x)m_B(b+x)$. If A continues deferral beyond $a+x$ and dies within the increment dx then B is free to choose between taking either the lump sum x or $\mathrm{B}(x)m_B(b+x)$. In the first case the reward for delaying termination is $x-\mathrm{B}(x)m_B(b+x)$ and in the second it is zero.

[15] Under Actuarial Fairness, while both A and B are alive, A is indifferent between terminating or continuing with deferral. However, on B's prior death, the expected residual life of the partnership is reduced from $M(a+x,b+x)$ to $m_A(a+x)$, meaning that continuation of deferral would be sub-optimal behaviour for A.

$m_A(a+x) < M(a+x,b+x)$. Now let $V(x)dx$ denote the conditional increase in expected NPV to the partnership for delaying termination of deferral by this increment dx. Then bringing the above together

$$V(x) = -1 - \mathrm{B}(x) + M(a+x,b+x)d\mathrm{B}(x) + r_A(a+x)\max[x - \mathrm{B}(x)m_B(b+x),0] \\ + r_B(b+x)\max[x - \mathrm{B}(x)m_A(a+x),0] \tag{17}$$

Now suppose that A has deferred for a period x_0 at the point at which actuarially fair rates are to be brought in. Then this is achieved by setting $V(x)=0$ [16] [17] for all $x > x_0$ subject to $\mathrm{B}(x_0) = \beta x_0$ where β is the uniform accrual rate that has previously been applied. Then for all $x > x_0$

$$\beta(x) = \frac{d\mathrm{B}}{dx} = \frac{1 + \mathrm{B}(x) - r_A(a+x)\max[x - \mathrm{B}(x)m_B(b+x),0] - r_B(b+x)\max[x - \mathrm{B}(x)m_A(a+x),0]}{M(a+x,b+x)} \tag{18}$$

It is worth noting that if the lump sum option were not available then the relevant equation would simply be

$$\beta(x) = \frac{dB}{dx} = \frac{1+\mathrm{B}(x)}{M(a+x,b+x)} \tag{19}$$

and the resulting marginal rates would now be higher to compensate for removal of that option, which would in any case only be invoked on A's prior death if $m_B(x) < x/\mathrm{B}(x)$ and on B's if $m_A(x) < x/\mathrm{B}(x)$.

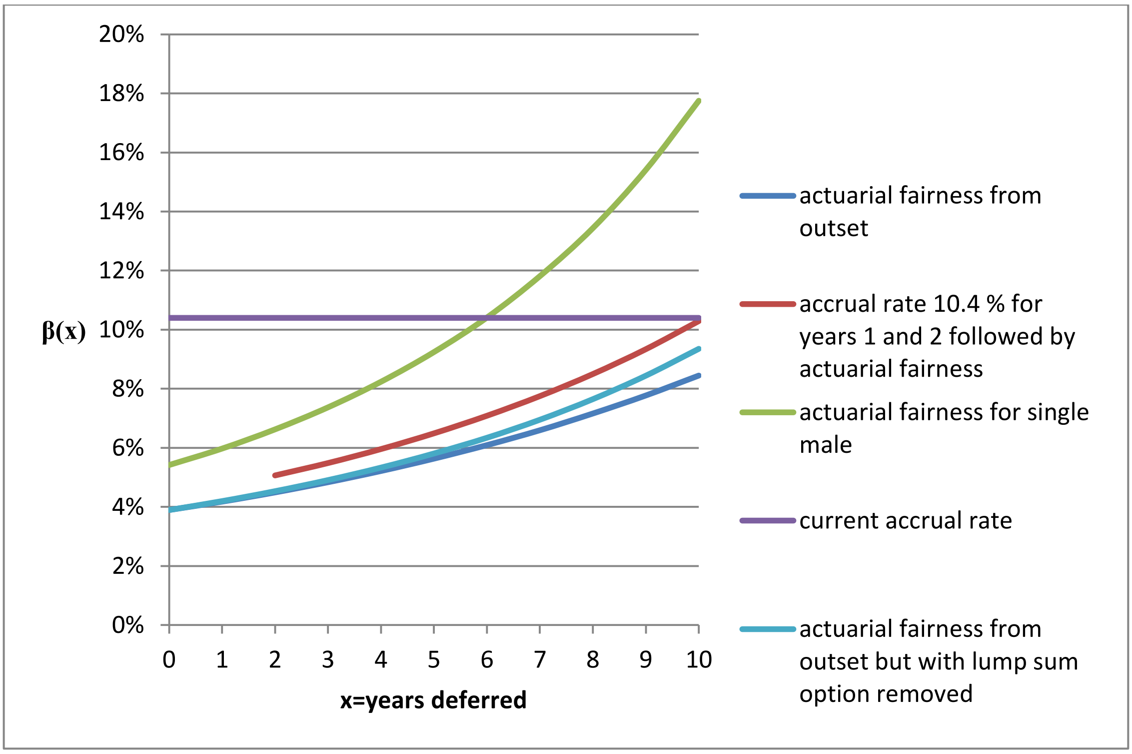
By way of example we take the situation where A is a man who achieves SPA of 65 before 6 April 2016 and whose wife is 63 at that time and who was already taking her own state pension at that point. Then we solve (18) using Euler's method with step size of 1 year, subject to $\mathrm{B}(2) = 0.104\text{x}2 = 0.208$ for the case of someone who started deferring two years before the introduction of actuarially fair rates of accrual, and for comparison only, a man who would have experienced actuarially fair rates (with and without partner) from the outset, had that been possible. The resulting instantaneous accrual rates are shown in Figure 6. As expected, the marginal rates are well below those for a single SPA male of 65, which increases still further the extent to which the current 10.4% departs from actuarial fairness. Removal of the lump sum option is seen to have no significant impact on rates for deferral periods of less than 6 years.

[16] An alternative approach is to equate NPVs of different deferral periods. While this works well for the case of a single deferrer, it is more onerous for the inherited case considered here, and the marginality argument is preferred leading more easily to the derived condition.

[17] We can now prove the previous assertion that on B's prior death a rational A would not continue deferral. From (8), the conditional rate of increase of expected NPV for A following B's death is $V(x) = -1 - \mathrm{B}(x) + \beta(x)m_A(a+x) < -1 - \mathrm{B}(x) + \beta(x)M(a+x,b+x) < 0$. The second inequality arises because under actuarial fairness, the terms containing $r_A(a+x)$ and $r_B(b+x)$ in an augmented version of (17) that includes the possibility of continued deferral by A, are still both non-negative. Since $V(x) < 0$, A would not continue to defer.

FIGURE 6

Actuarially fair rates for male RP deferrer (a=65) with female wife (b=63)



VIII Adverse selection

Adverse selection can occur where an insured person has information pertaining to the riskiness of a contract between insurer and insured, which the insurer does not possess. Thus, a prospective deferrer who believes that their expected residual life (life expectancy) is greater than that of the general population is in a position to take advantage of this asymmetry. This has been termed *active* selection when the perception of increased longevity leads to a conscious decision to defer. There is also *passive* selection where a person possessing certain socio-economic attributes (for example wealth, education) which are correlated with deferral, may also benefit from any correlation between those attributes and life expectancy. Moral hazard is where an insured person subsequently embarks upon behaviour that is disadvantageously risky to the insurer. For example, it is possible that a deferrer drawing the extra pension that is not available to a non-deferrer may choose to use that money for lifestyle and health decisions that tend to prolong life. A further example would be for a single RP deferrer to convert an existing personal relationship into a legal one so that the extra pension is now payable till the last survivor of the partnership dies; Figure 6 demonstrated the considerable extent of moral hazard in this case.

The deferral decision does bear some resemblance to the purchase of an annuity which can therefore inform this study. There is a difference in that a conventional single annuity premium is here replaced by a sequence of notional payments representing sacrificed pension during deferral. Many annuities give a constant periodic payment until death occurs. Others escalate or index the regular payment, and it is those scenarios that more closely mirror deferral, given that the basic state pension and extra pension are indexed according to the triple lock and CPI respectively. Annuities are often classified as compulsory or voluntary. Prior to 2015, the majority of members of defined contribution schemes bought an annuity. This has led to these being known as compulsory annuities. A non-pension related annuity is often termed a voluntary one. The small proportion of pre-2015 pensioners who chose not to buy an annuity but to initially drawdown their pension pot, illustrates that the description compulsory/voluntary is slightly ambiguous.

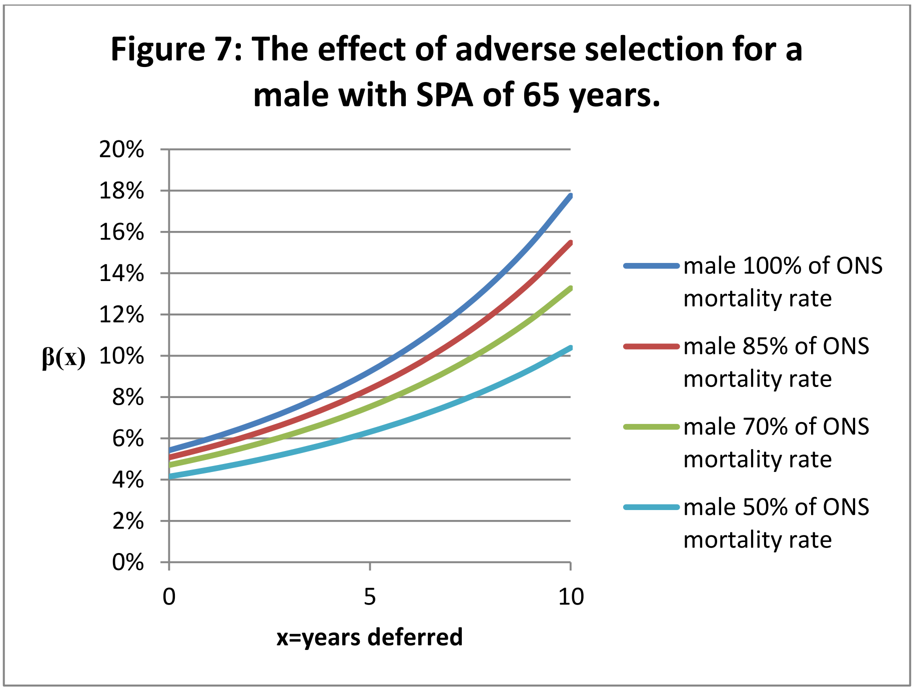
All empirical studies concur that there is evidence of adverse selection in the annuity market, both UK and internationally, and we might reasonably expect the same for those choosing to defer their state pension. Several, including those by Poterba (2001), Finklestein and Poterba (2002), and Finklestein and Poterba (2004) observe that adverse selection is greater with voluntary annuities. An international perspective is given by Mitchell and McCarthy (2002) who model how the degree of adverse selection varies by country and type of policy (compulsory/voluntary; level/escalating). Comparing the US and UK they find that voluntary annuitants have similar mortalities and those rates are much less than the population rates. Internationally, they found that mortality rates were at least 25% less for annuitants compared with the general population. Overall they did not find a significant difference between compulsory and voluntary adverse selection, but emphasised that this did not mean that there was no such a difference within individual countries, and they also cautioned as to the rather unclear nature, internationally speaking, as to what constituted a compulsory policy. Fong (2002) finds that the degree of adverse selection in Singapore is less than in several western countries. As there was minimal social security there, the explanation may be that there was relatively high demand for annuities by individuals who had built up a principal in their individual and compulsory central provident funds. Finkelstein and Poterba (2004) use a proportional hazards model to test for adverse selection with respect to three aspects: the size of the premium, the degree of 'back-loading', and the length of the guarantee period. Back-loading occurs when an annuity provides for escalating or indexed payments; the later years pay more, and such policies are therefore relatively more attractive to those with lower mortality rates and therefore greater life expectancies. Conversely, annuities that offer a guaranteed period of payment are relatively more popular with those that have a higher mortality rate. They did find strong evidence for more adverse selection the greater the degree of back-loading and evidence of higher mortality rates amongst those who selected annuities with guarantee periods. In contrast to some other studies they did not find a statistically significant difference between voluntary and compulsory annuities, the same conclusion reached by Mitchell and McCarthy (2002) when they looked at pooled data across several countries. Canon and Tonks (2016) highlight the difficulty of quantifying adverse selection as insurance companies' mortality data is to certain extent proprietary information. The alternative is to attempt an inference through examination of an annuity's money's worth. They demonstrated that back-loaded annuities had a lower money's worth than front-loaded ones for the UK compulsory market during 1994-2012. These studies give a general picture that adverse selection is always present, is quite likely to be greater with indexed/escalating annuities, and may be larger in the voluntary market. The quantitative assessments of adverse selection differ between studies, reflecting differences in scope, period covered, and country.

Now assume that Government wishes to calculate actuarially fair rates for deferrers. Unfortunately, the DWP does not collect age at death for deferrers[18] and so there is no direct way of obtaining mortality rates for them. In the absence of such data one may be guided by the voluntary and escalating annuity experience. The literature shows there is reasonably good qualitative but not quantitative consistency between the various studies. We might start with Mitchell and McCarthy's (2002) findings of at least a 25% reduction in mortality rates and calculate the effect of mortality rates being 85%, 70%, and 50% of the population rates. Table 2 shows how life expectancy changes for these three scenarios.

Table 2: Life expectancy in years at age 65 and age specific mortality rates

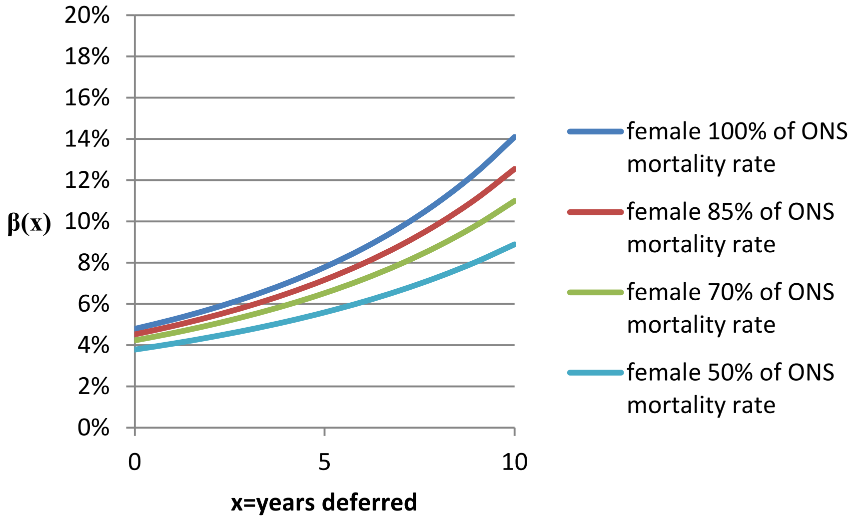
Gender \ mortality rate as % of population rate	100%	85%	70%	50%
Male	18.45	19.70	21.25	24.09
Female	20.87	22.11	23.63	26.42

By way of example we show in Figures 7 and 8 the corresponding actuarially fair marginal accrual rates for the case where $\lambda_1(x) = 0$ for all x, that is where the DRNPU is zero. We observe that adverse selection does indeed alter these rates quite significantly, particularly for larger deferral periods.



[18] Letter dated 17 December 2018 from Guy Opperman, Minister for Pensions and Financial Inclusion.

Figure 8: The effect of adverse selection for a female with SPA of 65 years.



IX Cost Neutrality vis-a-vis Actuarial fairness

Since the nominal discount rates for the Exchequer and deferrer are unlikely to be the same, it is inevitable that actuarial fairness for deferrers and cost neutrality for the Exchequer cannot be simultaneously achieved. In this section we will explore the consequences of a mismatch in discount rates. We let $S_\theta(t)$ denote the probability that a deferrer will survive to at least age t, given the age specific mortality rates for deferrers are $100\theta\%$ of those for the general population and we take the discount rate net of pension uprating rate (DRNPU) to be λ_1 for deferrers and λ_2 for the Exchequer respectively.

A government can achieve actuarial fairness while taking full account of adverse selection by setting the cumulative accrual function to be $B(x,\lambda_1,\theta)$ where

$$[1+B(x,\lambda_1,\theta)]\int_x^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_1 v}dv = \int_0^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_1 v}dv \tag{20}$$

Alternatively, it can achieve cost neutrality to the Exchequer by setting the accrual function to $B(x,\lambda_2,\theta)$ where

$$[1+B(x,\lambda_2,\theta)]\int_x^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_2 v}dv = \int_0^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_2 v}dv \tag{21}$$

In the former case the expected cost to the Exchequer of such a policy, expressed as a proportion of the expected cost of no deferral is

$$C_1(x,\theta) = \frac{[1+B(x,\lambda_1,\theta)]\int_x^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_2 v}dv}{\int_0^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_2 v}dv} = \frac{1+B(x,\lambda_1,\theta)}{1+B(x,\lambda_2,\theta)} \tag{22}$$

while the expected lifetime benefits to a deferrer expressed as a proportion of that received under no deferral is obviously

$$P_1(x,\theta) = 1 \tag{23}$$

In the latter case, the expected lifetime benefits received by a deferrer expressed as a proportion of expected benefits under no deferral is

$$P_2(x,\theta) = \frac{1 + B(x,\lambda_2,\theta)}{1 + B(x,\lambda_1,\theta)} = \frac{1}{C_1(x,\theta)} \tag{24}$$

while the expected cost to the Exchequer expressed as a proportion of that with no deferral is obviously given by

$$C_2(x,\theta) = 1 \tag{25}$$

Now consider a government that attempts to achieve actuarial fairness in the mistaken belief that there is no adverse selection. In that case the accrual function is set as $B(x,\lambda_1,1)$ where

$$[1 + B(x,\lambda_1,1)]\int_x^\infty \frac{S_1(a+v)}{S_1(a)}e^{-\lambda_1 v}dv = \int_0^\infty \frac{S_1(a+v)}{S_1(a)}e^{-\lambda_1 v}dv \tag{26}$$

and the expected lifetime benefits received, again expressed as a proportion of those achieved under no deferral is

$$P_3(x,\theta) = \frac{1 + B(x,\lambda_1,1)}{1 + B(x,\lambda_1,\theta)} \tag{27}$$

while the expected cost to the Exchequer expressed as proportion of the cost under no deferral is

$$C_3(x,\theta) = \frac{1 + B(x,\lambda_1,1)}{1 + B(x,\lambda_2,\theta)} \tag{28}$$

and so

$$P_3(x,\theta) = \frac{C_3(x,\theta)}{C_1(x,\theta)} \tag{29}$$

Conversely, now assume that a government attempts to achieve cost neutrality on the same mistaken assumption that there is no adverse selection. In that case the accrual function is set as $B(x,\lambda_2,1)$ where

$$[1 + B(x,\lambda_2,1)]\int_x^\infty \frac{S_1(a+v)}{S_1(a)}e^{-\lambda_2 v}dv = \int_0^\infty \frac{S_1(a+v)}{S_1(a)}e^{-\lambda_2 v}dv \tag{30}$$

The expected cost to the Exchequer expressed as a proportion of that under no deferral is

$$C_4(x,\theta)=\frac{[1+B(x,\lambda_2,1)]\int_x^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_2 v}dv}{\int_0^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_2 v}dv}=\frac{1+B(x,\lambda_2,1)}{1+B(x,\lambda_2,\theta)} \tag{31}$$

while the expected benefit to a deferrer expressed as proportion of the expected benefit under no deferral is

$$P_4(x,\theta)=\frac{[1+B(x,\lambda_2,1)]\int_x^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_1 v}dv}{\int_0^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_1 v}dv}=\frac{1+B(x,\lambda_2,1)}{1+B(x,\lambda_1,\theta)} \tag{32}$$

and so

$$P_4(x,\theta)=\frac{C_4(x,\theta)}{C_1(x,\theta)} \tag{33}$$

Finally, suppose a government uses (as it does currently) a uniform marginal accrual rate of β. Then the expected cost to the Exchequer expressed as a proportion of that with no deferral is

$$C_5(x,\theta)=\frac{[1+\beta x]\int_x^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_2 v}dv}{\int_0^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_2 v}dv}=\frac{1+\beta x}{1+B(x,\lambda_2,\theta)} \tag{34}$$

while the expected lifetime benefits to a deferrer expressed as a proportion of that received with no deferral is

$$P_5(x,\theta)=\frac{[1+\beta x]\int_x^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_1 v}dv}{\int_0^\infty \frac{S_\theta(a+v)}{S_\theta(a)}e^{-\lambda_1 v}dv}=\frac{1+\beta x}{1+B(x,\lambda_1,\theta)}=\frac{C_5(x,\theta)}{C_1(x,\theta)} \tag{35}$$

X The impact of adverse selection and non-symmetric discount rates.

We show in Table 3 the effect of adverse selection for a male with SPA of 65 years when the discount rate net of pension uprating rate (DRNPU) is zero for both deferrer and Exchequer, that is when $\lambda_1=\lambda_2=0$.

Table 3: Percentage changes in Exchequer costs and Deferrer benefits for specimen θ and x for a male with SPA of 65 years with DRNPU of 0% p.a. for both deferrer and Exchequer, with changes greater than 15% highlighted.

θ	1				0.7				0.5			
x	0	2	5	10	0	2	5	10	0	2	5	10
$(C_1\text{-}1)100\%$= Cost to	0	0	0	0	0	0	0	0	0	0	0	0
Exchequer while achieving actuarial fairness and adjusting for adverse selection												
$(P_2\text{-}1)100\%$=Gain to	0	0	0	0	0	0	0	0	0	0	0	0
Deferrer while Exchequer achieves cost neutrality and adjusts for adverse selection												
$(C_3\text{-}1)100\%$= Cost to Exchequer	0	0	0	0	0	1.5	4.4	11.1	0	2.8	7.8	20.3
$(P_3\text{-}1)100\%$=Gain to Deferrer	0	0	0	0	0	1.5	4.4	11.1	0	2.7	7.8	20.3
while Exchequer plans for actuarial fairness but neglects adverse selection												
$(C_4\text{-}1)100\%$= Cost to Exchequer	0	0	0	0	0	1.5	4.4	11.1	0	2.7	7.8	20.3
$(P_4\text{-}1)100\%$=Gain to Deferrer	0	0	0	0	0	1.5	4.4	11.1	0	2.7	7.8	20.3
while Exchequer plans for cost neutrality but neglects adverse selection												
$(C_5\text{-}1)100\%$= Cost to Exchequer	0	-0.3	-4.8	-21.0	0	1.2	-0.6	-12.2	0	2.4	2.7	-5.0
$(P_5\text{-}1)100\%$=Gain to Deferrer	0	-0.3	-4.8	-21.0	0	1.2	-0.6	-12.2	0	2.4	2.7	-5.0
while Exchequer uses a 5.8% accrual rate												
$(C_5\text{-}1)100\%$= Cost to Exchequer	0	7.9	12.2	2.0	0	9.5	17.1	13.3	0	10.8	21.0	22.7
$(P_5\text{-}1)100\%$=Gain to Deferrer	0	7.9	12.2	2.0	0	9.5	17.1	13.3	0	10.8	21.0	22.7
while Exchequer uses a 10.4% accrual rate												

We observe the exact equality of Exchequer costs and deferrer gains, the potentially high Exchequer penalties when adverse selection is ignored, and similarly high penalties for the current 10.4% accrual rate, even for moderate deferral periods.

A more realistic analysis might acknowledge that a deferrer is subject to the same uncertainty as the Exchequer plus an additional individual survival risk. That merits a higher discount rate, so that $\lambda_1 > \lambda_2$. We consider the following hypothetical example where pensions are indexed at the inflation rate which is assumed equal to the deferrer's nominal discount rate, representing a situation where the purchasing power of a pension remains constant over a pensioner's lifetime. Thus the deferrer's discount rate net of pension uprating (DRNUP) is $\lambda_1 = 0$. Suppose the Exchequer decides to set its DRNPU to be based upon the price of a zero coupon 20 year indexed linked gilt, which is say £149. Then $\lambda_2 = -\ln(1.49)/20 = -0.02$. We show in Table 4 the corresponding results. Since $\lambda_1 > \lambda_2$ it is intuitive, and can also be proved mathematically, that $C_1(x,\theta) > 1$ and $P_2(x,\theta) < 1$. This means that when adverse selection is acknowledged, the achievement of actuarial fairness will always incur cost to the Exchequer while obtaining cost neutrality for the Exchequer will deliver some unfairness to deferrers, no matter what the deferral period.

Table 4: Percentage changes in Exchequer costs and Deferrer benefits for specimen θ and x for a male with SPA of 65 years with DRNPU of 0% p.a. for deferrer and -2% p.a. for Exchequer, with changes greater than 15% highlighted.

θ	1				0.7				0.5			
x	0	2	5	10	0	2	5	10	0	2	5	10
(C_1-1)100%= Cost to	0	2.3	6.0	12.7	0	2.2	5.8	12.2	0	2.2	5.6	11.9
Exchequer while achieving actuarial fairness and adjusting for adverse selection												
(P_2-1)100%=Gain to	0	-2.3	-5.6	-11.2	0	-2.2	-5.5	-10.9	0	-2.1	-5.3	-10.6
Deferrer while Exchequer achieves cost neutrality and adjusts for adverse selection												
(C_3-1)100%= Cost to Exchequer	0	2.3	6.0	12.7	0	3.8	10.4	24.7	0	5.0	13.9	34.6
(P_3-1)100%=Gain to Deferrer	0	0	0	0	0	1.5	4.4	11.1	0	2.7	7.8	20.3
while Exchequer plans for actuarial fairness but neglects adverse selection												
(C_4-1)100%= Cost to Exchequer	0	0	0	0	0	1.5	4.2	10.6	0	2.6	7.5	19.5
(P_4-1)100%=Gain to Deferrer	0	-2.3	-5.6	-11.2	0	-0.8	-1.5	-1.4	0	0.4	1.8	6.8
while Exchequer plans for cost neutrality but neglects adverse selection												
(C_5-1)100%= Cost to Exchequer	0	2.0	0.9	-11.0	0	3.5	5.1	-1.5	0	4.6	8.5	6.4
(P_5-1)100%=Gain to Deferrer	0	-0.3	-4.8	-21.0	0	1.2	-0.6	-12.2	0	2.4	2.7	-5.0
while Exchequer uses a 5.8% accrual rate												
(C_5-1)100%= Cost to Exchequer	0	10.4	18.9	14.9	0	12.0	23.8	27.2	0	13.3	27.8	37.3
(P_5-1)100%=Gain to Deferrer	0	7.9	12.2	2.0	0	9.5	17.1	13.3	0	10.8	21.0	22.7
while Exchequer uses a 10.4% accrual rate												

Comparing C_1 and C_3, we note the clear consequences to the Exchequer of ignoring adverse selection while aiming for actuarial fairness. The penalty is larger in Table 4 than in Table 3 because the Exchequer is additionally disadvantaged by a lower DRNPU. Naturally, this results in some considerable advantage to deferrers; see P_3 where there is the same positive gain for a deferrer in Tables 3 and 4, rather than a zero gain had adverse selection been taken into account.

When the Exchequer aims for cost neutrality but ignores adverse selection we see from C_4 that neutrality turns into extra cost for the Exchequer; for example 7.5% for a 5 year deferrer with 50% of the population age specific mortality rates. Note for example how a deferrer loss of 5.3% (P_2) changes into a gain of 1.8% (P_4) for the deferrer.

Turning to the current SP uniform accrual rate of 5.8%, we see that with both 30% and 50% reductions in age specific mortality rates, when the deferral period is 5 years, there are respective losses of 0.6% and gains of 2.7% gain for the deferrer, which is not too far short of actuarial fairness. However, because of the lower DRNPU for the Exchequer there is a greater departure from cost neutrality for the Exchequer, that is an extra 5.1% and 8.5% for 5 year deferrals. The 21% loss to a 10 year deferrer with no adverse selection is large, indicative of significant deferrer unfairness, should the pensioner choose to defer for that period. However, adverse selection is a more likely scenario and it is interesting to see how this is mitigated to 12.2% and 5.0% losses respectively, under 30% and 50% reductions in mortality rates.

As we have noted in previous sections of the paper with the RP 10.4% accrual rate, it is no exaggeration to conclude that both the extra cost to the Exchequer and corresponding gain to the deferrer are disproportionally 'off the scale'.

XI Summary, Conclusions, and Policy suggestions

In this paper we have first reviewed the literature on deferral and early take-up up of pension and concluded that much of it relates to US and European situations. Most of the models are in discrete time, some using simulation where the rules of the scheme are more complex than for the UK state pension. This paper focusses on actuarial fairness in the UK. This is achieved when an individual's lifetime expectation of net present value of pension income is the same no matter how long he/she defers for. It also addresses cost neutrality for the Exchequer, whereby the expectation of net present value is independent of deferral period. Cost neutrality and actuarial fairness are not necessarily simultaneously achievable because the two parties may have different discount rates.

In section III, a continuous time formulation is developed for achieving actuarial fairness over all deferral periods for an unattached (single) person who has reached State Pension Age (SPA). In section IV numerical results for the marginal accrual rates are derived under the assumption that the pension uprating rate is the same as the deferrer's nominal discount rate. For those achieving state pension age after 5 April 2016 the current uniform accrual rate of 5.8% does not appear to depart too far from actuarial fairness for deferral periods less than 3 years for both males and females, but it is increasingly disadvantageous to males deferring longer than that (Figures 1 and 2). However, it is emphasised that that this conclusion is premised upon the assumption that the deferrer's discount rate net of pension uprating (DRNPU) is zero. In section 5 that assumption is relaxed. Accordingly, if from the deferrer's perspective, the real value of pensions decreases with time (a positive DRNPU) then the uniform 5.8% accrual rate begins to introduce more unfairness to deferrers the longer they defer (Table 1 and Figure 4). For those achieving state pension age before 6 April 2016 and subject to a uniform accrual rate of 10.4% it is apparent from Figure 4 that the policy is excessively advantageous to males who defer under a wide range of DRNPU. In other words it does relatively disadvantage those who choose not to or who are unable to defer. Females have greater life expectancy and that relative disadvantage of non-deferral obviously becomes even greater.

Providing IT systems can be adapted to review and apply accrual rates, perhaps on an annual basis, the objective of fairness to deferrers can be achieved, using the methodology derived here. A first step might be to prescribe cumulative accrual functions that are piecewise linear with say break points at say 3, 6, and 10 years deferral. From our review of the international experience it would appear that the UK would then lead the world in doing so, although it is notable that the US has different accrual and decrement rates for late and early take-up respectively. The methodology described in this paper can easily be extended to early take-up of pension should this seem desirable at some future point. Although we have concentrated on the UK state pension it is also applicable for example to defined benefit schemes. Implementing this type of policy does raise the question as how to treat current deferrers without worsening their historical position, in the sense that if they choose to continue deferring they should be in neither a better or worse position to that at the time accrual rates are changed. This is addressed in section VI. In section VII we have supplied a mathematical analysis for those who achieve SPA before 6 April 2016 and who have marriage or civil partners. For these individuals, Figure 6 demonstrates the further advantage that this group enjoys; an excessively high uniform accrual rate plus the payment to last survivor of a partnership.

We have concentrated on pre-tax pension income. A difficulty would arise in trying to achieve actuarial fairness with respect to post-tax pension receipts as individuals have differing sources of income and engage in varying levels of tax smoothing.

Deferral is a voluntary decision by people. That means that those who do so are self-selected At present no data on mortality rates is collected for deferrers by DWP or ONS. Accordingly, in Section VIII the literature on adverse selection for annuities was reviewed in order to assess what reduction in mortality rates for UK state pension deferrers might be. Assuming a DRNPU of zero, Figures 7 and 8 show that if such reduced mortality rates are to be acknowledged, then marginal accrual rates would need to be decreased quite significantly. Of course, that does mean that a minority who do not enjoy this increase in life expectancy are disadvantaged.

Unfortunately, one cannot necessarily assume that the DRNPU for deferrers is the same as that for the Exchequer. In fact, the former is likely to be larger, as a deferrer has to endure his/her individual survival risk whereas the Exchequer does not. This means that achieving actuarial fairness will incur extra cost for the Exchequer as the NPV of their payments will be larger than those received by the deferrer. In Tables 3 and 4 we show how this asymmetry in DRNPUs, together with adverse selection can impact greatly upon costs and gains to both the Exchequer and deferrer.

From this study we conclude:

(a) It is impossible to achieve technical actuarial fairness or cost neutrality for the Exchequer using a uniform accrual rate such as the current 5.8% and 10.4% ones. At the moment with the 5.8% accrual rate the actual degree of actuarial unfairness is not too severe although males wanting to defer more than 3 years might be worse off. That of course may change with future discount and pension uprating rates. Ideally, Government should stop using a uniform accrual rate and introduce a cumulative accrual rate commensurate with the period deferred, the estimated degree of adverse selection, discount rates, and pension uprating rates.

(b) It seems likely that for a wide set of scenarios on pension uprating rates and discount rates that the existing uniform 10.4% accrual rate is unsustainable. It is unfair to those who cannot avail of it and it imposes extra unnecessary expenditure on Government. Therefore, it should, subject to preserving historical benefits, be changed in line with accrual rates for those who achieve SPA after 5 April 2016. It is hard to see what the substantive argument is for treating pre-6 April 2016 individuals more favourably than others. It is unfortunate that this was not addressed earlier, not only because of the inherent unfairness but also because it incurs unnecessary Exchequer expenditure. There was a good opportunity to do so when the 5.8% rate was implemented on 6 April 2016.

(c) To improve future policy decisions in (a), Government should collect data on the numbers of deferrers; their deferral period, whether they take extra pension or lump sum, and age at death.

(d) The numerical results obtained in this paper do depend quite crucially upon the DRNPU. Here we have assumed it to be static and deterministic. Further research needs to be done in understanding its dynamic and stochastic behaviour over time, as intimated in section III. This would be of benefit not only to the question of deferral, but more generally to the whole question of pension payments and their long term cost to the Exchequer.

Acknowledgement

The author acknowledges with thanks the insightful comments made by anonymous reviewers of an earlier version of this paper.

References

Beveridge, W. (1942), 'Social Insurance and Allied Services', Parliamentary Paper, Cmd. 6404. London: The Stationary Office.

Blake, D. and Mayhew, L. (2006), 'On the sustainability of the UK State Pension System in the light of Population Ageing and Declining Fertility', *The Economic Journal*, vol. 116, pp. 286-305.

Bozio, A., Crawford, R., and Tetlow, G. (2010), 'The History of state pensions in the UK: 1948 to 2010', Institute of Fiscal Studies (IFS), Briefing Note BN 105.

Cannon, E. and Tonks, I. (2016) Cohort Mortality Risk or adverse selection in annuity markets, Journal of Public Economics, 141, 68-81

Coile, C., Diamond, P., Gruber J., and Jousten, A. (2002), 'Delays in Claiming Social Security Benefits', *Journal of Public Economics*, vol. 84, 357-385.

Coleman, N., McLeod, R., Norden, O. and Coulter, A. (2008) State Pension Deferral: public awareness and attitudes, Department of Work and Pensions, Research Report Number 526

Dagpunar, J.S. (2015), 'Deferring a state pension - is it worthwhile?', *Significance*, vol. 12, 2, pp. 30-35.

Dagpunar, J. S. (2017), 'Optimum Strategies for UK state pension deferral with particular reference to partner's right to inherit benefits', The Pensions Institute, Cass Business School, City University, London, Discussion paper PI-1703,. Available from https://www.pensions-institute.org/papers.html

Dagpunar, J.S. (2018), A Critique of the paper by Kanabar, R. and Simmons, P. (2016), 'To Defer or not to defer? UK state pension and work decisions in a lifecycle model', Applied Economics, 48, 58, 5699-5716. *Applied Economics*, 50, 60, 6569-6575. Available at https://doi.org/10.1080/00036846.2018.149003

Duggan, J. and Soares, C. (2002), 'Actuarial nonequivalence in early and delayed Social Security Benefit Claims', *Public Finance Review*, vol. 30, 3, 188-207.

Farrar, S., Moizer, J., and Hyde, M. (2012), 'The value of incentives to defer the UK state pension', *Pensions*, vol. 17, 1, 46-62.

Finklestein, A. and Poterba, J. (2002), 'Selection Effects in the United Kingdom Individual Annuities Market', *The Economic Journal*, 12, pp. 28-50.

Finklestein, A. and Poterba, J. (2004), 'Adverse Selection in Insurance Markets: Policyholder Evidence from the UK Annuity Market', *Journal of Political Economy*, 112, 1, pp. 183-208.

Fong, W.M. (2002), 'On the cost of adverse selection in individual annuity markets: Evidence from Singapore*, Journal of Risk and Insurance*, 69, pp. 193-207

Government Actuary's Department (2014), 'Report by the Government Actuary on the actuarially fair rate of increments for those reaching State Pension age on or after 6 April 2016 and choosing to defer their State Pension beyond State Pension age', Government Actuary's Department. Available from http://data.parliament.uk/DepositedPapers/Files/DEP2014-1112/GAD_State_Pension_Deferral_Report.pdf

Heiland, F., and Yin, N. (2014), 'Have we finally achieved actuarial fairness of social security retirement benefits and will it last*?',* Michigan Retirement Research Center, University of Michigan, working paper no. 2014-307. Available from http://www.mrrc.isr.umich.edu/publications/papers/pdf/wp307.pdf

Kanabar, R. and Simmons, P. (2016), 'To defer or not to defer? UK state pension and work decisions in a lifecycle model', *Applied Economics*, vol. 48, 58, pp. 5699-5716.

Mamon, R. (2004), 'Three ways to solve for bond prices in the Vasicek model', *Journal of Applied Mathematics and Decision Sciences*, vol. 8, 1, pp. 1-14.

Medijainen, M. (2011), 'Incentives to retire by old-age pension policy in Estonia', *Estonian Discussions on Economic Policy*, vol. 19, pp. 101-120.

Meneu, R., Devesa, E., Devesa, M., Domingguez, I., and Encinas, B. (2016), 'Adjustment mechanisms and intergenerational actuarial neutrality in pension reforms', *International Social Security Review*, vol. 69, pp. 87-107.

Mirer, T.W. (1998), 'The optimal time to file for social security benefits', *Public Finance Review*, vol.26, 6, pp. 611-636.

Mitchell, O.S. and McCarthy, D. (2002), 'Estimating International Adverse Selection in Annuities', *North American Actuarial Journal*, 6:4,pp. 38-54

Moizer, J., Farrar, S., and Hyde, M. (2017), 'UK State Pension deferral incentives and sustainability', *Applied Economics*, published online 6 November 2017

Office of the Superintendent of Financial Institutions Canada (2017), Canada Pension Plan Actuarial Adjustment Factors as specified in the 27th Actuarial Report on the Canada Pension Plan as at 31

December 2015, Actuarial Study no. 18, April 2017, Available from http://publications.gc.ca/collections/collection_2017/bsif-osfi/IN5-1-18-2017-eng.pdf

Oksanen, H. (2005), 'Actuarial neutrality across generations applied to public pensions under population ageing: effects on government finances and national saving', European Commission Directorate-General for Economic and Financial Affairs, Economic Papers, no. 230, July 2005.

Pensions Commission (2004) Pensions: Challenges and Choices: The first report of the Pensions Commission, October 2004, Appendix F, page 142

Poterba, J. (2001), 'Annuity Markets and Retirement Security, *Fiscal Studies*, 22,3, pp. 249-270

Queisser, M. and Whitehouse, W. (2006), 'Neutral or Fair? Actuarial Concepts and Pension-System Design', *OECD Social, Employment and Migration Working Papers*, no. 40, OECD Publishing, Paris. Available from http://dx.doi.org/10.1787/351382456457

Rasdal, K. (2013), 'Norwegian couples and the take-up of pension benefits', MPhil thesis, Department of Economics, University of Oslo.

Rose, C. (2015), 'The return on investment for delaying social security beyond age 62', *Journal of Financial Planning*, vol. 28, 4, pp. 50-58.

Shoven, J. and Slavov, S. (2014), 'Does it pay to delay social security?', *Journal of Pension Economics and Finance*, vol. 13, 2, pp. 121-144.

Stubbs, J. and Adetunji, J. (2016), 'UK pension changes in 2015: some mathematical considerations', *The Mathematical Gazette*, vol. 100, pp. 193-202.

Thurley, D. (2010), 'Deferred retirement increments, House of Commons Library, SN/BT-2868, London: House of Commons.

Thurley, D. (2016), 'State Pension uprating - 2010 onwards', House of Commons Library, SN-05649, London: House Of Commons.

Thurley, D. (2017), State Pension Deferral. House of Commons Library, CBP-02868, London : House of Commons.

Vasicek, O. (1977), 'An equilibrium characterization of the term structure', *Journal of Financial Economics*, vol. 5, pp. 177-188.