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Efficiency, Risk Aversion and Portfolio Insurance: An Analysis of Financial Asset Portfolios held by Investors in the United Kingdom

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EFFICIENCY, RISK AVERSION AND PORTFOLIO INSURANCE: AN ANALYSIS OF FINANCIAL ASSET PORTFOLIOS HELD BY INVESTORS IN THE UNITED KINGDOM

*David Blake**

Using data for the United Kingdom, we show that investors in six different wealth ranges hold mean-variance efficient portfolios of financial assets. This result permits us to estimate coefficients of relative risk aversion for investors in each wealth range. We find that these coefficients are much higher than most previous studies have found. This implies that investors (i) are unwilling to hold risky assets *unless* they are compensated with a sufficiently high risk premium and (ii) are willing to pay for portfolio insurance. The general non-availability of portfolio insurance in the United Kingdom appears to indicate a supply-side rather than a demand-side failure.

There has been a small number of earlier studies of asset portfolios held by UK investors. These have mainly been concerned with the effects of age, sex, income or wealth level on the asset composition of these portfolios (e.g. Revell (1962), Atkinson and Harrison (1978), Shorrocks (1982), Inland Revenue (1993) and Banks *et al.* (1994)). However, none of these studies has examined explicitly the efficiency of UK asset portfolios. In this paper we use data drawn from the Financial Research Survey conducted between April 1991 and March 1992 and summarised in Banks *et al.* (1994) to investigate the efficiency of asset portfolios across different wealth ranges over this period. We also derive estimates of the coefficients of relative risk aversion for investors in each of the wealth ranges and use these estimates to determine whether there is a potential demand for portfolio insurance by UK investors.

In Section I, we establish the theoretical framework for this analysis. In Section II, we examine the returns generated by financial assets in the United Kingdom between 1946 and 1991 as a basis for forecasting returns and risks over the portfolio holding period. In Section III, we present the results of our analysis on portfolios held by UK investors, and we draw conclusions in the final section.

I. THEORETICAL CONSIDERATIONS

A. Portfolio Efficiency and Portfolio Insurance

Under the assumption that asset returns are multivariate normally distributed, it is possible to show that only mean-variance efficient portfolios of assets are optimal in the sense of being consistent with expected utility maximisation (see,

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e.g., Ingersoll (1987, pp. 96–7)). It is also possible to show that the minimum-variance portfolio having expected return μ is given by:

$$\boldsymbol{\theta}(\mu) = \lambda \boldsymbol{\Omega}^{-1} \mathbf{i} + \gamma \boldsymbol{\Omega}^{-1} \mathbf{m}, \quad (1)$$

where

$$\left. \begin{aligned} \lambda &= \frac{C - \mu B}{D}, & \gamma &= \frac{\mu A - B}{D}, \\ A &= \mathbf{i}' \boldsymbol{\Omega}^{-1} \mathbf{i}, & B &= \mathbf{i}' \boldsymbol{\Omega}^{-1} \mathbf{m}, \\ C &= \mathbf{m}' \boldsymbol{\Omega}^{-1} \mathbf{m}, & D &= AC - B^2. \end{aligned} \right\} \quad (2)$$

Here $\boldsymbol{\theta}$ is the vector of N portfolio weights, \mathbf{m} is the vector of expected returns on portfolio assets, $\boldsymbol{\Omega}$ is the covariance matrix of returns on assets, \mathbf{i} is the unit vector and λ and γ are Lagrangean multipliers associated with the constraints $\mathbf{i}'\boldsymbol{\theta} = 1$ and $\mathbf{m}'\boldsymbol{\theta} = \mu$ respectively.¹

Similarly it is possible to show that the maximum-expected return portfolio having variance σ^2 is given by:

$$\boldsymbol{\theta}(\sigma^2) = \boldsymbol{\Omega}^{-1} \left(\frac{\mathbf{m} - \alpha \mathbf{i}}{2\beta} \right), \quad (3)$$

where

$$\left. \begin{aligned} \alpha &= \frac{(B - AB\sigma^2) + [(B - AB\sigma^2)^2 - (A - A^2\sigma^2)(C - B^2\sigma^2)]^{\frac{1}{2}}}{(A - A^2\sigma^2)}, \\ \beta &= \frac{B - \alpha A}{2} \end{aligned} \right\} \quad (4)$$

are Lagrangean multipliers associated with the constraints $\mathbf{i}'\boldsymbol{\theta} = 1$ and $\boldsymbol{\theta}'\boldsymbol{\Omega}\boldsymbol{\theta} = \sigma^2$ respectively.

The equation of the efficient set is:

$$\begin{aligned} \sigma^2 &= \boldsymbol{\theta}(\mu)' \boldsymbol{\Omega} \boldsymbol{\theta}(\mu) \\ &= \boldsymbol{\theta}(\mu)' \boldsymbol{\Omega} (\lambda \boldsymbol{\Omega}^{-1} \mathbf{i} + \gamma \boldsymbol{\Omega}^{-1} \mathbf{m}) \\ &= \lambda + \gamma \mu \\ &= \frac{A\mu^2 - 2B\mu + C}{D} \end{aligned} \quad (5)$$

and its slope (which measures the marginal rate of transformation (MRT) between mean and standard deviation), is:

$$\text{MRT} = \frac{d\mu}{d\sigma} = \frac{D\sigma}{2\mu - B}. \quad (6)$$

If utility is an isoelastic function of wealth exhibiting constant relative risk aversion:

$$U(W) = \frac{W^{1-\phi} - 1}{1-\phi}, \quad (7)$$

¹ Equation (1) does not impose any short holding restrictions, so that elements of $\boldsymbol{\theta}$ can be negative.

if portfolio returns are normally distributed with mean μ and variance σ^2 , and if investors maximise expected utility then indifference curves are determined as a local approximation by the equation:

$$\mu = \bar{U} + \frac{1}{2}\phi\sigma^2 \quad (8)$$

where \bar{U} is an index of expected utility. The slope of this equation (which measures the marginal rate of substitution (MRS) between mean and standard deviation) is given by:

$$\text{MRS} = \frac{d\mu}{d\sigma} = \phi\sigma, \quad (9)$$

where ϕ is the coefficient of relative risk aversion (CRRA) (see Pratt (1964), Arrow (1970)). The optimal portfolio is that for which (6) and (9) are equal, which implies that the CRRA is given by:

$$\phi = \frac{D}{2\mu - B}. \quad (10)$$

Expected utility maximising investors ought to select asset portfolios consistent with (10) holding. If they do, we can readily derive their coefficient of relative risk aversion, ϕ . Given ϕ , we can immediately derive the proportionate risk premium, π , namely the maximum proportion of total wealth that investors would be prepared to pay to avoid risk (see Pratt (1964))²:

$$\pi = \frac{1}{2}\phi\sigma^2. \quad (11)$$

Gibbons, Ross and Shanken (1989) (hereafter GRS) have proposed a test of the significance of the difference between the actual portfolio (θ_a) held by an investor and a corresponding efficient portfolio (either $\theta(\mu)$ or $\theta(\sigma^2)$), based on the difference between the slopes of arrays from the origin through the two portfolios in expected return-standard deviation space. If the actual portfolio is an efficient portfolio, the two slopes will be the same; if the actual portfolio is inefficient, the slope of the efficient portfolio will be significantly greater. GRS define a quantity:

$$K \equiv \frac{1 + (\mu_e/\sigma_e)^2}{1 + (\mu_a/\sigma_a)^2} - 1, \quad (12)$$

where (μ_e/σ_e) is the slope of an array from the origin through an efficient portfolio (either $\theta(\mu)$ or $\theta(\sigma^2)$) and (μ_a/σ_a) is the slope of an array from the origin through the actual portfolio (θ_a). GRS show, under the assumption that asset returns are multivariate normal and under the null hypothesis that the actual portfolio is an efficient portfolio, that:

$$F = \frac{M(M - N - 1)K}{N(M - 2)} \quad (13)$$

² Equation (11) is an approximation which is valid for small risks (see, e.g., Eeckhoudt and Gollier (1995, ch. 4)).

has a central F distribution with degrees of freedom N and $(M - N - 1)$, where N is the number of assets in the portfolio and M is the number of time series observations used to estimate μ and σ^2 .³

Risk-averse investors can also hedge the downside risk in their portfolios through the purchase of portfolio insurance, in effect the purchase of a put option on their wealth holdings (Leland (1980)). Suppose that an investor decides to buy an at-the-money put option with an exercise price equal to the current value of his or her wealth, W_0 . The put premium is determined using the Black and Scholes (1973) formula for an at-the-money call option together with the put-call parity formula (see Stoll (1969)):

$$\begin{aligned} P &= C - W_0(1 - e^{-rT}) \\ &= W_0\{[N(d_1) - 1] - e^{-rT}[N(d_2) - 1]\}, \end{aligned} \quad (14)$$

where

$$d_1 = \frac{r\sqrt{T}}{\sigma} + \frac{1}{2}\sigma\sqrt{T}, \quad (15)$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

r is a risk-free interest rate, T is the expiry period of the option, and $N(d_1)$ and $N(d_2)$ are cumulative normal distribution functions evaluated at d_1 and d_2 .⁴ If investors are offered portfolio insurance which costs as a proportion of their wealth (P/W_0) less than π (from (11)), we would expect them to take it.

B. Forecasting Returns and Risks

Up till now, we have said nothing about how the expected returns (\mathbf{m}) and the covariances between the returns ($\mathbf{\Omega}$) on assets might be determined. To some extent this depends on the degree of sophistication of investors, but in the absence of any survey data indicating how investors forecast returns and risks, two possibilities suggest themselves.

The simplest strategy is to use the sample means (m_i) and covariances (σ_{ij}) from historical data on asset returns ($i = 1, N$); in other words, to use the first two unconditional moments.⁵ This strategy would be a reasonable one if the distribution of returns is stationary, with returns exhibiting mean reversion.

³ MacKinlay (1985) has shown, using simulation evidence, that this F test is fairly robust even when asset returns are not normal but have distributions that are leptokurtic relative to the normal. See below.

⁴ While the original Black-Scholes model was derived under the assumption that investors are risk-neutral, it is possible to show that a risk-neutral valuation of the option is still valid if, as we are assuming, the average investor exhibits constant relative risk aversion (see, e.g., Rubinstein (1976), Breeden and Litzenberger (1978) and Brennan (1979)). Equation (14) also assumes that there are no indivisibilities or transactions costs in the provision of portfolio insurance, otherwise the cost would be higher than indicated by P in (14). Furthermore, as is standard in the literature, we ignore any impact that the introduction of portfolio insurance has on the distribution of returns on the underlying securities.

⁵ In addition the first moment should be the arithmetic mean not the geometric mean, since as Kolbe *et al.* (1984) point out: 'The quantity desired is the rate of return that investors expect over the next year for the random annual rate of return on the market. The arithmetic mean is the unbiased measure of the expected value of repeated observations of a random variable, not the geometric mean.'

However, the strategy would not be reasonable if returns are non-stationary and would in any case not be consistent with investors having rational expectations.

This suggests a second possibility, namely that we should consider the first two conditional moments from a time series model of the returns data, such as an autoregressive distributed lag-autoregressive conditional heteroscedasticity (ARDL-ARCH) model:

$$r_{it} = a_0^i + a_1^i t + \sum_{k=1}^N \sum_{j=1}^P a_{kj}^i r_{k,t-j} + u_{it}, \quad (16)$$

$$u_{it} = \epsilon_{it} \left(b_0^i + \sum_{j=1}^Q b_j^i u_{i,t-j}^2 \right)^{\frac{1}{2}}, \quad \begin{matrix} i = 1, N \\ t = \text{Max}(P, Q) + 1, M, \end{matrix} \quad (17)$$

where the ϵ_{it} are multivariate standard normal. This modelling framework is sufficiently general to allow the underlying return distributions to be either stationary or non-stationary, cointegrated or non-cointegrated, and to exhibit intermittent periods of volatility and tranquility via the ARCH(Q) process in (17) (see, respectively, Dickey and Fuller (1979), Engle and Granger (1987), and Engle (1982)). Rational expectations then dictates that we take into account all information up to time M in forecasting returns and risks:

$$m_{i,M+1} = a_0^i + a_1^i (M+1) + \sum_{k=1}^N \sum_{j=1}^P a_{kj}^i r_{k,M+1-j}, \quad (18)$$

$$\begin{aligned} \sigma_{ij,M+1} &= b_0^i + \sum_{j=1}^Q b_j^i u_{i,t-j}^2 && \text{if } i = j \\ &= c_{ij} \left(b_0^i + \sum_{k=1}^Q b_k^i u_{i,t-k}^2 \right)^{\frac{1}{2}} \left(b_0^j + \sum_{k=1}^Q b_k^j u_{j,t-k}^2 \right)^{\frac{1}{2}} && \text{if } i \neq j, \end{aligned} \quad (19)$$

where c_{ij} is the unconditional covariance between ϵ_{it} and ϵ_{jt} .

II. AN ANALYSIS OF FINANCIAL ASSET RETURNS

Table 1 examines the characteristics of the returns on the three main classes of financial assets, interest-bearing accounts (IBAs), bonds and shares, held by UK investors over the period 1946–91. The data are drawn from Barclays de Zoete Wedd (1992).⁶ As might be expected, shares had the highest average return at 16.33%, but bonds at 6.43% had a lower average return than IBAs at 7.81%. Yet comparing standard deviations, the returns on bonds (with a standard deviation of 14.23%) were nearly six times as volatile as those on IBAs (with a standard deviation of 2.53%); on the other hand, shares were more than twice as volatile as bonds (with a standard deviation of 29.53%). But comparing coefficients of variation, bonds appear to be riskier than shares

⁶ The Financial Research Survey collects information on asset allocations, not on individual holdings of shares and bonds. However, most personal sector investors hold their risky assets in the form of unit trusts, so it is reasonable to use broadly-based value-weighted indices of share and bond prices, such as those supplied by BZW, to represent the returns on asset holdings. There is also no alternative to using the same indices for all wealth ranges, since the survey provides no information on how individual security holdings differ between wealth ranges.

Table 1

*Characteristics of the Returns on Financial Assets in the United Kingdom 1946-91**

	IBAs†	Bonds‡	Shares§
Mean return (arithmetic)	7.8095	6.4324	16.3346
Standard deviation of return	2.5262	14.2288	29.5290
Coefficient of variation	0.3235	2.2120	1.8078
Covariance matrix			
IBAs	6.3817	14.2259	11.0492
Bonds	14.2259	202.4588	251.3242
Shares	11.0492	251.3242	871.9618
Tests for normality			
Constant only (critical value, 5.99)	5.10	15.55	133.21
Constant plus dummies** (critical value, 5.99)	—	0.81	1.13
Autocorrelation at lag†† (critical value, 0.29)			
1	0.88	0.01	-0.24
2	0.77	0.04	-0.19
3	0.72	-0.14	-0.11
4	0.68	0.13	0.15
5	0.66	0.40	0.07
6	0.53	0.19	-0.21
7	0.44	0.10	0.02
Box-Pierce Q statistic‡‡ (critical value, 14.1)	149.88	11.24	8.17
Tests for stationarity§§			
Difference stationarity	-1.56	-5.65	-8.11
(critical value, number of lagged dependent variables)	(-4.00, 0)	(-4.79, 2)	(-4.01, 1)
Trend stationarity*** (critical value, ± 2.01)	-0.64	-0.11	0.01
Tests for mean reversion			
Variance ratio statistic at lag††† (critical value, 1.96)			
2	0.12	-3.55	-3.38
3	-0.57	-2.67	-3.07
4	-0.83	-2.76	-2.92
5	-0.91	-2.64	-2.54
6	-0.46	-2.28	-2.18
7	-0.28	-2.07	-2.16

* Source: Barclays de Zoete Wedd (1992).

† Gross return on interest-bearing accounts with building societies.

‡ Gross return (coupons plus capital gains) on government bonds.

§ Gross return (dividends plus capital gains) on equities.

|| Based on the test for skewness and kurtosis given in Jarque and Bera (1980), the test statistic under the null hypothesis is of no skewness and no excess kurtosis is χ^2 -distributed with 2 D.F.

** The dummy variables are $D7475$ (with -1 for 1974, +1 for 1975, 0 elsewhere), $D77$ (with +1 for 1977, 0 elsewhere), and $D82$ (with +1 for 1982, 0 elsewhere).

†† Under the null hypothesis of zero autocorrelation, the test statistic is asymptotically normally distributed with zero mean and standard deviation $M_t^{-1/2}$.

‡‡ Defined by $Q = M \Sigma_{j=1}^P \rho_j^2$ (where ρ_j is the j th autocorrelation coefficient) and χ^2 -distributed with $P = 7$ D.F. under the null hypothesis of zero autocorrelation up to lag $P = 7$.

§§ Based on the regression $\Delta r_{it} = d_0^t + d_1^t t + d_2^t r_{i,t-1} + \sum_{j=1}^S d_{2+j}^t \Delta r_{i,t-j} + \xi_{it}$ where Δ is the first-difference operator.

||| Under the null hypothesis that r_{it} is stationary in differences (and hence is generated by a unit root process) against the alternative hypothesis that r_{it} is stationary in levels, which is the one-sided test $H_0: d_2^t = 0, H_1: d_2^t < 0$, the t-statistic on d_2^t in §§ has the non-standard t-distribution given in Dickey and Fuller (1979).

*** Under the null hypothesis that r_{it} is stationary about a linear trend (and hence is generated by a linear trend) against the alternative hypothesis that r_{it} is stationary in levels, which is the two-sided test $H_0: d_1^t \neq 0, H_2: d_1^t = 0$, the t-statistic on d_1^t in §§ has a standard t-distribution with 46 D.F..

(continued)

††† The variance ratio statistic is defined by

$$VR(i, k) = \left(\frac{\sigma_{ik}^2}{\sigma_{i1}^2} - 1 \right) \left[\frac{2(2k-1)(k-1)}{3Mk} \right]^{-\frac{1}{2}},$$

where

$$\sigma_{i1}^2 = \frac{1}{M-1} \sum_{t=1}^M \left[(r_{it} - r_{i,t-1}) - \frac{1}{M} (r_{iM} - r_{i0}) \right]^2$$

is the unbiased estimator of ω_i^2 under the assumption that

$$r_{it} = d_0^i + r_{i,t-1} + \xi_{it}$$

and $\xi_{it} \sim \text{NID}(0, \omega_i^2)$, and where

$$\sigma_{ik}^2 = \frac{1}{k(M-k)(M-k+1)} \sum_{t=k}^M \left[(r_{it} - r_{i,t-k}) - \frac{k}{M} (r_{iM} - r_{i0}) \right]^2$$

is an unbiased estimator of the k th differences of r_{it} and also an unbiased estimator of ω_i^2 under the assumption that r_{it} follows a random walk. Under the null hypothesis of no mean reversion, $H_0: VR(i, k) = 0$, the test statistic $VR(i, k)$ is asymptotically $N(0, 1)$.

over the period. The table also presents the covariance matrix of the raw returns; we shall refer to this matrix later but for the moment we will note that the matrix indicates that all the raw returns are positively correlated.

The next part of the table tests whether the returns are normally distributed. We do this by examining the residuals from an OLS regression of the returns on a constant term for skewness and kurtosis. The returns on IBAs are consistent with being generated from a normal distribution (namely with zero skewness and kurtosis of three) but the returns on bonds and shares exhibit leptokurtosis with respect to the normal. This turns out to be due to two outlying share returns and four outlying bond returns. In the case of shares, the stock market crash and recovery in 1974 and 1975 led to share returns of -49.4% in 1974 and 149.6% in 1975. Bonds behaved in a similar way with returns of -15.2% in 1974 and 36.8% in 1975. Bonds also delivered exceptional performance in 1977 and 1982 with returns of 44.8% and 51.3% respectively. The first case is explained by the favourable reaction of the bond market to the austerity programme forced on the Labour government following the IMF visit the previous year; the second case is the favourable response to victory in the Falklands War. If four bond dummy variables (for 1974, 1975, 1977 and 1982) and two share dummy variables (for 1974 and 1975) are included in the regression, the residuals are normally distributed. So once allowance is made for extreme values, annual asset returns are normally distributed.

We next examine the autocorrelation function (ACF) for the three returns. The ACF for IBAs shows the familiar declining pattern generated by a non-stationary series, while those for bonds and shares indicate stationary series; this is confirmed by the Box and Pierce (1970) Q -statistics. However, in the case of bonds, there is a statistically significant spike at lag 5, and a spike that is almost significant at lag 1 for shares, indicating the possibility of autoregressive processes of fifth and first orders for bonds and shares respectively.

We now undertake tests for both difference stationarity and trend stationarity using a sequential procedure outlined in Dolado *et al.* (1990). The tests clearly indicate that both bonds and shares are stationary in levels, while IBAs contain a unit root but no additional non-stationarity due to trend. These results are confirmed by tests for mean reversion using the variance ratio statistics proposed by Cochrane (1988) and Lo and MacKinlay (1988, 1989). The variance ratio statistic is not significantly different from zero at up to seven lags in the case of IBAs, a result that is consistent with the return on IBAs being generated by a unit root process. However, the variance ratio statistics are significantly different from zero for bonds and shares indicating the presence of mean reversion in these two series (which is consistent with evidence from the United States, see, e.g., Poterba and Summers (1988)). So we can conclude that the returns on IBAs are $I(1)$ while the returns on bonds and shares are $I(0)$. This immediately rules out the possibility that the three series form a cointegrated system.

Given these preliminary findings, we proceed to estimate ARDL models for the three series. We imposed the unit root restriction on IBAs by estimating in first differences and our initial specifications included up to six lags of each variable. The final specifications are presented in Table 2 (dummy variables were included in the bonds and shares equations corresponding to the outliers discussed above but their coefficients are not listed in the table). Turning first to the diagnostic statistics, we see that there is reasonable goodness-of-fit, the joint significance of the included variables is high, and that residuals from these regressions are both stationary and non-autocorrelated. However, the residuals from the IBAs equation indicate the presence of an ARCH process. Further experimentation indicated that this was a first-order process, so the IBAs ARDL equation assuming an ARCH(1) process was estimated using a four-step GLS estimator outlined in Greene (1993, p. 440). It is these GLS estimates that are presented in the table. The bonds and shares equations, because they contained no ARCH process, were estimated by OLS.

The final specifications of the ARDL-ARCH models are:

$$r_{It} = 0.2268 + r_{I,t-1} - 0.0410r_{B,t-1} + u_{It}, \quad (20)$$

where

$$u_{It} = \epsilon_{It}(0.5009 + 0.8331u_{It}^2, t-1)^{\frac{1}{2}},$$

$$r_{Bt} = 4.0204 + 0.2587r_{B,t-5} + \text{dummies} + u_{Bt}, \quad (21)$$

$$r_{St} = 17.8073 - 0.3398r_{S,t-1} + 0.6561r_{B,t-1} + \text{dummies} + u_{St}. \quad (22)$$

So the return on IBAs is generated by a random walk with positive drift (reflecting the gradual rise in nominal interest rates over the post-war period) together with a small negative spillover effect from the bond market in the previous year. The residuals from this equation follow an ARCH(1) process with zero mean and conditional variance:

$$\sigma_{It}^2 = 0.5009 + 0.8331u_{It}^2, t-1. \quad (23)$$

The return on bonds is generated by a fifth-order autoregressive process,

Table 2

*Models Generating the Returns on Financial Assets in the United Kingdom 1946–91
and Forecast Returns and Covariances for 1992*

	IBAs*	Bonds†	Shares†
Autoregressive distributed lag model			
Constant (t-ratio)	0.2268 (1.68)	4.0205 (2.57)	17.8073 (5.24)
IBAs (–1)	1.000 (Fixed)	—	—
Bonds (–1)	–0.0410 (4.78)	—	0.6561 (2.70)
Bonds (–5)	—	0.2587 (2.39)	—
Shares (–1)	—	—	–0.3398 (2.71)
R ²	0.5705	0.6384	0.5899
F-statistic (critical value, D.F.1, D.F.2)	10.16 (4.07, 1, 43)	15.89 (2.64, 4, 36)	19.65 (2.84, 3, 41)
ADF(1) (critical value, –3.50)	–5.71	–6.88	–5.93
AUTO(1) (critical value, 3.84)	0.83	0.65	0.75
ARCH(6) (critical value, 12.60)	13.98	15.1	7.25
Autoregressive conditional heteroscedasticity model			
Constant (t-ratio)	0.5009 (2.28)	—	—
Squared residual(–1)	0.8331 (2.55)	—	—
R ²	0.1294	—	—
AUTO(1) (critical value, 3.84)	1.41	—	—
Expected returns and risks			
Expected returns (1992)	5.6119	8.2373	24.4650
Expected covariance matrix (1992)			
IBAs	8.5960	–3.8065	–0.0575
Bonds	–3.8065	75.3962	44.3321
Shares	–0.0575	44.3321	368.8743

* Estimated using the four-step GLS procedure outlined in Greene (1993, p. 440).

† Estimated by OLS.

reflecting the 5-year interest rate cycle in the United Kingdom. The return on shares is generated by a first-order autoregressive process with a negative coefficient (confirming the strong mean reversion found above), together with a positive spillover effect from the bond market in the previous year.

These models can be used to calculate the expected returns for IBAs, bonds and shares and the covariance matrix of returns for 1992 using (18) and (19), noting that the c_{ij} in (19) are the unconditional covariances between ϵ_{It} , u_{Bt} and u_{St} in (20), (21) and (22) respectively.⁷ They are listed in the last part of Table 2. Comparing the expected covariance matrix for 1992 with the historical

⁷ The forecasting equations (20) to (23) filter out the effect of past extreme values. It is sensible to do this if it is believed that 1991–2 is unlikely to be an atypical year.

covariance matrix for the raw returns listed in Table 1, we note the following: the absolute values of the conditional variances and covariances are (not surprisingly) smaller than their unconditional equivalents, with the exception of the conditional variance of IBAs which is larger, reflecting the importance of the ARCH(1) term; and while the unconditional covariances between all assets are positive, the conditional covariances between IBAs and bonds and between IBAs and shares are negative. This will turn out to be important for determining the efficiency of UK portfolios. The next section investigates this.

III. AN ANALYSIS OF FINANCIAL ASSET PORTFOLIOS

In this section, we use the relationships we have derived above to test whether UK investors' portfolios are efficient for the period 1991–2. The data are drawn from Banks *et al.* (1994).

Table 3 presents results for six different financial wealth ranges⁸, with mid-range wealth levels varying from £252 to £100,000⁹ (see row 1). Row 2 shows the percentage of the total population in each wealth range. Average financial wealth¹⁰ in the United Kingdom in 1991–2 was about £3,000; median financial wealth was even lower at about £450. The average is higher than the median because of a very small proportion (around 1%) of very rich people with assets above £36,800. Three-quarters of the population had average financial assets below £3,500 and 95% of the population had average financial assets below £15,000.

Financial assets are held almost exclusively in the form of interest-bearing accounts, bonds and shares. The asset allocation across these three categories in different wealth ranges is shown in rows 3–5 of the table. Almost the entire population (99%) held more than half their financial assets in the form of IBAs; indeed 75% of the population held at least 80% of their financial assets in the form of IBAs. This demonstrates that the majority of the British population place great emphasis on the liquidity and security of their financial assets: this is to be expected, since IBAs are used for transactions purposes as well as for investment purposes.¹¹ Only the top 1% of the population held more than half (70.6%) of their total financial wealth in the form of investment assets (bonds and shares) rather than transactions assets. In all wealth ranges, holdings of investment assets are overwhelmingly in the form of shares rather than bonds: only 1% of total financial assets for the poorest investors and

⁸ 25% of the population have financial wealth below £50 which is held entirely in the form of cash or IBAs. These individuals are excluded from this analysis.

⁹ In the absence of a known upper limit to the top wealth range, we assume a mid-range level of £100,000.

¹⁰ Excluding housing and pension wealth. Data limitations force us to ignore the fact that housing and pension assets are also part of an optimal investment portfolio.

¹¹ In practice, it is impossible to distinguish IBAs used exclusively for transactions purposes from those used exclusively for investment purposes. However, the UK *National Accounts* show that the personal sector has a relatively stable liquid assets-to-wealth ratio in the long run and that this ratio responds in the short term to changes in relative returns and risks. This justifies the use of the mean-variance framework developed in Section I above for determining simultaneously the optimal portfolio weights in IBAs, bonds and shares.

Table 3

Asset Allocation, Portfolio Characteristics, Risk Aversion Estimates and Cost of Portfolio Insurance in Different Financial Wealth Ranges in the United Kingdom, 1991-2

	Financial wealth ranges (£)							
	50-454	455-3,499	3,500-7,994	7,995-14,999	15,000-36,800	36,800+		
1. Mid-range wealth (£)	252	1,977	5,702	11,452	25,900	100,000		
2. Percentage of total population in range	25	25	15	5	4	1		
Asset allocation* (percentages)								
3. IBAs†	86.5	82.1	78.8	67.2	53.4	29.4		
4. Bonds‡	1.0	1.4	2.0	3.2	5.0	5.9		
5. Shares§	12.5	16.5	19.2	29.6	41.6	64.7		
Portfolio characteristics (percentages)								
6. Expected return	7.99	8.76	9.28	11.28	13.59	17.96		
7. Standard deviation	3.50	3.99	4.38	6.08	8.25	12.60		
8. Coefficient of variation**	43.75	45.60	47.16	53.89	60.74	70.11		
9. Marginal benefit of risk††	4.17	1.57	1.33	1.18	1.06	1.00		
10. Marginal cost of return‡‡	0.24	0.64	0.75	0.85	0.94	1.00		
Risk aversion estimates								
11. Marginal rate of transformation§§	1.665	1.371	1.262	1.088	1.027	0.993		
12. Implied coefficient of relative risk aversion (%)	47.60	34.33	28.82	17.90	12.44	7.88		
13. Risk premium (%)***	2.91	2.74	2.76	3.31	4.24	6.25		
Cost of portfolio insurance								
14. Put premium (P) (£)†††	0.20	2.77	11.50	65.19	306.84	2633.19		
15. Put premium per unit of wealth (%)†††	0.08	0.14	0.20	0.57	1.18	2.63		
16. Portfolio-insured returns (%)§§§	7.91	8.62	9.08	10.71	12.41	15.33		

* Source: Banks *et al.* (1994, tables 3.4 and 3.5); excludes housing and pension wealth.

† Interest-bearing accounts cover bank and building society accounts, national savings and TESSAs (tax-exempt special savings accounts). ‡ Bonds cover gilts, local authority bonds and bond unit trusts; estimated at one-third of the 'other' category in table 3.5. § Shares cover direct shareholdings, unit trusts, investment trusts and PEPs (personal equity plans); unit trusts and investment trusts comprise two-thirds of the 'other' category in table 3.5. || Derived from Table 2. ** Defined as (standard deviation $\times 100$) \div expected return. †† Defined as (expected return in current wealth range - expected return in previous wealth range) \div (standard deviation in current wealth range - standard deviation in previous wealth range). Since for wealth levels below £50, the entire sum is held in IBAs, we have for this wealth range, an expected return of 5.61% and a standard deviation of return of 2.93%. ‡‡ Defined as the reciprocal of row 9. §§ Calculated from (6). || Calculated from (10). *** Calculated from (11) $\times 100$. ††† Calculated from (14). §§§ Calculated as the difference between the Expected return (row 6) and the Put premium per unit of wealth (row 15).

under 6% for the richest investors.¹² But as a result of the privatisations in the 1980s, even the poorest members of society hold some shares, somewhere between 13 and 19% of total holdings. Richer investors hold between 30 and 65% of total holdings in shares.

Rows 6–10 of Table 3 show some of the risk-return characteristics of the asset portfolios in different wealth ranges. These characteristics are based on forecasts for the returns and covariances on IBAs, bonds and shares for 1992 from the models given in (20)–(23); the forecasts are given at the bottom of Table 2.¹³ The expected gross return on IBAs, bonds and shares for 1992 were 5.61%, 8.24% and 24.47% respectively.¹⁴ Standard deviations of the forecast returns were 2.93%, 8.68% and 19.21% respectively, while correlations between IBAs and bonds, IBAs and shares, and bonds and shares were –14.95%, –0.10% and 26.58% respectively. These low correlations will turn out to be very useful for portfolio diversification purposes.

As might be anticipated, given the asset allocation in different wealth ranges, both the expected return on assets and their risk (as measured by portfolio standard deviation) increase with the level of wealth. The poorest investors expected a return on their portfolios of 7.99% by taking on a risk level of 3.50%, while the wealthiest investors expected a return of 17.96% but at the cost of taking on a risk level of 12.60% (rows 6 and 7). The coefficient of variation (row 8) shows that risk per unit of return increases with the level of wealth: rich investors have portfolios exhibiting more risk per unit of return generated than do poor investors. This is confirmed by rows 9 and 10. Row 9 shows the marginal benefit of risk, in other words, the additional return generated by an additional unit of risk as one moves into a higher wealth range. Each percentage point increase in risk generates an additional 4.17 percentage points of return for poor investors, but only 1.00 percentage point for rich investors. Another way of looking at this is via the marginal cost of return (row 10), which is the inverse of the marginal benefit of risk: each percentage point

¹² However, columns 4 and 5 also show that the ratio of bonds to shares is fairly constant across wealth ranges (ranging from 8% when wealth is £252 to 12% when wealth is £25,900). This is to be expected since Table 2 shows that the covariances between the returns on IBAs and those on bonds and shares are very low. This means that the conditions for two-fund separation almost hold. This would imply that the optimal portfolio can be chosen along the capital market line as a combination of IBAs and a common portfolio of risky assets (about 10% in bonds and 90% in shares).

¹³ We assume that investors have a one-year investment horizon: they structure their portfolios between April 1991 and March 1992 based on forecasts of returns and risks for the end of 1992.

¹⁴ We ignore the effects of transactions costs and taxes in the following analysis. This is unlikely to lead to significant distortions in our results. Transactions costs (commissions and spreads) are paid only when shares and bonds are bought or sold, and most personal sector investors are unlikely to be active investors. Also transactions costs are negligible for transactions between IBAs or for new share and bond issues such as those involved with privatisations. Banks *et al.* (1994, table 3.14) show that the main changes in the composition of assets between 1987–8 and 1991–2 were switches between different types of IBAs and share purchases via privatisations, neither of which involve significant transactions costs. Income tax is payable on investment income (at the basic rate of 25% on taxable income up to £23,700 in 1991–2 and at the higher rate of 40% on taxable income above this sum) and capital gains tax is payable on realised capital gains on shares (at the same rate as the highest rate of income tax payable by an individual, but with the first £5,500 exempt from any capital gains tax in 1991–2, and with gilts free from capital gains tax altogether). Inland Revenue statistics show that of around 26 m taxpayers in the United Kingdom, only 2.4 m (or 9%) pay higher rate tax and only 90,000 (0.35%) pay capital gains tax.

increase in return costs an additional 0.24 percentage points of risk for poor investors; but 1.00 percentage point for rich investors.

In Table 4, we present evidence on the efficiency of the portfolios held in the six wealth ranges discussed in Table 3. The minimum-variance portfolios (see (1)) are given in the columns labelled $\min \sigma^2$, while the maximum-expected return portfolios (see (3)) are given in the columns labelled $\max \mu$. Row 4 shows, for each wealth range, the additional expected return that can be achieved by moving from the actual to the maximum-expected return portfolio: expected returns can be improved by up to 3.4%. Row 5 shows, for each wealth range, the reduction in standard deviation that can be achieved by moving from the actual to the minimum-variance portfolio: risk can be reduced by between 0.2% and 5.1%. These differences appear to be quite small, and row 6 shows that they are not statistically significant. We can therefore conclude that the actual portfolios held by UK investors are not statistically distinguishable from efficient portfolios if investors form their expectations using the forecasting models given in (20)–(23).

Investors in all wealth ranges could however improve marginally the actual efficiency of their portfolios by slightly reducing their holdings of IBAs and shares and increasing the weighting in bonds. Table 1 showed that bonds appeared to be an inefficient asset with an average return of 6.43% and standard deviation of 14.23%, being dominated by IBAs with an average return of 7.81% and standard deviation of 2.53%. But we have seen that once we have taken into account the time series properties of the assets, the unanticipated component of the return on bonds has a negative correlation with that of IBAs (-14.95%) and a small positive correlation with that of shares (26.58%) and hence is an ideal asset to include in the portfolio for the purpose of diversification. We also showed that bonds are a particularly useful asset for timing the five-year interest rate cycle in the United Kingdom.

Since the actual portfolios are close to being efficient, we can use the actual portfolio allocation to derive point estimates of the CRRA in each wealth range. This is done in rows 11–13 of Table 3. Row 11 shows the marginal rate of transformation between expected return and standard deviation for the actual portfolio in each wealth range. We know that an optimal portfolio must be an efficient portfolio and that an optimal portfolio satisfies $MRS = MRT$ and so we can use our estimates of the MRT to calculate the implied coefficients of relative risk aversion in row 12 (using (6), (9) and (10)). The CRRAs range from 47.60 for poor investors to 7.88 for rich investors, so, as expected, rich investors are considerably less risk averse than poor investors. However, rich investors take on substantially more risk than poor investors and the risk premium that investors would be prepared to pay (as a percentage of their total wealth) to avoid risk altogether depends on both their attitude to risk and the degree of risk taken on. The estimated risk premium in each wealth range is listed in row 13 (see (11)). The poorest investors are prepared to pay up to 2.91% of their wealth while the richest investors are prepared to pay up to 6.25% of their wealth to avoid risk. With the exception of a move from the first to the second wealth ranges, the risk premium rises with the level of risk

Table 4
Minimum-Variance and Maximum-Expected Return Portfolios in Different Financial Wealth Ranges

Financial wealth ranges (£)											
50-454			455-3,949			3,500-7,904			7,905-14,999		
Min σ^2	Max μ		Min σ^2	Max μ		Min σ^2	Max μ		Min σ^2	Max μ	
Asset allocation (%)											
1. IBAs	76.34	74.98	72.43	71.58	69.80	69.11	59.65	59.36	47.93	47.86	25.76
2. Bonds	12.83	12.76	12.63	12.58	12.49	12.45	11.95	11.93	11.33	11.32	10.15
3. Shares	10.83	12.26	14.94	15.84	17.72	18.44	28.40	28.70	40.74	40.82	64.16
Portfolio characteristics (%)											
4. Expected return	7.99	8.26	8.76	8.93	9.28	9.42	11.28	11.34	13.59	13.60	17.98
(% increase)	—	(3.4)	—	(1.9)	—	(1.5)	—	(0.1)	—	(0.0)	(0.0)
5. Standard deviation	3.32	3.50	3.86	3.99	4.27	4.38	6.03	6.08	8.24	8.25	12.60
(% reduction)	(5.1)	—	(3.3)	—	(2.5)	—	(0.8)	—	(0.1)	—	(0.2)
6. F-statistic*	1.33	0.84	0.82	0.47	0.63	0.35	0.20	0.11	0.04	0.02	0.02

* Test of the significance of the difference between the efficient portfolio and the actual portfolio, calculated from (13) and distributed as F with 3 and 42 D.F. under the null of no significant difference; critical value for $F_{0.05}(3,42) = 2.83$.

taken on. We can conclude from this that, although the degree of relative risk aversion falls as individuals become richer, richer individuals take on relatively more risk and this has the effect of raising their risk premium.

We can use these estimates of the risk premia to determine whether investors are likely to take out portfolio insurance.¹⁵ The risk premia indicate the maximum percentage of wealth that investors are prepared to pay to avoid risk altogether. Investors would be prepared to pay more than this to avoid downside risk but leave themselves exposed to any upside potential in their asset portfolios. Row 14 of Table 3 shows the value of the put premiums that investors with mid-range wealth levels would have to pay to avoid downside risk; row 15 shows the values of these premiums as a proportion of total wealth. An individual with financial assets of £252 would have to pay only £0.20 or 0.08% of his/her wealth to avoid the downside risk associated with his/her asset allocation: this is clearly because the bulk of the portfolio (86.5%) is in capital-certain assets. On the other hand, an investor with financial assets of £100,000 would have to pay as much as £2,633.19 or 2.63% of his/her wealth to avoid downside risk: a much higher proportion of total wealth on account of the much greater investment (64.7%) in the riskiest asset category, shares. Clearly in each wealth range, the cost of portfolio insurance is less than the maximum that investors would be willing to pay to avoid risk. Therefore, we would expect the demand for portfolio insurance to be high in all wealth ranges. Row 16 shows the portfolio-insured returns in each wealth range (i.e. the expected return (row 6) subtracting the cost of the put premium (row 15)). Despite the finding that the proportionate cost of the put premium increases with wealth, the expected returns increase at a faster rate, so that the portfolio-insured returns also increase monotonically with wealth.

The fact that investors in all wealth ranges, including relatively unsophisticated investors, hold mean-variance efficient portfolios is somewhat surprising, but this outcome is sensitive to the assumption made about expected returns and risks. We have assumed that investors forecast returns and risks using an ARDL-ARCH model based on the time series properties of the underlying assets. If, on the other hand, we had assumed that investors were less sophisticated than this and merely used the average returns and covariances from the historical sample (see Table 1), we would have got a very different set of results. In particular we would have found that, because bonds appear to be an inefficient asset, the actual portfolios were not efficient portfolios in any wealth range and that efficient portfolios would involve short holdings of bonds. In other words, investors would (if they were able to) issue bonds and invest the proceeds in additional holdings of IBAs and shares. For example, in the case of poor investors, the minimum-variance portfolio would have the following asset allocation: 110.4% in IBAs, -19.6% in bonds, and 9.2% in shares; while in the case of the richest investors, the corresponding portfolio would be: 138.7% in IBAs, -88.2% in bonds and 49.5% in shares. Apart from the fact that these asset allocations are not in practice feasible, they

¹⁵ The risk-free interest rate used in this calculation is 5.61%, the same as the expected return on IBAs.

indicate the vital importance of assessing portfolio efficiency within the context of a sensible forecasting model for returns and risks: we are conducting a joint test of efficiency and the forecasting model.

Given our estimates for the CRRAs across different wealth ranges, how do these compare with those found in other studies? Most previous studies have been conducted using US data. For example Blume and Friend (1975), using the Federal Reserve survey on the financial characteristics of consumers, found that the asset allocation was constant across different wealth ranges and concluded that this was consistent with relative risk aversion being constant at different wealth levels. In contrast, Cohn *et al.* (1975), using survey data collected by a stockbroker from its clients, found that relative risk aversion fell as wealth increased. This is the same result that we find. But what about the size of the CRRA? Arrow (1970) argued that on theoretical grounds the CRRA should be around unity (which is consistent with utility being logarithmic). Tobin and Dolde (1971), in a study of life cycle behaviour with borrowing constraints, found that a value of 1.5 fitted observed life cycle savings patterns. Friend and Blume (1975), using data on individual portfolio holdings, estimated the CRRA to be around two. Mehra and Prescott (1985) used these low levels for the CRRA to demonstrate the existence of an equity premium puzzle, namely that with CRRAs of between one and two, the excess return on equities above the risk-free interest rate should be at most 0.35% rather than the 6% that is conventionally observed in advanced financial systems. Mehra and Prescott concede that with a sufficiently large CRRA virtually any sized equity premium could be obtained. Our study finds CRRAs ranging from 7.88 to 47.60 across different wealth ranges, with a weighted-average value of 35.04. More recent US studies have also estimated larger values for the CRRA than previously found. For example Mankiw and Zeldes (1991) using the same data as Mehra and Prescott estimate the CRRA to be 26.3.

IV. CONCLUSION

Investors in the United Kingdom do not hold identical portfolios of financial assets.¹⁶ Rich investors hold much more equity in their portfolios and thereby take on much more risk than poor investors. However, we have shown in this paper that investors in all wealth ranges hold portfolios with asset allocations that are mean-variance efficient if they take into account the time-series properties of asset returns and determine the expected returns and risks on assets accordingly and, in turn, select their asset holdings on the basis of these expected returns and risks. Of particular interest is the holding of bonds which by themselves appear to be an inefficient asset: our analysis shows that bonds have a small but important role in helping to diversify risks in UK investors' portfolios.

Since actual portfolios seem to lie near to the efficient frontier, we were able to use the risk-return characteristics of the actual portfolios to infer both the

¹⁶ Although proportionately they hold fairly similar portfolios of risky financial assets.

coefficient of relative risk aversion and risk premium in each wealth range. We estimated much higher degrees of risk aversion than most previous studies have found, but as expected, poor investors are considerably more risk averse than rich investors, but they take on much less risk. As a consequence poor investors are prepared to pay nearly 3 % of their wealth to avoid taking any risk, whereas rich investors are prepared to pay 6.25 % of their wealth. Clearly investors would be prepared to pay even more than this to insure their portfolios, that is, to avoid downside risk but preserve upside potential. We found that very poor investors had to pay only 0.08 % of their wealth for portfolio insurance whereas very rich investors had to pay 2.63 %. So in each wealth range, there is evidence that the demand for portfolio insurance is strong. The fact that portfolio insurance is not widely available to private client investors is indicative of a supply-side failure rather than a demand-side failure. Finally, we note that the very high degree of risk aversion that we have estimated indicates that investors are reluctant to hold equities *unless* they are compensated with a sufficiently high risk premium.

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