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Andrew Clare, James Seaton, Peter N. Smith and Stephen Thomas

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The Pensions Institute
Cass Business School
City University London
106 Bunhill Row
London EC1Y 8TZ
UNITED KINGDOM
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Sequencing, Perfect Withdrawal Rates and Trend Following Investing Strategies: Making the Decumulation Experience more Predictable

Andrew Clare*, James Seaton*, Peter N. Smith† and Stephen Thomas*

*Cass Business School, City University London

†University of York.

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Abstract

We use the concept of Perfect Withdrawal Rates to compare decumulation investment strategies for a 100% equity portfolio with and without a trend-following overlay to show that the important concept of Sequence risk can be ameliorated by a technique which successfully smooths returns: two strategies with the same mean, variance, Sharpe ratios and cumulated returns will offer very different decumulation experiences if the sequence (or order) of returns differ. Using US data from 1872-2014 we show that 20-year (constant) withdrawal rates can vary between 4% and 12% p.a. but are raised considerably by smoothing returns using simple trend-following. The distribution of Perfect Withdrawal Rates shifts to the right, reducing left-tail possibilities. We show that knowing the Cyclically Adjusted Price-Earnings ratio (CAPE ratio) for the S&P 500 index at the beginning of the decumulation process is useful for predicting the sustainable withdrawal rate for that period.

Keywords: Decumulation; Sequence Risk; Perfect Withdrawal Rates; Trend-Following; CAPE (Cyclically Adjusted Price Earnings' ratio)

JEL Classification: G10, 11, 22.

1. Introduction

In the important book 'The Retirement Plan Solution' (Ezra et al, 2009), there is a careful dissection of 'The Reinvention of Defined Contribution' in pension savings and decumulation, a topic of growing importance in many parts of the world as companies retreat from defined benefit (DB) schemes and encourage investment and withdrawal decisions by individuals. The book devotes a mere dozen pages (out of 200) to the topic of investing (though there are a few more pages on returns' history), and indeed it is relegated to the authors' so-called ''3rd dial'' after Personal Spending' (i.e. withdrawal rates) and Longevity Protection. Yet we will argue that it is Asset Allocation and Investment Strategy which is the glue at the centre of all decumulation experiences. Similarly, economists increasingly focus on ever more creative decumulation strategies; for instance, combining deferred annuities, state benefits, guaranteed annuity-type income along with flexible income sourced from differing degrees of risky investment. But researchers are generally silent on the type of investment strategy for a successful decumulation experience with risky assets, (e.g. Merton, 2014), preferring to create a risk-free benchmark of index-linked bonds (Sexaeur, Peskin and Cassidy, 2012), or utilising combinations of bonds and equities often in target date or glidepath commercial solutions which offer period specific conclusions and ignore the diversification lessons of undergraduate finance[1].

In this paper we use Perfect Withdrawal Rates (PWR) (Suarez et al, 2015) to investigate the decumulation experience since 1872 for a US investor with a 20-year investing horizon and a 100% equity portfolio: we show how applying a simple (absolute) trend following investing strategy leads to a far better range of withdrawal outcomes relative to a long-only equity portfolio. The main findings can be extended to different decumulation periods and differing asset allocation strategies and carry a simple but powerful message: seeking investment strategies with lower *sequencing risk* facilitates higher withdrawal rates and hence a better decumulation experience. In general using trend-following techniques to reduce maximum drawdown leads to such improved outcomes, whatever the universe of assets used for portfolio construction.

Alternatively derivative strategies may be used to control tail risk and reduce sequencing risk: Milevsky and Posner (2014) use traded equity options to create a 'longevity extension overlay' by selling calls and buying puts to give a zero-cost collar solution; the efficacy of such a strategy relative to shifting asset allocations is an important question as yet not investigated. Strub (2013) also examines alternative methods of reducing tail risk, including cash and options based tail hedging strategies. The cash based methods are shown to significantly increase risk adjusted returns for the S&P500 and reduce drawdowns, while the options-based strategy suffers a decrease in performance from 2003 on due to the increase in the cost of puts with respect to calls after that date. Basically options can be expensive whereas switching into cash at various times following a trend-following or similar rule is effectively 'free'.

[1] ''Paul Resnick of Finametrica refers to the portfolio planning component as '…perhaps the last great frontier for real advisers' (Resnick, 2015, http://www.riskprofiling.com/blog/October-2015/sequenceriskbeatup).''

So we wish to investigate two fundamental aspects of the decumulation experience (though much of what follows applies to the accumulation phase as well). Firstly, decisions regarding asset allocation are central to the concept of *de-risking* in retirement: indeed the latter is defined in terms of the former, i.e. hold less equities in later years. This is undoubtedly accepted wisdom both academically via life-cycle models and in practice in the form of 'Target Date', 'Lifestyle', or 'Glide-Path' funds (see Ezra et al, 2009, p. xvii, who report that for the 12-month period ending 31st December 2008, the average return for the largest 3 target date US fund families for 2010 (the nearest to retirement) was -24%). Whilst less bad than overall equity markets which returned -43%, this is hardly comforting to an imminent retiree).

However this preference for de-risking is being challenged from a number of directions: the academic financial planning literature of around 20 years ago had discovered that de-risking by abandoning equities could lead to far worse outcomes. As Cooley et al (1998), say:

> *'…… investors who expect long payout periods should choose an asset allocation that is at least 50% common stock and a lower withdrawal rate. Conversely, a higher withdrawal rate appears to be sustainable for shorter payout periods, such as 15 or 20 years, provided the portfolio has a substantial percentage of stocks. Investors who plan to inflation adjust withdrawals should choose lower withdrawal rates and invest at least 50% of the portfolio in stocks. Finally, the lower withdrawal rates of 3% and 4% recommended by some analysts appear to be excessively conservative for portfolios with at least 50% stock, unless the investor wishes to leave a substantial portion of the initial retirement portfolio to his/her heirs.'*

Blanchett et al (2012) show similar findings for 20 countries using 113 years of data. More recently, and very much from a practical financial planning/advisory perspective, Finametrica (2015) and others point out that with retirees living 30+ years this de-risking may prove disastrous in terms of lost wealth and consequently lower drawdown rates achieved; in short, investors would be missing out on the upswings in equity markets. How to manage the exposure to risk in equity (or other) asset markets is the next key question and one might point to the challenge from several directions regarding the allegedly positive relation between risk and return. The Smart Beta experience points to low volatility stocks and low beta stocks enjoying high returns, and this dates back to the 1970's. Falkenstein (2012) draws these findings together for a wide range of asset classes. We offer even more persuasive evidence on this point.

Secondly, it is reasonable to say that most of the literature, possibly inspired by Bengen's (1994) '4% rule' focusses on the relation between withdrawal rates/drawdown strategies and longevity of funds versus life expectancy, given some (usually arbitrary) investment return series. It is then a simple matter to find what sort of withdrawal rates are consistent with different returns' experience and life expectancy/planning horizons, 'the ultimate great unknown', for an individual at least! The literature then evolves via a multiplicity of exercises in changing key parameters, generally suggesting that sustainable withdrawal rates lie between 3.5% and 4.5% (of initial balance) for a country such as the US, for example Finke et al, (2013). The empirical analysis focussing on investment strategies usually involves either a historical 'real life' investing period, with the appropriate ex-post sustainable withdrawal experience, or the use of Monte Carlo methods to draw a large number of random investment returns from say, US capital market history, the construction of sustainable withdrawal rates for each drawing for a specified planning period, and, given a large number of different draws, a probability distribution for the withdrawal rate itself. In this case it is possible to make probability statements regarding the latter concept and associated statistics regarding failure rates. Gerrard et al (2004) and Milevsky and Young (2004) consider the optimal choice of withdrawal rate (i.e. consumption) and financial investment decision within a world of stylised behaviour of asset returns.

Variations on this theme involve introducing a bequest motive or allowing withdrawal rates to 'adapt' each year as new information becomes available such as investment return experience or life expectancy (i.e. planning period) changes. Such adaptive rules can take on a myriad of complex rules (see Spitzer et al, 2008 and Mitchell, 2009).

In this paper we want to shift the focus back to investment strategy and employ the concepts of Perfect Withdrawal Rate and Sequence Risk to allow comparison of competing strategies in a systematic and relevant way:

- The *Perfect Withdrawal Rate* (PWR) is the withdrawal rate that effectively exhausts wealth at death (or at the end of a fixed, known period) if one had *perfect foresight* of all returns over that period. Note that a similar concept has been put forward by Blanchett et al (2012).

- *Sequence Risk*, (sometimes called Sequential Risk), is the risk of experiencing bad investment outcomes at the *wrong time*: typically the *wrong time* is towards the *end* of the accumulation phase and at the *beginning* of the decumulation period, i.e. it is symmetric around the time of retirement.

This concept of sequencing risk is of particular interest to the decumulation industry (e.g., see Okusanya, 2015, or Chiappinelli and Thirukkonda, 2015); these papers basically point out the importance of 'path dependency' of investment returns (i.e. the order in which they occur). This concept is at least as important to the retirement journey as the total return earned by the investment, yet portfolio construction, both academic and practical, has typically focussed on total return and volatility, possibly constructing Sharpe or similar performance statistics as a way of comparing strategies. Typically studies show, using simple arithmetical examples, that higher withdrawal rates are always possible when the worst investment years occur later in the decumulation period (for *any given set* of returns).The natural reaction to Sequence Risk has been to de-risk a portfolio as one approaches 'retirement' (from both an accumulation and decumulation experience), along the lines of 'glidepath' or similar strategies.

Here our aim is to use a long series of monthly total US equity returns (back to 1872) to show the relation between Sequence Risk and the achievable PWR for a simple 100% equity strategy plus a basic trend-following alternative, while advocating the use of maximum drawdown as a performance statistic for selecting appropriate investment strategies. This will generally be positively related to Sequence Risk in practice, though not so in theory: we suggest that while the literature acknowledges that standard deviation is not relevant to the concept of Sequence Risk, a low maximum loss investing experience is a *necessary* (though *not* sufficient) condition for low Sequence Risk.

In summary, we use PWR as a measure to compare investment strategies and show that strategies with low maximum drawdown have superior sequencing experiences and higher PWR. For example, two strategies may have same means, variance (and Sharpe) but very different sequences of returns — the one with the lower maximum drawdowns will *usually* generate higher total wealth after a fixed period: in turn this may be described as having a higher PWR.

So what sort of strategies will be associated with lower sequence risk? We introduce a simple *trend following* strategy in this paper for US equities, though Clare et al (2016) use it in a multi asset context elsewhere, and the results are very much the same. Such strategies smooth returns and lead to higher PWRs, along with reduced volatility, but most importantly there is always a greatly reduced *Maximum Drawdown* experience, which in practice will usually be associated with a higher withdrawal rate experience.

An additional question is whether equity market valuation is useful information for predicting the PWR journey at any point in time? In other words, does, say, a high cyclically-adjusted Shiller PE ratio suggest an overvalued market followed by equity price falls and a bad sequence of returns, leading to subsequent lower future PWRs? We find strong evidence for this form of market-timing, consistent with Blanchett et al (2015). Finally, how do we handle longevity risk? We conduct our analysis with a fixed planning period of 20 years to avoid unnecessary complication and allow us to focus on the investment process. The 20-year deferred annuity is our preferred longevity risk hedging tool (see Merton, 2014 and Sexauer et al, 2012). Laibson (2009) points out that cognitive function decline is well set in among over 50% of US adults by the time they are in their 80's and that there should be more help for people in making the right financial decisions, perhaps via enhanced automaticity: deferred annuities could help here. Our analysis can be extended to handle different longevity assumptions but at this stage will divert attention away from our main focus on investment strategy.

2. Background: Approaches to calculating Withdrawal Rates/Amounts (PWR/PWA)

The concept of PWR[2] (Suarez et al, 2015, see also Blanchett et al, 2012 for a similar approach involving perfect foresight of both investment returns and longevity) to create withdrawal strategies from retirement portfolios and is based not on heuristics and/or empirical testing but on analytics. The authors construct a probability distribution for the PWR and apply it sequentially, deriving a new measure of *sequencing risk* in the process. We use these ideas to show that a particular class of investment strategies (both simple and transparent) can offer superior (Perfect) Withdrawal Rates across virtually the whole range of returns' environments. Basically this smoothing of returns will lead to a better decumulation experience across virtually all investing timeframes.

Retirees (here we focus on decumulation though sequencing risk applies similarly to savers in the accumulation phase) want to use their funds to support as high as possible standard of living but without depleting their wealth so quickly that the years ahead become difficult to finance: this is called 'failure risk'. The mirror image of this is withdrawing 'too little' money and hence leaving 'excessively high' balances at the end of the planning period (or, indeed, lifespan): this is called

[2] When returns are log-normal, the PWR can actually be expressed as an analytic function, the Gamma distribution. In fact, the inverse of the PWR is often called the Stochastic Present Value (SPV) of $1 of income for life, which is another "term" that is used quite a bit is a relatively new way (Milevsky and Robinson, 2005). We are grateful to Moshe Milevsky for drawing our attention to this.

‘surplus risk’, and implies an unnecessarily restricted standard of living throughout the decumulation years.

Researchers have typically derived ‘rules’ which determine the withdrawal amounts in each period based on the retiree’s age, portfolio of assets, and preference for near-term consumption versus a later higher potential failure rate: of course, inflation, taxes, liquidity requirements, precautionary balances etc. also influence choices but the former trio are generally considered of primary importance in driving the appropriate withdrawal rate (e.g. Cooley et al, 1998, Blanchett et al, 2012). This literature is essentially heuristic and empirical in nature, varying withdrawal rates, investment portfolios, almost always between equities and bonds rather than multi-assets, (and hence the pattern of returns) and time-horizons for consumption (usually age-related).

The literature on optimal withdrawals in retirement can be traced back to Bengen (1994), where he presents the concept of “the 4% rule”: basically he shows that a 4% withdrawal rate from a retirement fund, adjusted for inflation, is ‘usually’ sustainable for ‘normal’ retirement periods. Cooley, Hubbard, & Walz (1998, 1999, 2003, and 2011) then confirmed this finding, with similar findings using overlapping samples of historical stock and bond returns.

A crucial distinguishing feature of these “first generation” papers is that they rely on a constant real withdrawal amount throughout the decumulation phase, with no ‘adaptive’ behaviour as circumstances change. A number of studies have introduced ‘adaptive’ rules: Guyton & Klinger (2006) manipulate the inflationary adjustment when return rates are too low, modifying the withdrawal amount, while Frank, Mitchell, & Blanchett (2011) use adjustment rules that depend on how much the rate of return deviates from the historical averages. Zolt (2013) similarly suggests curtailing the inflationary adjustment to the withdrawal amount in order to increase the portfolio’s survival rate where appropriate.

Basically these withdrawal rates ‘adapt’ to the changing circumstances.

Of course an important addition is to treat the planning horizon length as a stochastic variable (instead of fixed). The aim here (quite sensibly!), is to ensure that the funds in the retirement account “outlive” the retiree: Stout and Mitchell (2006) use mortality tables to make sure that the uncertain retirement period is considered, while Stout (2008) decreases the withdrawal amount whenever the account balance falls below a measure of the present value of the withdrawals yet to be made, (and increases it when the balance is above this measure). Mitchell (2011) similarly uses thresholds to initiate such adjustments.

A more theoretically coherent approach treats the selection of withdrawal amounts as a lifetime-utility maximization problem. Milevsky and Huang (2011) consider the total discounted value of the utility derived across the entire retirement period, where this length of retirement is a stochastic variable and the subjective discount rate is a given, while Williams and Finke (2011) use a similar model with more realistic portfolio allocations. Blanchett, Kowara, and Chen (2012) measure the relative efficiency of different withdrawal strategies by comparing the actual cash flows provided by each strategy to the flows that would have been feasible under perfect foresight.

3. The Perfect Withdrawal Amount (PWA)

The concept of PWA is introduced in a world of no taxes or inflation: annual withdrawals are made on the first day of each year and annual investment returns accrue on the last day of the year. For any given series of annual returns there is one and only one constant withdrawal amount that will leave the desired final balance on the account after *n* years (the planning horizon). The final balance could be a bequest or indeed zero. Suarez et al (2015) suggest that this is equivalent to finding the fixed-amount payment that will fully pay off a variable-rate loan after *n* years.

The basic relationship between account balances in consecutive periods is:

$$K_{i+1} = (K_i - w)\ (1+r_i) \qquad (1)$$

where, K_i is the balance at the beginning of year *i*, *w* is the yearly withdrawal amount, and r_i is the rate of return in year *i* in annual percent. Applying Eq. (1) chain-wise over the entire planning horizon (*n* years), we obtain the relation between the starting balance K_S (or K_1) and the ending balance K_E (or K_n):

$$K_E = (\{[(K_S - w)\ (1+r_1) - \mathrm{w}]\ (1+r_2) - \mathrm{w}\}\ (1+r_3) \dots - w)\ (1+r_n) \qquad (2)$$

And we solve equation (2) for *w* to get:

$$w = [K_S \prod_{i=1}^{n}(1+r_i) - K_E] / \sum_{i=1}^{n} \prod_{j=i}^{n}(1+r_j) \qquad (3)$$

Equation (3) provides the constant amount that will draw the account down to the desired final balance if the investment account provides, for example, a 5% return in the first year, 3% in the second year, minus 6% in the third year, etc., or any other particular sequence of annual returns. This figure is called the Perfect Withdrawal Amount (PWA).

Quite simply, if one knew in advance the sequence of returns that will come up in the planning horizon, one would compute the PWA, withdraw that amount each year, and reach the desired final balance exactly and just in time.

In this context there would be a fixed withdrawal as future rates are known. Numerous studies provide examples of a sequence of, say, 30 years of returns generated possibly with reference to an historical period or via Monte Carlo simulations, and offer the unique solution of the PWA: it involves withdrawing the same amount every year, giving the desired final balance with no variation in the income stream, no failure and no surplus. As we noted above, Blanchett, Kowara and Chen (2012) present a measure similar to PWA called Sustainable Spending Rate (SSR). Suarez et al (2015) point out that the PWA is a generalization of SSR, with SSR being the PWA when the starting balance is $1 and the desired ending balance is zero.

So every sequence of returns is characterised by a particular PWA value and hence the retirement withdrawal question is really a matter of "guessing" what the PWA will turn out to be (eventually) for each retiree's portfolio and objectives. So the problem now becomes how to estimate the probability distribution for PWAs from the probability distribution for the returns on the assets held in the retirement account.

Note that the analysis so far offers a number of useful insights into sequencing risk measurement: firstly, Equation (3) can be restated in a particularly useful way since the term $\prod_{i=1}^{n}(1+r_i)$ in the numerator is simply the cumulative return over the entire retirement period, (call it R_n).

The *denominator*, in turn, can be interpreted as a measure of sequencing risk:

$$\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_i) = (1+r_1)(1+r_2)(1+r_3)\ldots(1+r_n) + (1+r_2)(1+r_3)\ldots(1+r_n) + (1+r_3)(1+r_4)\ldots(1+r_n) + \ldots.$$

$$\ldots..+ (1+r_{n-1})(1+r_n) + (1+r_n) \quad (4)$$

The interpretation of this is straightforward: for any given set of returns equation (4) is *smaller* if the *larger* returns occur *early* in the retirement period and *lower* rates occur at the *end*. This is because the later rates appear more often in the expression. Suarez et al (2015) suggest the use of the reciprocal of equation 4 to capture the effect of sequencing: so let $S_n = 1/\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_i)$. This rises as the sequence becomes more favourable, and even though one set of returns appearing in 2 different orders will have the same total return (i.e. R_n with different S_n values), so the PWA rates will be different.

We emphasised earlier that whereas in most finance contexts *total* return is the key variable, in both accumulation and decumulation the *order* of returns also matters. An example will make this clearer: suppose we have 3 sets of returns in Table 1; clearly the mean, volatility and Sharpe (and indeed Maximum Drawdown) are the same but the sequences differ as is evidenced by the different values of SeqRisk, which is $1/S_n$ in the notation above, with lower values of this metric associated with higher PWRs.

This allows a useful,highly intuitive simplification of Equation (3) in the form of Equation (5), such that the PWA depends positively on total return, R_n, the starting amount, K_s,and the measure of sequence risk, S_n, and negatively on the final amount, K_e:

$$w = (R_n K_s - K_E)\, S_n \quad (5)$$

This representation emphasises that it is not simply the total returns that matter but the order in which the component returns occur: if 'good' returns come early in the sequence then the PWA will be larger than if they occur later.

Other studies have tried to account for sequencing risk (Frank and Blanchett, 2010; Frank, Mitchell, & Blanchett, 2011; Pfau, 2014), often developing proxy variables to measure the correction required due to the sequencing issue. Suarez et al (2015) suggest that equation (5) comes directly from the simplest, most natural interpretation of the problem—i.e. S_n is not a proxy but a measure of what they term '*orientation'* (return rates going up, going down, up a little then down a lot, etc.), and this is the crucial concept for assessing sequencing.

Finally we should note that *w* (the PWA) can be transformed into a withdrawal *rate* by dividing equation (5) by K_s

$w/K_s = R_n S_n - S_n(K_E / K_S)$ (6)

Note that if we have a bequest motive then we simply now need to know the fraction of the initial sum to be bequeathed to calculate the PWR: as Suarez et al (2015) point out (p. 6), in contrast to simplistic financial planning solutions, to set aside a bequest sum beforehand is not necessary as these funds can also generate returns and be used for consumption. Setting aside sums is simply a special case of the above general form, equation (6). We show later that similarly setting aside cash for, say 3 years of consumption, along with a subsequent withdrawal strategy, can be handled easily within the PWR framework.

4. Constructing a Probability Distribution for PWRs for an all equity portfolio: the S&P500, 1872-2014

Much of the planning literature aims to make probability statements regarding the chance of running out of funds given any particular withdrawal rate and planning horizon. So we now create a probability distribution for the PWR/PWA using a long-run of monthly equity returns extracted from the Shiller website[3]. This all-equity portfolio may be considered rather unlikely as an investment choice in practice but it serves to illustrate our key points regarding choice of investment strategy and adding, say, bonds to provide 'de-risking'/diversification' simply reinforces our findings. A surprising result may well be that a 100% equity portfolio is not such a bad idea *providing* one overlays it with trend-following.

First of all, assuming indeed that we have perfect foresight, what would the PWR look like through time assuming a 20 year decumulation period? This is shown in Figure 1 where, as throughout this paper, we assume a zero bequest intention. We focus here on the blue line which shows the PWR generally varying between 8 and 12% but occasionally straying as low as 4% in 1930 and as high as 16% in 1950! Indeed for several years around 1980-1990 it is well above 12%. This suggests two things for us: there is a huge variation in the ability to withdraw cash from a retirement pot depending on the accident of one's birth date. Secondly, *all* of the rates are above 4%, giving very long term succour to Bengen's (1994) 4% rule (at least over 20 year periods). The reason for the brown line superimposed on Figure 1 will become apparent when we address the choice of investment strategies.

What about *real* PWRs? In Figure 2 we use the same data to generate a history of real PWRs. Unsurprisingly the basic inferences are similar: the lowest PWR is just above 4% (the blue line), with peaks in 1920 and 1950 at around 14%.The main message is once again the huge variation in rates over time: the accidents of birth date are crucial to the ability to consume in retirement. Note once again that these PWRs are *constant* withdrawal rates over the 20-year period as perfect foresight is assumed.

Now we know what the history of PWRs would look like with perfect foresight for the 100% S&P500 portfolio, we can construct a probability distribution for this particular investment strategy: we

[3] http://www.econ.yale.edu/~shiller/data.htm

begin with 100% invested in this equity portfolio (while realising that taking more diversified portfolios will lead to a less dispersed distribution). We take the real returns on the S&P500 for the period 1872 through 2014 and use a Monte Carlo engine to draw 20 years of 12 monthly values at random from this set (240 as a sequence), one at a time with replacement sampling. These were then interpreted as the monthly returns in a 20-year planning horizon, in the same order as they were drawn.

This process is repeated 20,000 times and we computed the cumulative return (R_n) and sequencing factor (S_n) for each series of returns obtained. This provided us with 20,000 (R_n, S_n) pairs; Figure 3 below presents the frequency distribution of the PWA formula (equation (5)) evaluated at each of these 20,000 (R_n, S_n) pairs, using $1 million as starting balance and with $0 as desired ending balance. Table 2 contains the same information but with cut-off points for various probabilities listed. We focus solely on *real* PWRs from here on as clearly it is the inflation adjusted experience which matters.

Figure 3 and Table 2 are broadly comparable with Figures 3 and 4 in Suarez et al (2015), albeit with real PWRs and a 20 year horizon. We can interpret the Figures as follows: there is a 1% chance of a real PWR of 2.95% or less; a 10% chance of a real PWR of 5.01% or less; and a 50% chance of a real PWR of 8.64% or more. Given that the final amount is $0, any overshoot in withdrawing results in ruin: hence we could say that 50% of the Monte Carlo withdrawal runs produced real PWRs less than 8.64% so that failure risk for withdrawing over 8.64% is indeed 50%. Similarly failure risk for withdrawing over 5% p.a. is about 10% (i.e. 10% of runs produced real PWRs of over 5.01%).

Of course we could introduce a bequest motive which simply means a positive final balance target: Suarez et al (2015) show that this moves the PWR distribution to the right (implying a higher risk of failing to meet the bequest).The inverse of failure risk is surplus risk and this can be estimated by inverting the roles of PWR and the end balance. For a given end balance and PWR we can say that a surplus accrues a certain percentage of time reflecting the occurrence of PWRs greater than that chosen. In fact, in the Suarez et al (2015, p. 9) example, with a nominal perfect withdrawal amount of $43,000 p.a. (i.e. a 4.3% withdrawal rule in their case), 74% of the Monte Carlo runs ended up with more money than they began with; in 58% of the runs the final balance was double the starting balance; and it would have a 12% probability of ending up with 10 times the initial sum!

5. Sequential Application of the PWA/PWR Formula

Of course in practice individuals do not maintain the constant withdrawal amount and update their behaviour in the face on new information, so-called *adaptive* withdrawals: so it is assumed that we make a withdrawal at the beginning of retirement year 1 and live off this money for the next twelve months until, at the very end, the sum accrued during the previous year is actually credited to our account. Then the PWA/PWR distribution needs to be recalculated using equation (5), with the balance actually showing as the new starting balance and shortening the time horizon by one period. The shortening of the horizon is attained by substituting in a new set of 20,000 (R_n, S_n) pairs as before but this time drawing return rate sequences that are shorter by one year. From this new distribution we choose the withdrawal amount for year 2; we could change the risk-tolerance profile as necessary year by year. And this process is simply repeated every year.

So the *process* by which the withdrawal amount is selected is *always the same*, but not the withdrawal amount itself. PWA incorporates all new information into the set of data available for the next analysis, and this updating will create adjustment pressures that, in all likelihood, will end up with some modification to the withdrawal amount/rate. Of course, one can change the planning horizon to take on new information, say, regarding life expectancy. Hence, adjusting equation (6) for no bequests gives equation (7), which can be evaluated for various planning horizons, *n*, as desired:

$$w/K_s = R_n S_n \tag{7}$$

This calculation repeated for different planning horizons effectively creates a new Figure 3 and Table 2 for each one, allowing the advisor to discuss appropriately updated PWAs year by year as new investment experiences take place and planning horizons (possibly) shift. A bad investment year will lead to an increase in the failure rate at the pre-existing PWR and hence possible recalibration of a desired PWR if the probability of failure at the previous rate is now unacceptable (to the client). This is the statistically correct approach since it recalculates the probability distribution of the PWA each year using all new information.

This is a simple approach, much more so than some of the adaptive rules found in the previous literature (Bernard, 2011; Blanchett & Frank, 2009; Guyton, 2004; Mitchell, 2011; Pye, 2000; Robinson, 2007; Stout & Mitchell, 2006). For example, in Guyton & Klinger (2006) we find that,

> *"withdrawals are to increase from year to year to make up for inflation, except that there is no increase after a year where the portfolio's total return is negative and when that year's withdrawal rate would be greater than the initial withdrawal rate"* (p. 5), or *"when a current year's withdrawal rate has risen more than 20 percent above the initial withdrawal rate, the current year's withdrawal is reduced by 10 percent; this rule expires 15 years before the maximum age to which the retiree wishes to plan"* (p. 7).

Of course if a bequest is required we return to equation (6):

$$w/K_S = R_n S_n - S_n(K_E/K_S) \tag{6}$$

This now has a simple interpretation whereby the investment performance to date is reflected in the updated K_s and the withdrawal rate with no bequest (i.e. R_n, S_n) less the sequence measure (S_n) times the final (bequest) amount divided by the (updated) initial wealth: the larger the intended bequest, the lower the PWR.

A final introductory remark regarding PWR: when we discuss confidence ranges for PWR, since the process is updated (i.e. is adaptive), we now mean that there is a certain probability *not* that we will run out of funds but that one will *not have to reduce* the PWR in future to achieve a given bequest target, which may of course be zero.

6. Trend Following, Maximum Drawdown and Sequence Risk

What influences sequence risk? Clearly from equations (6) and (7) the sequence risk measure, S_n, influences the PWR directly: equation (6) shows that the more favourable sequencing, S_n, gives a higher PWR. More favourable sequencing is associated with relatively good returns early in the planning period (see the Aviva and Standard Life references earlier): in particular, the avoidance of heavy losses in the early phases of decumulation is crucial for high PWAs.

But if asset returns are fairly unpredictable, how can we secure a favourable S_n? One very straightforward solution is to acknowledge that while the *order* of returns cannot be predicted, it may be possible to produce investment strategies which offer substantially reduced volatility of returns, or, more precisely, much reduced drawdown in returns since reduced volatility in itself is not enough to secure a high PWA. Indeed, whilst there is no precise mathematical relationship between maximum drawdown and sequence risk we suggest that a low maximum drawdown should be associated in practice with more favourable sequence outcomes.

While diversifying across asset classes should nudge portfolio returns in the desired direction with improved risk-return, and possibly lower maximum loss experiences, there is an even more powerful technique which can be applied to individual asset classes with dramatic effect: this is simple *'trend-following'*, whereby one invests in an asset when it is in an uptrend (defined as a current value above some measure of recent past average), and switched into cash when the current value is below such an average. This has a long history of application (Hurst et al, 2012) and is explored in a series of papers by Clare et al (2013, 2014, 2016) and Faber (2007) and more recently by Asness et al (2015).

Our basic hypothesis is that applying a simple trend-following overlay to any series of asset returns dampens volatility, typically maintains or increases returns over longer periods, and substantially reduces maximum drawdown for that series: this is directly related to reduced sequencing risk. In this paper our empirical investment strategy involves 100% US equity investment in the form of the S&P 500 (e.g. see Suarez et al (2015) for a similar equity portfolio example over a far shorter time period). We replace this with a simple-trend following adjustment to the 100% equity strategy based on comparing a month-end index value to a simple average of the previous 10 months' end-months' values. The results are not sensitive to the choice of trend definition (see Clare et al, 2016): as an illustration Table 3 compares the performance of the S&P 500 with and without trend-following. Clearly, average return is noticeably higher with trend following (200 bps p.a.) but even more important from our point of view is the one-third reduction in volatility and the halving of maximum drawdown! This pattern is repeated for most asset classes and is described in detail by Faber (2007) and Clare et al (2016).

So how would the descriptive statistics above impact the distribution of PWRs if we now have a portfolio which is 100% S&P500 with a simple trend-following overlay? We present the comparison in Figure 4 which now sets out the distribution of the PWRs with and without trend-following using similar Monte Carlo techniques as in our earlier Figure 3. There is a substantial shift to the right in the distribution and it is much more concentrated around its median value of around 9%: the final column of Table 4 reinforces this conclusion with far higher PWRs around 90% of the time; in fact at lower probability levels the PWRs are nearly double those for the 100% equity strategy: trend-following reduces maximum loss and sequence risk in tandem, and this results in a noticeably higher PWR in virtually all cases except the relatively few high PWR instances (i.e. about 10% of the time).

An alternative way of motivating the use of a trend-adjusted equity return as an investment strategy is to simply plot the maximum drawdown of the 100% equity series against the maximum drawdown of the trend-adjusted version: this is done in Figure 5. Clearly the S&P line has far steeper losses than the trend-following S&P, especially around well-known historical eras of financial crashes such as the 1930's, the early 1970's and 2008/9.

Reducing maximum losses in an investment strategy would seem to allow higher withdrawal rates relative to strategies with similar cumulative returns but higher drawdowns: and a trend-following overlay will serve to reduce such drawdowns.

7. The PWR and Market Valuation: Can Valuation Measures help in securing higher withdrawals?

If a simple trend-following investment strategy facilitates superior withdrawal rates most of the time, it is natural to ask whether other market timing or valuation indicators can help us choose withdrawal amounts to give similarly ''improved'' solutions? In particular, measures such the Cyclically Adjusted Price Earnings ratio (CAPE) ratio (Shiller, 2001) have been shown to have some predictive power for longer-run equity returns. Figure 7 shows the time-series plot of beginning period CAPE (right-hand axis) against the 20-year real PWR: if the earnings' yield is high (and hence CAPE is low) it is indicative of future good equity returns and hence we would expect a higher perfect foresight PWR (for the subsequent 20-year period), and this is seen clearly in Figure 7, with low points for CAPE in 1920, 1930 and 1980 being associated with high (subsequent) PWRs. But does this casual observation carry over to a linear regression model?

One way to systematically assess the usefulness of knowing the CAPE ratio at any given moment of time is to examine the historical relation between the CAPE and associated PWR by a simple linear regression equation: this we do in Figure 7 which is a scatter diagram of all CAPE, PWR combinations. In Figure 8 we offer a slight variation on this by relating the CAPE values to the PWRs associated with the S&P returns *adjusted for trend* as in Section 6. The equations themselves are shown on the Figures: in Figure 7 both coefficients have Newey-West adjusted t-values greater than 4, while in Figure 8 both coefficients are above 7.The goodness-of-fit R squared values are 37% and 42% respectively.

In passing we observe some interesting features of these Figures 7 and 8: firstly, there is a clear relation between higher CAPE values and lower subsequent PWRs; secondly, the lower bound for trend-adjusted returns' PWRs for all CAPE values is 6% versus 4% for the unadjusted series; and finally, the scatter is much more concentrated for the trend-adjusted PWRs, suggesting a closer relation between valuation and PWRs .We estimate simple linear regressions (not shown here) to summarise the PWR/CAPE relations in our analysis of various sub-periods below. Blanchett et al (2012) introduce both bond yields and the CAPE ratio as indicators of market valuation: in focussing on trend-adjusting and PWRs we take a rather different approach.

8. Applying PWR, Trend-Following and Valuation techniques to two very different historical periods: 1973- and 1995-

A common feature of the financial planning literature is to take different periods of financial history and explore sustainable withdrawal rates in different environment (e.g. Chatterjee, 2011). To this end we examine two very different historical periods, the 20 year period from 1973 and the 20 year period from 1995.

We examine the use of PWRs in three different environments with and without trend-adjusted equity returns:

i) Perfect foresight of returns

ii) Monte Carlo generated returns from our full data set up to whatever date is being considered.

iii) The PWR associated with the information from the simple regression lines summarising the relation between inverse CAPE and PWR in Figures 7 and 8: i.e., use the existing CAPE ratio at any moment in time to 'predict' the PWR from the simple regression lines in Figures 7 and 8.

Table 5 contains the results for the 20 years beginning in 1995 for the 100% equity portfolio while Table 6 contains the same results for the trend-adjusted returns. Beginning with the unadjusted results in Table 5 we see from the second column that equity returns for the first 6 years were pretty robust indeed, suggesting the likelihood of low sequencing risk; this is indeed the case and the perfect foresight real PWR is 10.781% giving a real PWA of $10,781 p.a. for each of the 20 years.

We now use the median of the distribution of Monte Carlo results for the PWR for each year updating the withdrawal rate sequentially with one less year at a time-i.e. adaptive withdrawal given one year less in the planning horizon and new information on available wealth given both the investment return and the withdrawal for that year. Figure 5 shows the *median* real PWR for planning horizons of 20 through to 1 year. Unsurprisingly the PWR converges on 100% as we start the final year. After the initial 5 years of good investment performance the investment pot reaches over $188,000 by the end of year 5 (i.e. 16 years left to go). Things then take a turn for the worse in 2008 with a 39% fall in the S&P, leading to a fall in the PWA from $10,629 to under $6,000 for 2009 (with 6 years remaining).

The final set of results uses adaptive withdrawal but with the PWR associated with the CAPE at the beginning of each year as given by the linear regression from Figure 7.The inverted CAPE values are given in column 3: the fairly low withdrawal rates in the early years, together with robust investment returns, leads to wealth reaching over $216,000 by the end of 1999. Together with the CAPE-driven PWRs, this leads to higher withdrawal amounts in the final years than those suggested by the Monte Carlo method. It would indeed appear that knowing today's CAPE ratio led to a superior withdrawal experience.

We now ask how the pattern of PWRs would differ if we repeat the calculations in Table 6 but with trend-adjusted equity returns? Firstly we note a perfect foresight PWR higher than for the simple equity returns, not an unexpected result given our PWR distributions in Figure 4. Note how in column 2 the trend-adjusted strategy leads to far better returns of 1.1% compared to -39% in 2008 and -4.4% compared to -22% in 2002.This facilitates a much higher withdrawal in the final 6 years relative to no trend-following, sometimes by several thousands of dollars per annum. The same result is found if we use a valuation ratio to guide our withdrawal rate in the last few columns of Table 6.

What happens if we now repeat the exercise over a period of financial history characterised by poor returns in the early years, for example the 20 years beginning 1973? Table 7 shows that 1973 and

1974 saw falls in the S&P of over 23% and 34% respectively, suggesting high sequencing risk. Although returns recovered later in the period the damage was done: the perfect foresight PWR was only 4.591% for the 20 year period, again emphasising our earlier point that accidents of birth date have a major bearing on one's income in retirement. Both the median Monte Carlo and CAPE valuation metric lead to substantial reduction in real withdrawals relative to Table 5.

But what if we now use trend-adjusted equity returns and repeat this exercise over this historical period? Table 8 contains these results for trend-adjusted returns. First of all note the absence of really severe negative returns in column 2, which allows the perfect foresight PWR to rise by a third to over 6.148% pa. Similarly the Monte Carlo and valuation-based results suggest much higher withdrawals are possible, particularly in the final years, relative to the trend-adjusted returns. It would thus appear that simple market-timing using a trend-following technique can offer a far superior withdrawal experience in decumulation.

9. PWA experience year-by-year since 1971 with 20-year planning horizons.

As a final check on the different performance of 100% equity portfolios and adjusting for trend-following we compare the Monte Carlo and Valuation withdrawal methods on a year-by-year basis since 1971 with 20-year planning horizons. We present the perfect foresight PWA, and features of the PWA distribution, namely the median, maximum and minimum values. In addition we offer a new measure of how difficult it is to get the unknown PWA in practice: we call this

SQ = (Standard Deviation of the Distribution of PWA Outcomes) / Perfect Foresight PWA

In other words, the more spread out the distribution of possible values relative to a scaling factor, namely the PWA, the more 'noisy' the possible sequence of outcomes, and the lower the PWA. The bottom line in Table 9 presents the average figures for the columns: note how similar the 2 methods appear to be on average, including the *SQ* measures.

We now repeat the exercise with trend-adjusted S&P returns in Table 10: immediately we can see a far more concentrated distribution for these trend adjusted PWAs then the unadjusted ones .The minimum value for the Monte Carlo exercise is also much higher with trend adjustment. The *SQ* measures are now roughly half those for the simple 100% equity strategy.

Clearly, yet again, the simple act of trend-adjusting the S&P equity index level as a simple investment strategy allows a much better withdrawal experience.

10. What if the client wants three years of spending set aside?

One strategy that has some popularity with financial advisors is when retirement commences to take the first few years of withdrawals, perhaps typically three, from the accumulated pension sum and place these in a cash account to achieve a 'certain', precautionary objective The remainder of the pension pot is invested in risk assets and decumulation from this begins when the initial cash withdrawals have been exhausted. We examine here how this strategy affects future withdrawals, and as such the standard of living, based on the two investment approaches described earlier. For the sake of simplicity, we assume that the initial cash withdrawals are held such that they maintain their real value whilst outside of the remaining investment (for instance, they could be held in short-term index-linked government bonds

Table 11 shows the real remaining value of the investment pot after 3 years assuming that cash was withdrawn at the beginning of the decumulation period at the start of each year commencing in 1872. We call the perfect withdrawal rate after upfront cash withdrawals have been exhausted and more conventional decumulation commenced as the *deferred PWR*. Table 11 shows different rates of upfront cash withdrawals. For example, a 10% per annum withdrawal rate means that 30% (3 years × 10%) of the investment pot is taken at the start of the drawdown period and placed in cash. Unsurprisingly, the more money that is withdrawn early the smaller the pot becomes after the initial 3 years are up. If very little cash is taken early (0% to 5%) then the investment pot is frequently larger after 3 years than when drawdown started. Large withdrawals such as 15% to 20% significantly eat into the pot with many years of decumulation remaining.

Table 12 expands on this by showing how the deferred PWR varies according to amount of upfront cash withdrawals. The far-right column of the table shows the initial PWR which is the constant proportion of the initial pot money that can be withdrawn if no additional upfront cash withdrawals are made. Firstly we note that the Initial PWR is higher under the trend-following method than the standard buy-and-hold and that these rates are approximately 9% to 10% per annum. If one takes small cash withdrawals upfront then amounts larger than the initial PWR can be taken a later date, e.g. 5% withdrawals for 3 years using trend following leads to an average deferred PWR of 12% compared to an initial PWR of 10%. Taking a lot of cash early, however, leads to substantially reduced future withdrawals. Three years of 20% withdrawals (i.e. putting 60% in cash at the start) and investing in US stocks results in an average deferred PWR of less than 5% compared to an initial PWR of nearly 9%.

The ranges between maximum and minimum PWRs show considerable variation and reflect the varying returns and/or sequencing risk that has been experienced during the past century and more. Figure 9 shows plots of the deferred and initial PWRs for both US stocks and trend-following US stocks. Retirees beginning decumulating around 1920 and 1950 would have had a very different experience to those commencing around 1930 and 1970 irrespective of the withdrawal strategy. It is clear from the charts, though, that taking too much cash in early years reduces living standards in later periods. Taking small amounts early has the reverse effect which might encourage retirees to perhaps consider some part-time work in the first years of decumulation in order to preserve the investment pot. In general, taking upfront cash above the initial PWR leads to a lower standard of living in the future and vice versa. This is slightly complicated by the early returns on the market. Taking a little more cash than the initial PWR suggests is beneficial when the market has a negative real return but too much withdrawn depletes the pot. If the market has positive real returns then cash is underperforming and thus multi-year upfront cash withdrawals at the PWR lead to lower future amounts than the initial PWR taken annually.

Thus far we have assumed that it has always been three years of upfront cash withdrawals. Table 13 allows this to vary between 0 and 5 years. The case of one year is very similar to the PWR calculation in that this assumes a cash withdrawal is made at the start of each annual period which then lasts for the next 12 months (and earns no real return during this time), whilst the case of 0 years is the PWR. The now familiar pattern is again apparent with longer periods of taking small upfront withdrawals (i.e. 5%) resulting in higher deferred PWRs and vice versa. From a practical standpoint, although we are assuming here that cash is taken, there is no compulsion on the retiree's part to spend it at the initially designated rate. For instance, suppose 5 years at 10% is withdrawn upfront. If the market falls by 20% in the first year of decumulation, they may decide to adapt to these new conditions and make the remaining 40% last for a further 5 years, i.e. spend the rest at 8% per year, and let the investment pot remain invested for an additional twelve months. Responding to changing conditions, be they good or bad, by adjusting behaviour makes a great deal of sense compared to rigidly adopting a set of withdrawal patterns.

11. Concluding Thoughts

In this paper we have drawn attention to a number of key features of the much neglected investment aspects of retirement planning and execution: we have also seen how the accidents of birthdate can dramatically impact retirement income: in particular, while the reduction of sequencing risk is recognised by financial planning professionals as an important aspect of the decumulation journey, there is relatively little awareness of it in the mainstream asset management and investing strategy literature, possibly because there is no widely accepted measure of it in practice. The challenge of creating investing strategies for the decumulation phase beyond the risk-free Treasury Inflation Protected Securities (TIPS) portfolios of, say, Sexaeur et al (2012) has barely begun: the choice would seem to be between controlling tail-risk with derivatives (Milevsky and Posner, 2014), versus portfolio timing adjustments into and out of cash (Strub, 2013). This study is firmly in the latter camp. Certainly the vague notion of 'derisking' via portfolio timing adjustment into cash using target-date or glidepath methods is largely untested and not rigorous.

We find:

i) derisking is not necessarily a good idea as one approaches the point of retirement or indeed enters the decumulation phase.

ii) simple market-timing adjustments in the guise of trend-following overlays to a 100% equity portfolio can substantially enhance the feasible withdrawal rate.

iii) there is a practical, empirical relation between sequence risk and the maximum drawdown of an investment strategy(tail risk) and this should be a major concern when creating retirement investing strategies: large drawdowns early in the investment life destroy withdrawal rates; we have also explored measures of sequence risk.

iv) there is useful information in market valuations in the form of the Cyclically Adjusted Price Earnings' (CAPE) ratio which can help assess 'over/undervaluation' of the equity market as a guide to future withdrawal rates.

v) additional research to include the familiar bonds and equity portfolios along with multi-asset portfolios leads to similar conclusions: smoothing asset returns by simple trend-following offers substantially enhanced withdrawal rates relative to unadjusted portfolio strategies. Such strategies are more straightforward than options' strategies and may be used by a wider array of investors.

vi) transactions costs have not been included in this analysis, but since withdrawals are once a year and the round trip trend switches are approximately every year and a half, and the asset is simply an equity index, then these should be very small and will not affect our basic conclusions.

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Table 1

Example of Sequencing Risk

Year	Return (%)	Return (%)	Return (%)
1	20	-20	0
2	10	-10	10
3	0	0	-10
4	-10	10	-20
5	-20	20	20
SeqRisk	3.98	5.98	4.92
PWR (%)	23.87	15.90	19.30

Table 2

Perfect Withdrawal Rate as Real Percentage of Initial Balance: 20,000 Simulations of 20-year Decumulation using Monthly S&P Real Returns: 1872-2014

Percentile	S&P Real PWR (%)
1	2.95
5	4.20
10	5.01
20	6.11
30	7.00
40	7.85
50	8.64
60	9.43
70	10.38
80	11.47
90	13.05
95	14.37
99	16.87

Table 3		
Performance of S&P 500 With and Without Trend Following: 1872-2014		
	S&P	Trend Following
Annualized Real Return (%)	6.82	8.84
Annualized Real Volatility (%)	14.29	9.86
Maximum Real Drawdown (%)	76.80	34.88

Table 4		
Perfect Withdrawal Rate as Real Percentage of Initial Balance: 20,000 Simulations of 20-year Decumulation using Monthly S&P Real Returns With and Without Trend Following: 1872-2014		
Percentile	S&P Real PWR (%)	TF S&P Real PWR (%)
1	2.95	5.57
5	4.20	6.61
10	5.01	7.21
20	6.11	8.03
30	7.00	8.67
40	7.85	9.23
50	8.64	9.80
60	9.43	10.38
70	10.38	11.01
80	11.47	11.81
90	13.05	13.00
95	14.37	14.04
99	16.87	16.22

Table 5

Example of 20-Year Decumulation Starting in 1995 with S&P 500 Investment

				Perfect Foresight PWA			**Monte Carlo Median PWA**				**Valuation Based PWA**			
Year	Real Ret (%)	EY Start	Year	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
20	35.0	5.02	1995	100,000	10,781	120,439	100,000	8.55	8,545	123,458	100,000	6.91	6,910	125,666
19	19.6	4.00	1996	120,439	10,781	131,140	123,458	8.92	11,013	134,473	125,666	6.55	8,233	140,437
18	29.6	3.61	1997	131,140	10,781	155,943	134,473	9.18	12,340	158,242	140,437	6.59	9,256	169,966
17	23.5	3.03	1998	155,943	10,781	179,278	158,242	9.58	15,153	176,717	169,966	6.55	11,140	196,154
16	18.4	2.58	1999	179,278	10,781	199,446	176,717	9.99	17,647	188,287	196,154	6.60	12,956	216,848
15	-8.8	2.26	2000	199,446	10,781	171,980	188,287	10.44	19,654	153,721	216,848	6.79	14,734	184,241
14	-14.2	2.68	2001	171,980	10,781	138,385	153,721	10.82	16,625	117,693	184,241	7.51	13,828	146,295
13	-22.0	3.28	2002	138,385	10,781	99,480	117,693	11.19	13,167	81,489	146,295	8.43	12,333	104,437
12	20.0	4.33	2003	99,480	10,781	106,445	81,489	11.63	9,476	86,421	104,437	9.71	10,143	113,160
11	9.3	3.76	2004	106,445	10,781	104,522	86,421	12.46	10,772	82,653	113,160	10.24	11,582	110,983
10	3.5	3.68	2005	104,522	10,781	97,067	82,653	13.22	10,929	74,269	110,983	11.27	12,504	101,974
9	11.4	3.78	2006	97,067	10,781	96,153	74,269	14.34	10,652	70,892	101,974	12.55	12,797	99,375
8	2.1	3.67	2007	96,153	10,781	87,203	70,892	15.70	11,131	61,043	99,375	13.84	13,753	87,458
7	-39.3	3.85	2008	87,203	10,781	46,397	61,043	17.41	10,629	30,607	87,458	15.53	13,582	44,851
6	26.6	6.51	2009	46,397	10,781	45,084	30,607	19.57	5,989	31,162	44,851	19.11	8,572	45,924
5	12.3	4.92	2010	45,084	10,781	38,534	31,162	22.83	7,115	27,014	45,924	21.61	9,922	40,443
4	-0.8	4.47	2011	38,534	10,781	27,521	27,014	27.82	7,515	19,337	40,443	26.20	10,595	29,600
3	14.8	4.87	2012	27,521	10,781	19,218	19,337	35.90	6,941	14,230	29,600	34.41	10,187	22,286
2	27.8	4.71	2013	19,218	10,781	10,781	14,230	52.07	7,410	8,715	22,286	50.71	11,302	14,037
1	15.0	4.02	2014	10,781	10,781	0	8,715	100.00	8,715	0	14,037	100.00	14,037	0

Table 6

Example of 20-Year Decumulation Starting in 1995 with S&P 500 Investment with Trend Following

				Perfect Foresight PWA			**Monte Carlo Median PWA**				**Valuation Based PWA**			
Year	Real Ret (%)	EY Start	Year	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
20	32.4	5.02	1995	100,000	12,308	116,082	100,000	9.75	9,746	119,473	100,000	8.21	8,213	121,503
19	19.6	4.00	1996	116,082	12,308	124,104	119,473	10.03	11,988	128,541	121,503	7.85	9,534	133,903
18	29.6	3.61	1997	124,104	12,308	144,850	128,541	10.35	13,301	149,311	133,903	7.91	10,587	159,775
17	10.6	3.03	1998	144,850	12,308	146,622	149,311	10.67	15,927	147,554	159,775	7.91	12,636	162,770
16	11.0	2.58	1999	146,622	12,308	149,052	147,554	10.97	16,186	145,782	162,770	8.03	13,064	166,132
15	-4.1	2.26	2000	149,052	12,308	131,198	145,782	11.42	16,649	123,896	166,132	8.28	13,753	146,199
14	1.8	2.68	2001	131,198	12,308	121,034	123,896	11.70	14,492	111,376	146,199	8.99	13,136	135,462
13	-4.4	3.28	2002	121,034	12,308	103,967	111,376	12.17	13,555	93,539	135,462	9.84	13,326	116,789
12	20.9	4.33	2003	103,967	12,308	110,861	93,539	12.63	11,818	98,841	116,789	10.98	12,821	125,749
11	1.8	3.76	2004	110,861	12,308	100,296	98,841	13.37	13,217	87,137	125,749	11.53	14,498	113,217
10	-0.1	3.68	2005	100,296	12,308	87,916	87,137	14.16	12,338	74,738	113,217	12.54	14,195	98,940
9	9.0	3.78	2006	87,916	12,308	82,437	74,738	15.11	11,290	69,179	98,940	13.78	13,638	93,007
8	1.1	3.67	2007	82,437	12,308	70,931	69,179	16.36	11,317	58,524	93,007	15.09	14,032	79,878
7	1.3	3.85	2008	70,931	12,308	59,367	58,524	18.08	10,579	48,554	79,878	16.84	13,448	67,274
6	18.2	6.51	2009	59,367	12,308	55,612	48,554	20.20	9,806	45,789	67,274	20.01	13,464	63,588
5	8.0	4.92	2010	55,612	12,308	46,748	45,789	23.40	10,714	37,865	63,588	22.66	14,410	53,089
4	-6.1	4.47	2011	46,748	12,308	32,332	37,865	28.12	10,646	25,552	53,089	27.33	14,510	36,217
3	9.6	4.87	2012	32,332	12,308	21,938	25,552	36.02	9,203	17,912	36,217	35.47	12,847	25,605
2	27.8	4.71	2013	21,938	12,308	12,308	17,912	51.98	9,311	10,991	25,605	51.51	13,188	15,868
1	15.0	4.02	2014	12,308	12,308	0	10,991	100.00	10,991	0	15,868	100.00	15,868	0

Table 7

Example of 20-Year Decumulation Starting in 1973 with S&P 500 Investment

				Perfect Foresight PWA			**Monte Carlo Median PWA**				**Valuation Based PWA**			
Year	Real Ret (%)	EY Start	Year	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
20	-23.5	5.36	1973	100,000	4,591	72,972	100,000	8.84	8,844	69,719	100,000	7.48	7,477	70,765
19	-34.2	7.41	1974	72,972	4,591	44,987	69,719	8.86	6,178	41,802	70,765	8.79	6,222	42,461
18	29.1	12.06	1975	44,987	4,591	52,147	41,802	8.87	3,707	49,177	42,461	11.39	4,837	48,568
17	16.8	9.76	1976	52,147	4,591	55,563	49,177	9.20	4,523	52,172	48,568	10.56	5,130	50,752
16	-12.2	8.62	1977	55,563	4,591	44,759	52,172	9.62	5,019	41,405	50,752	10.33	5,245	39,960
15	-1.1	10.33	1978	44,759	4,591	39,728	41,405	9.87	4,089	36,907	39,960	11.57	4,623	34,949
14	4.3	11.10	1979	39,728	4,591	36,646	36,907	10.25	3,782	34,548	34,949	12.41	4,336	31,928
13	15.7	11.43	1980	36,646	4,591	37,102	34,548	10.76	3,717	35,684	31,928	13.11	4,185	32,110
12	-10.5	10.65	1981	37,102	4,591	29,102	35,684	11.42	4,076	28,293	32,110	13.31	4,273	24,918
11	14.8	12.77	1982	29,102	4,591	28,142	28,293	12.09	3,422	28,555	24,918	15.24	3,796	24,250
10	18.7	11.81	1983	28,142	4,591	27,948	28,555	13.02	3,719	29,474	24,250	15.56	3,774	24,299
9	0.7	10.19	1984	27,948	4,591	23,532	29,474	14.14	4,168	25,493	24,299	15.57	3,783	20,669
8	26.6	10.42	1985	23,532	4,591	23,970	25,493	15.45	3,939	27,278	20,669	16.92	3,496	21,732
7	22.8	8.55	1986	23,970	4,591	23,793	27,278	17.21	4,695	27,725	21,732	17.48	3,799	22,016
6	-4.3	7.10	1987	23,793	4,591	18,367	27,725	19.62	5,438	21,318	22,016	18.93	4,167	17,073
5	13.8	7.47	1988	18,367	4,591	15,674	21,318	22.82	4,864	18,718	17,073	22.33	3,813	15,085
4	24.4	6.80	1989	15,674	4,591	13,790	18,718	27.72	5,189	16,833	15,085	26.85	4,050	13,731
3	-8.0	5.67	1990	13,790	4,591	8,466	16,833	35.86	6,036	9,937	13,731	34.37	4,719	8,293
2	18.4	6.31	1991	8,466	4,591	4,591	9,937	51.87	5,154	5,664	8,293	51.01	4,231	4,812
1	12.2	5.42	1992	4,591	4,591	0	5,664	100.00	5,664	0	4,812	100.00	4,812	0

Table 8

Example of 20-Year Decumulation Starting in 1973 with S&P 500 Investment with Trend Following

				Perfect Foresight PWA			**Monte Carlo Median PWA**				**Valuation Based PWA**			
Year	Real Ret (%)	EY Start	Year	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
20	-15.3	5.36	1973	100,000	6,148	79,522	100,000	10.20	10,203	76,086	100,000	8.81	8,806	77,270
19	-4.0	7.41	1974	79,522	6,148	70,429	76,086	10.25	7,800	65,545	77,270	10.13	7,830	66,652
18	8.3	12.06	1975	70,429	6,148	69,592	65,545	10.39	6,810	63,587	66,652	12.80	8,535	62,918
17	13.0	9.76	1976	69,592	6,148	71,671	63,587	10.63	6,757	64,199	62,918	11.86	7,462	62,648
16	-4.9	8.62	1977	71,671	6,148	62,304	64,199	10.96	7,036	54,354	62,648	11.58	7,258	52,669
15	-5.5	10.33	1978	62,304	6,148	53,075	54,354	11.20	6,087	45,619	52,669	12.78	6,731	43,417
14	-2.4	11.10	1979	53,075	6,148	45,820	45,619	11.54	5,266	39,402	43,417	13.53	5,874	36,658
13	13.4	11.43	1980	45,820	6,148	44,994	39,402	11.96	4,713	39,341	36,658	14.13	5,180	35,699
12	-3.9	10.65	1981	44,994	6,148	37,328	39,341	12.49	4,913	33,083	35,699	14.27	5,096	29,408
11	20.5	12.77	1982	37,328	6,148	37,579	33,083	13.17	4,357	34,621	29,408	15.92	4,681	29,801
10	18.7	11.81	1983	37,579	6,148	37,300	34,621	14.02	4,854	35,325	29,801	16.22	4,835	29,627
9	-1.3	10.19	1984	37,300	6,148	30,760	35,325	15.10	5,334	29,614	29,627	16.29	4,825	24,491
8	26.6	10.42	1985	30,760	6,148	31,147	29,614	16.32	4,832	31,362	24,491	17.51	4,288	25,567
7	22.8	8.55	1986	31,147	6,148	30,692	31,362	18.07	5,667	31,546	25,567	18.29	4,677	25,646
6	11.6	7.10	1987	30,692	6,148	27,386	31,546	20.28	6,399	28,059	25,646	19.97	5,123	22,899
5	2.5	7.47	1988	27,386	6,148	21,764	28,059	23.49	6,590	22,000	22,899	23.27	5,329	18,006
4	24.4	6.80	1989	21,764	6,148	19,430	22,000	28.11	6,185	19,677	18,006	27.84	5,012	16,167
3	-10.8	5.67	1990	19,430	6,148	11,845	19,677	36.16	7,114	11,203	16,167	35.49	5,738	9,301
2	7.9	6.31	1991	11,845	6,148	6,148	11,203	51.99	5,824	5,805	9,301	51.75	4,813	4,842
1	9.5	5.42	1992	6,148	6,148	0	5,805	100.00	5,805	0	4,842	100.00	4,842	0

Table 9

S&P: Comparison Between Monte Carlo and Valuation Withdrawal Methods for Initial $100,000 Portfolio

		Monte Carlo				Valuation			
Year	PWA ($)	Max ($)	Median ($)	Min ($)	SD/PWA (%)	Max ($)	Median ($)	Min ($)	SD/PWA (%)
1971	5,571	10,052	5,248	3,890	31.53	8,788	4,923	3,944	25.45
1972	5,235	9,754	5,091	3,777	29.95	8,419	4,747	3,843	23.67
1973	4,591	8,844	4,609	3,422	27.37	7,477	4,252	3,496	20.66
1974	6,173	8,626	6,440	4,771	20.16	8,552	5,866	4,888	14.69
1975	9,817	13,794	10,433	7,728	20.57	11,851	9,561	8,020	12.15
1976	8,261	11,397	8,924	6,344	20.21	9,945	8,149	6,870	11.77
1977	7,582	11,840	8,391	5,774	23.11	11,264	7,573	6,406	15.06
1978	9,208	16,134	10,434	6,979	27.73	15,877	9,297	7,886	20.73
1979	10,131	21,087	11,675	7,492	34.63	21,708	10,220	8,706	31.29
1980	10,692	24,817	12,143	7,618	42.32	27,033	10,661	9,146	43.68
1981	10,239	25,109	11,554	6,977	49.43	29,078	10,053	8,669	55.93
1982	12,587	30,727	14,317	8,267	51.15	34,362	12,091	10,559	59.85
1983	12,342	28,628	13,755	8,315	47.61	31,900	11,731	10,134	58.05
1984	11,593	25,887	12,445	7,906	44.34	28,632	10,833	9,352	54.20
1985	12,741	27,471	13,340	8,329	41.52	29,760	11,730	10,071	50.95
1986	11,288	23,194	11,277	8,393	37.80	24,586	11,006	8,709	47.20
1987	10,125	20,186	10,219	7,594	35.56	20,783	11,013	7,585	43.88
1988	11,513	22,569	12,247	8,443	34.30	22,328	13,639	8,432	41.46
1989	11,159	21,161	11,509	8,500	32.06	20,188	13,918	7,943	38.03
1990	9,660	18,149	9,907	5,400	33.16	16,676	11,653	6,735	34.93
1991	11,197	21,059	11,496	6,389	34.17	18,582	13,076	7,704	32.09
1992	10,272	18,986	10,361	5,775	33.81	16,170	11,645	7,163	28.46
1993	9,812	18,076	9,854	5,506	34.60	14,771	10,694	6,699	25.41
1994	9,806	18,023	9,826	5,498	35.19	14,100	10,678	6,502	22.41
1995	10,781	19,654	10,712	5,989	34.96	14,734	11,442	6,910	20.13
Average	**9,695**	**19,009**	**10,248**	**6,603**	**34.29**	**18,703**	**10,018**	**7,455**	**33.29**

Table 10									
S&P with Trend Following: Comparison Between Monte Carlo and Valuation Withdrawal Methods for Initial $100,000 Portfolio									
		Monte Carlo				Valuation			
Year	PWA ($)	Max ($)	Median ($)	Min ($)	SD/PWA (%)	Max ($)	Median ($)	Min ($)	SD/PWA (%)
1971	7,083	10,898	6,749	4,675	25.23	9,757	6,117	4,627	24.12
1972	6,895	11,002	6,631	4,713	23.83	9,699	5,749	4,648	23.55
1973	6,148	10,203	5,955	4,357	21.78	8,806	5,152	4,288	22.73
1974	7,572	10,051	7,511	5,607	15.93	10,883	6,479	5,480	20.63
1975	8,362	10,397	8,472	6,360	14.42	12,390	7,171	6,262	19.62
1976	8,236	10,457	8,452	6,387	14.42	11,164	7,362	6,393	16.49
1977	7,798	10,079	8,138	6,150	14.13	10,561	7,234	6,149	14.17
1978	8,748	11,546	9,314	7,034	13.94	11,466	8,352	6,987	11.89
1979	10,004	14,932	10,706	8,082	17.08	14,062	9,804	8,092	13.23
1980	11,229	16,897	11,982	8,979	19.41	16,639	11,191	9,114	16.39
1981	11,002	16,487	11,525	8,583	20.57	17,020	10,955	8,876	19.80
1982	12,662	18,571	13,320	9,685	20.14	19,415	12,671	10,141	20.91
1983	11,820	16,762	12,346	9,681	18.11	17,835	11,886	9,370	21.25
1984	11,068	15,351	11,288	8,847	16.79	16,547	11,198	8,732	21.01
1985	12,371	16,923	12,741	9,726	16.11	18,163	12,521	9,689	21.57
1986	10,930	14,531	11,023	8,634	14.43	15,529	11,182	8,406	21.21
1987	9,773	12,893	9,738	7,643	14.27	13,569	10,038	7,427	20.71
1988	9,493	12,582	9,306	7,440	14.61	12,920	10,529	7,149	20.56
1989	9,994	13,377	9,743	7,882	15.33	13,443	11,948	7,509	20.47
1990	8,699	11,693	8,389	6,881	16.23	11,649	10,330	6,466	19.89
1991	10,440	14,283	10,003	8,394	16.59	13,946	12,358	7,709	19.98
1992	10,553	14,395	10,068	8,449	16.79	13,787	12,474	7,631	19.21
1993	10,485	14,281	9,986	7,956	17.69	13,299	12,076	7,409	17.56
1994	10,654	14,502	10,092	8,047	18.49	13,053	11,884	7,330	15.63
1995	12,308	16,649	11,568	9,203	18.50	15,868	13,387	8,213	14.40
Average	**9,773**	**13,590**	**9,802**	**7,576**	**17.39**	**13,659**	**10,002**	**7,364**	**19.08**

Table 11

Pot Value at Start of Deferred PWR Decumulation after Taking 3 Years of Cash Withdrawals Upfront (% of Initial Pot Value)

	Withdrawal Rate Per Annum (%)				
	0	5	10	15	20
100% US Stocks					
Mean	127.44	108.33	89.21	70.09	50.98
Median	123.35	104.84	86.34	67.84	49.34
Maximum	223.80	190.23	156.66	123.09	89.52
Minimum	49.64	42.19	34.75	27.30	19.86
100% Trend Following					
Mean	132.68	112.78	92.88	72.97	53.07
Median	129.87	110.39	90.91	71.43	51.95
Maximum	217.58	184.95	152.31	119.67	87.03
Minimum	71.78	61.02	50.25	39.48	28.71

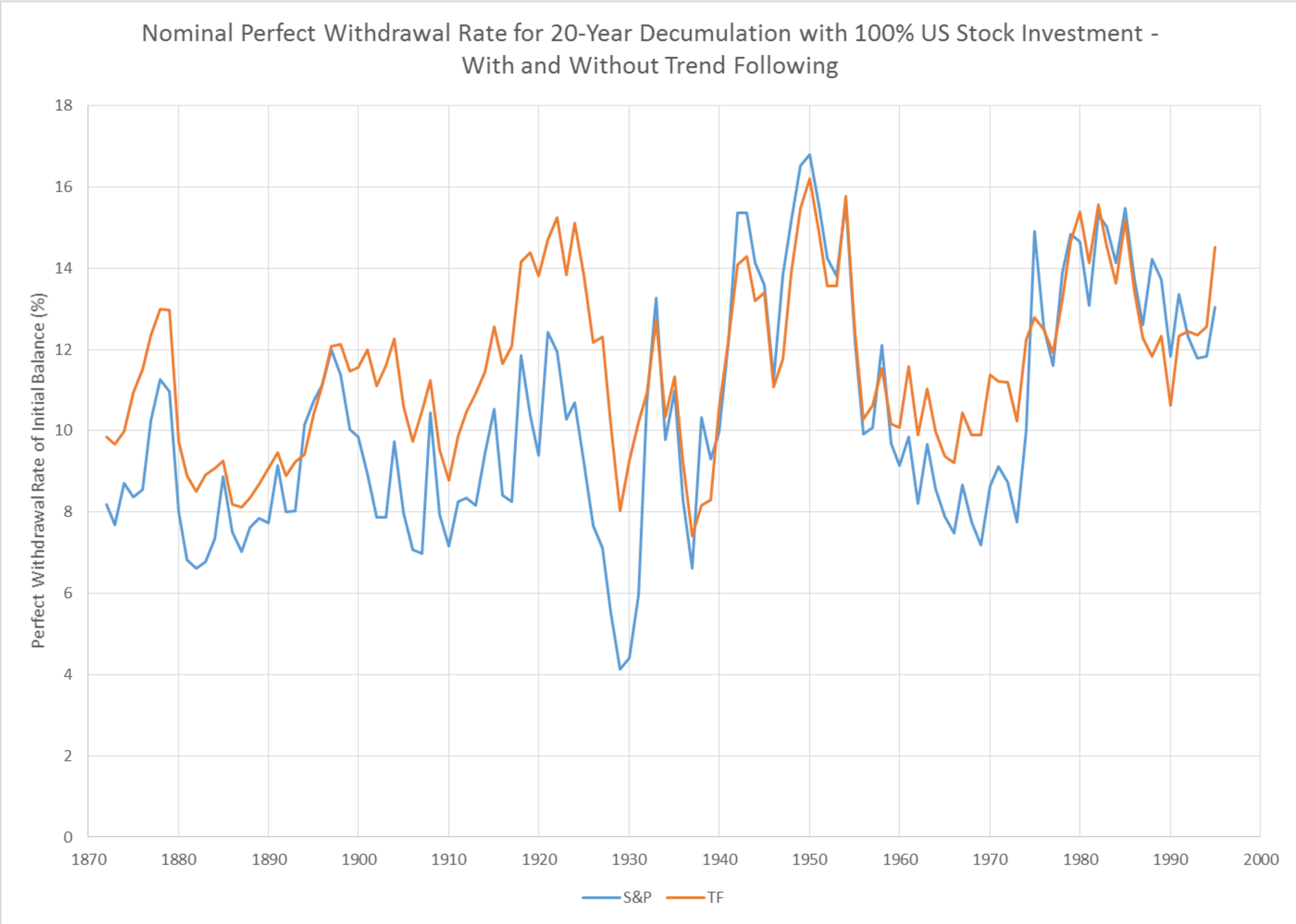
Table 12

Deferred PWR over Remaining Decumulation Period after Taking 3 Years of Cash Withdrawals Upfront (% of Initial Pot Value)

	Withdrawal Rate Per Annum (%)					Initial
	0	5	10	15	20	PWR (%)
100% US Stocks						
Mean	12.09	10.28	8.46	6.65	4.84	8.85
Median	11.59	9.85	8.11	6.37	4.64	8.73
Maximum	24.37	20.71	17.06	13.40	9.75	15.06
Minimum	5.02	4.27	3.52	2.76	2.01	4.28
100% Trend Following						
Mean	14.14	12.02	9.90	7.78	5.66	10.01
Median	13.38	11.37	9.37	7.36	5.35	9.81
Maximum	28.48	24.21	19.93	15.66	11.39	16.86
Minimum	7.05	5.99	4.94	3.88	2.82	5.81

Table 13																		
Deferred PWR over Remaining Decumulation Period after Taking Cash Withdrawals Upfront at Various Rates of Initial Pot Value per Year																		
	Number of Withdrawal Years																	
	5% Withdrawal Rate						10% Withdrawal Rate						15% Withdrawal Rate					
	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
100% US Stocks																		
Mean	8.85	9.31	9.79	10.28	10.79	11.31	8.85	8.82	8.70	8.46	8.09	7.54	8.85	8.33	7.61	6.65	5.39	3.77
Median	8.73	9.08	9.49	9.85	10.12	10.42	8.73	8.61	8.44	8.11	7.59	6.95	8.73	8.13	7.38	6.37	5.06	3.47
Maximum	15.06	16.84	18.75	20.71	22.72	24.80	15.06	15.95	16.67	17.06	17.04	16.53	15.06	15.07	14.58	13.40	11.36	8.27
Minimum	4.28	4.25	4.26	4.27	4.26	4.21	4.28	4.03	3.78	3.52	3.19	2.80	4.28	3.80	3.31	2.76	2.13	1.40
100% Trend Following																		
Mean	10.01	10.64	11.31	12.02	12.76	13.53	10.01	10.08	10.06	9.90	9.57	9.02	10.01	9.52	8.80	7.78	6.38	4.51
Median	9.81	10.33	10.82	11.37	11.84	12.42	9.81	9.79	9.62	9.37	8.88	8.28	9.81	9.24	8.42	7.36	5.92	4.14
Maximum	16.86	19.27	21.81	24.21	27.53	30.93	16.86	18.26	19.38	19.93	20.65	20.62	16.86	17.24	16.96	15.66	13.77	10.31
Minimum	5.81	5.86	5.94	5.99	6.14	6.34	5.81	5.55	5.28	4.94	4.60	4.23	5.81	5.24	4.62	3.88	3.07	2.11

Figure 1.



Nominal Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment - With and Without Trend Following
Perfect Withdrawal Rate of Initial Balance (%)
0
2
4
6
8
10
12
14
16
18
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
S&P
TF

Figure 2.

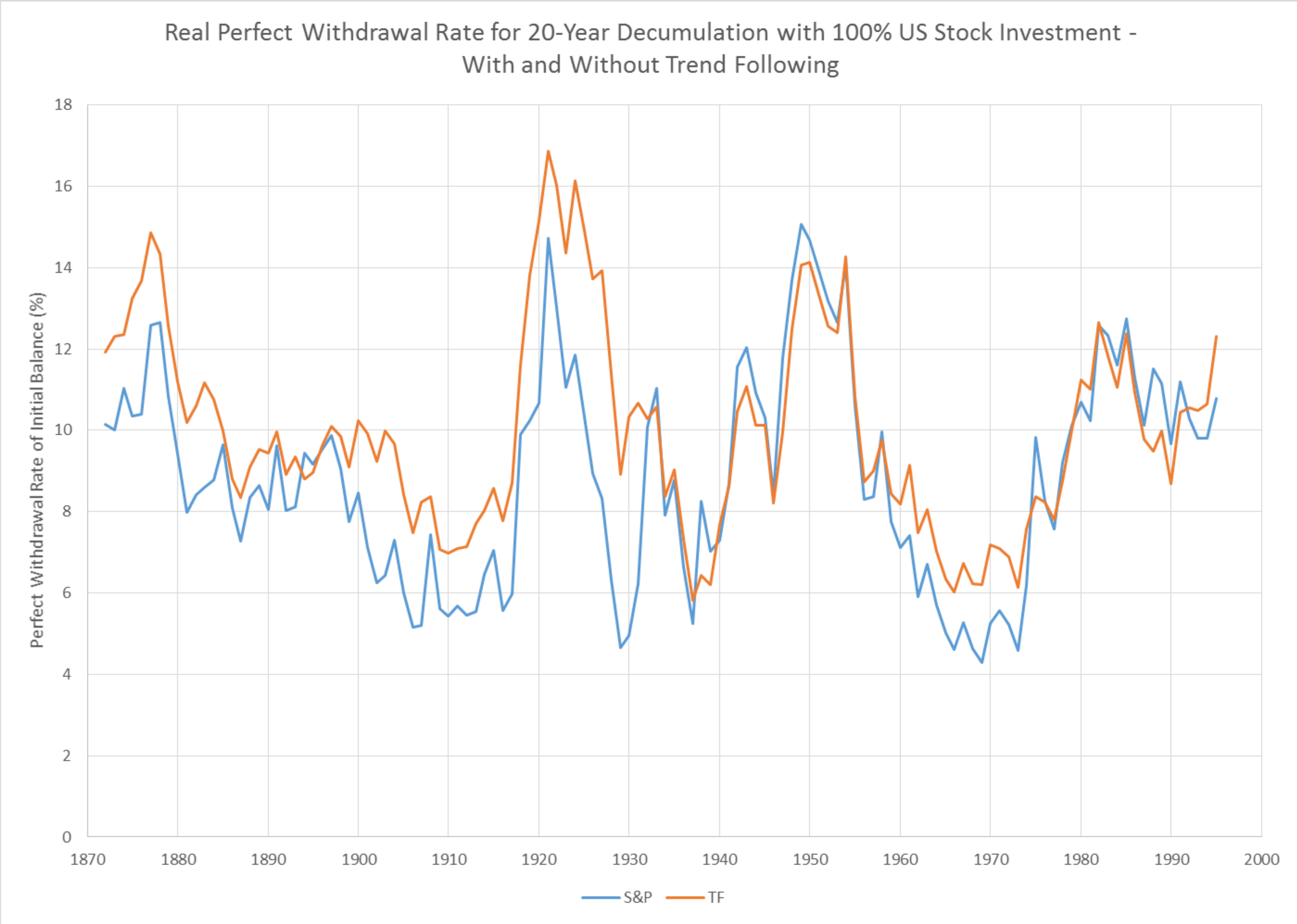

Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment - With and Without Trend Following
Perfect Withdrawal Rate of Initial Balance (%)
18
16
14
12
10
8
6
4
2
0
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
S&P
TF

Figure 3.

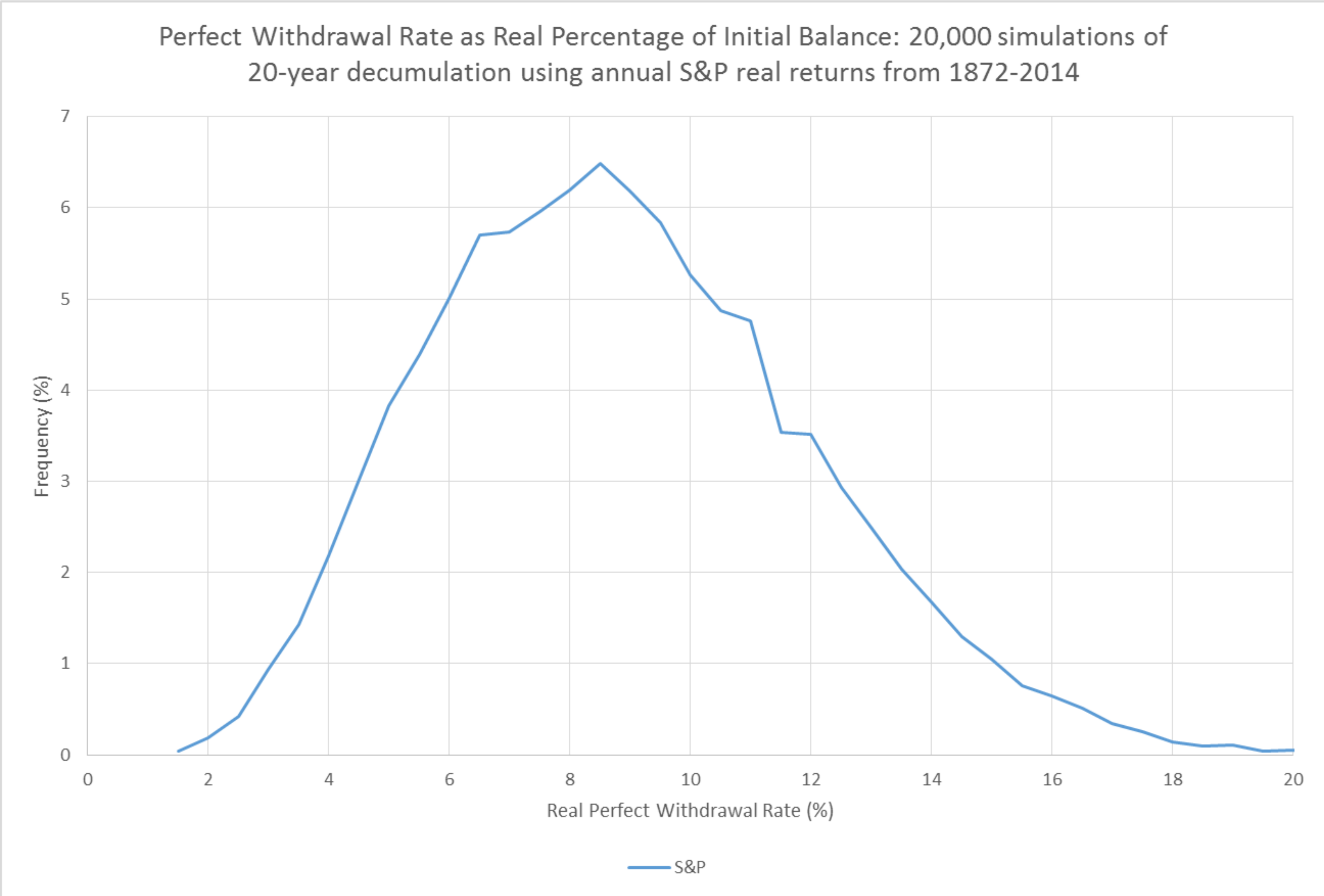

Perfect Withdrawal Rate as Real Percentage of Initial Balance: 20,000 simulations of 20-year decumulation using annual S&P real returns from 1872-2014
Frequency (%)
0
1
2
3
4
5
6
7
0
2
4
6
8
10
12
14
16
18
20
Real Perfect Withdrawal Rate (%)
S&P

Figure 4.

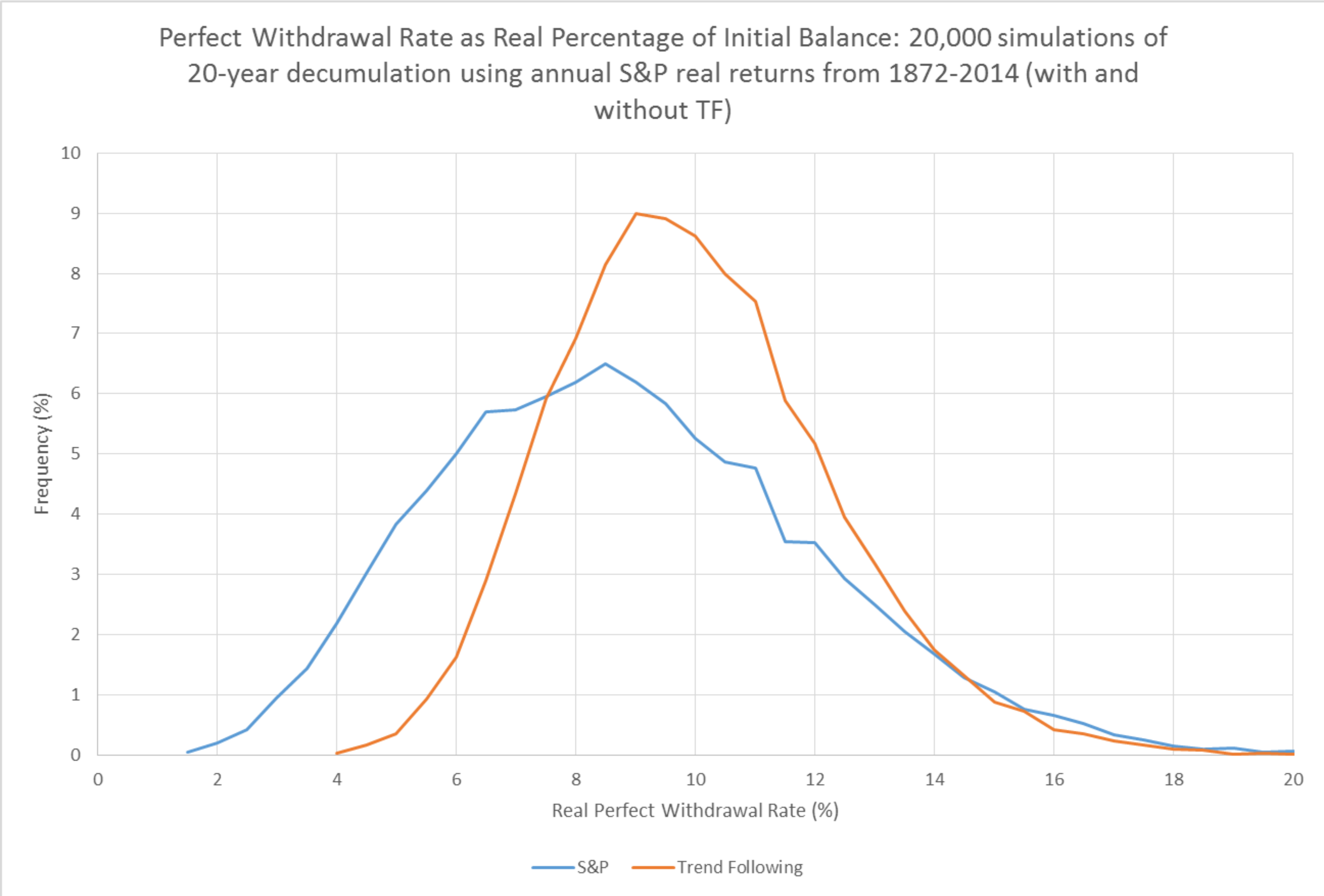
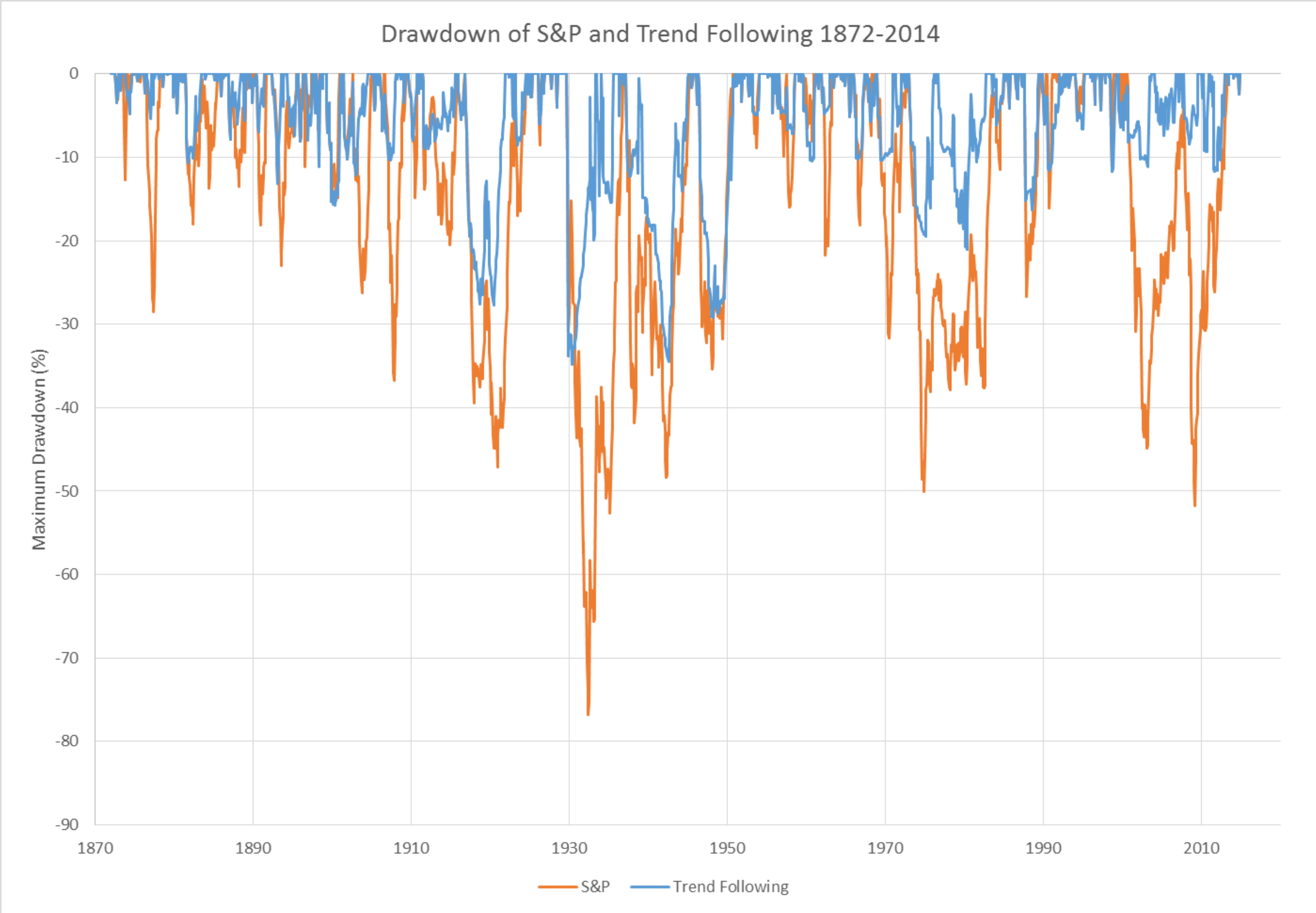
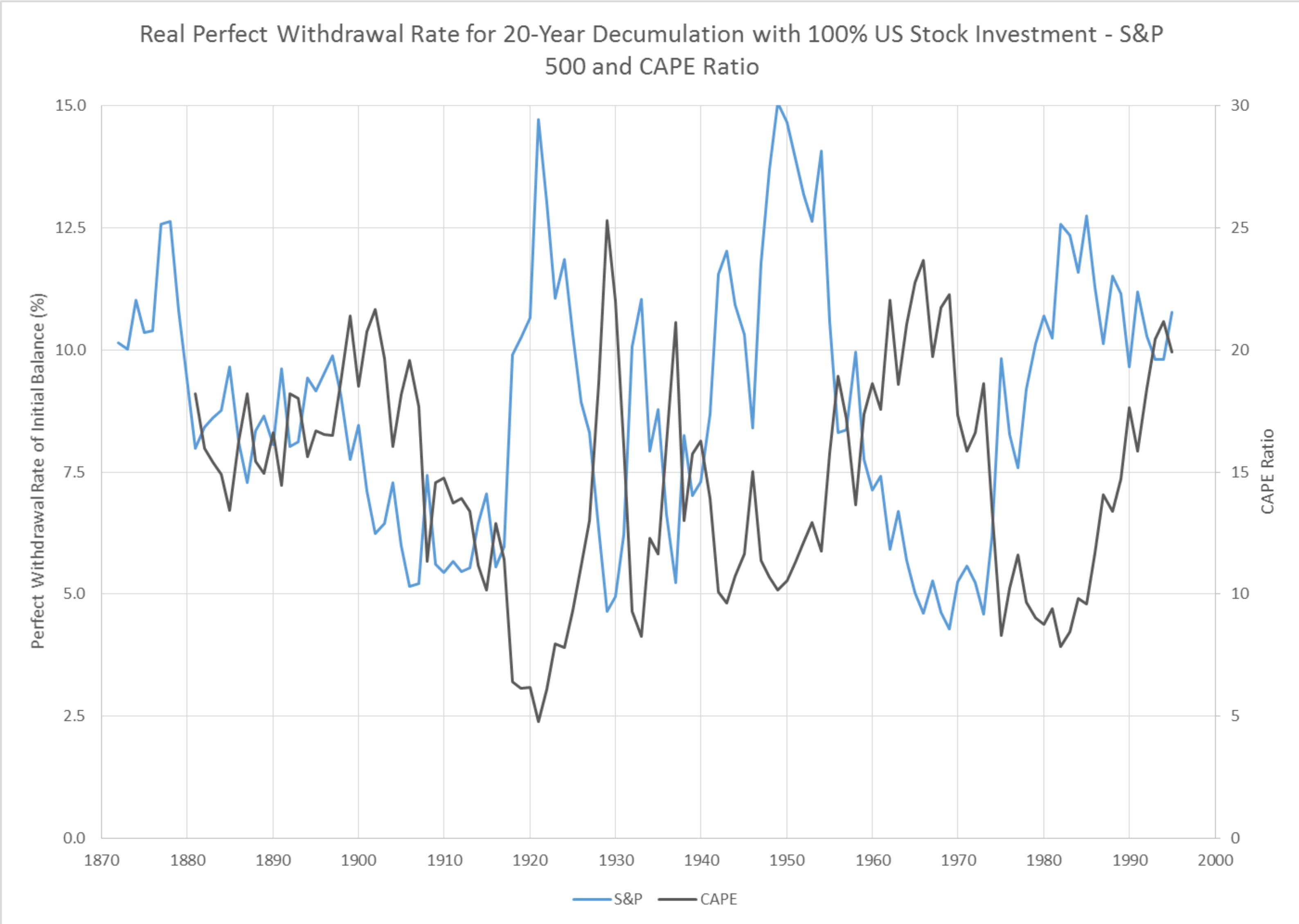

Perfect Withdrawal Rate as Real Percentage of Initial Balance: 20,000 simulations of 20-year decumulation using annual S&P real returns from 1872-2014 (with and without TF)
Frequency (%)
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
16
18
20
Real Perfect Withdrawal Rate (%)
S&P
Trend Following

Figure 5.



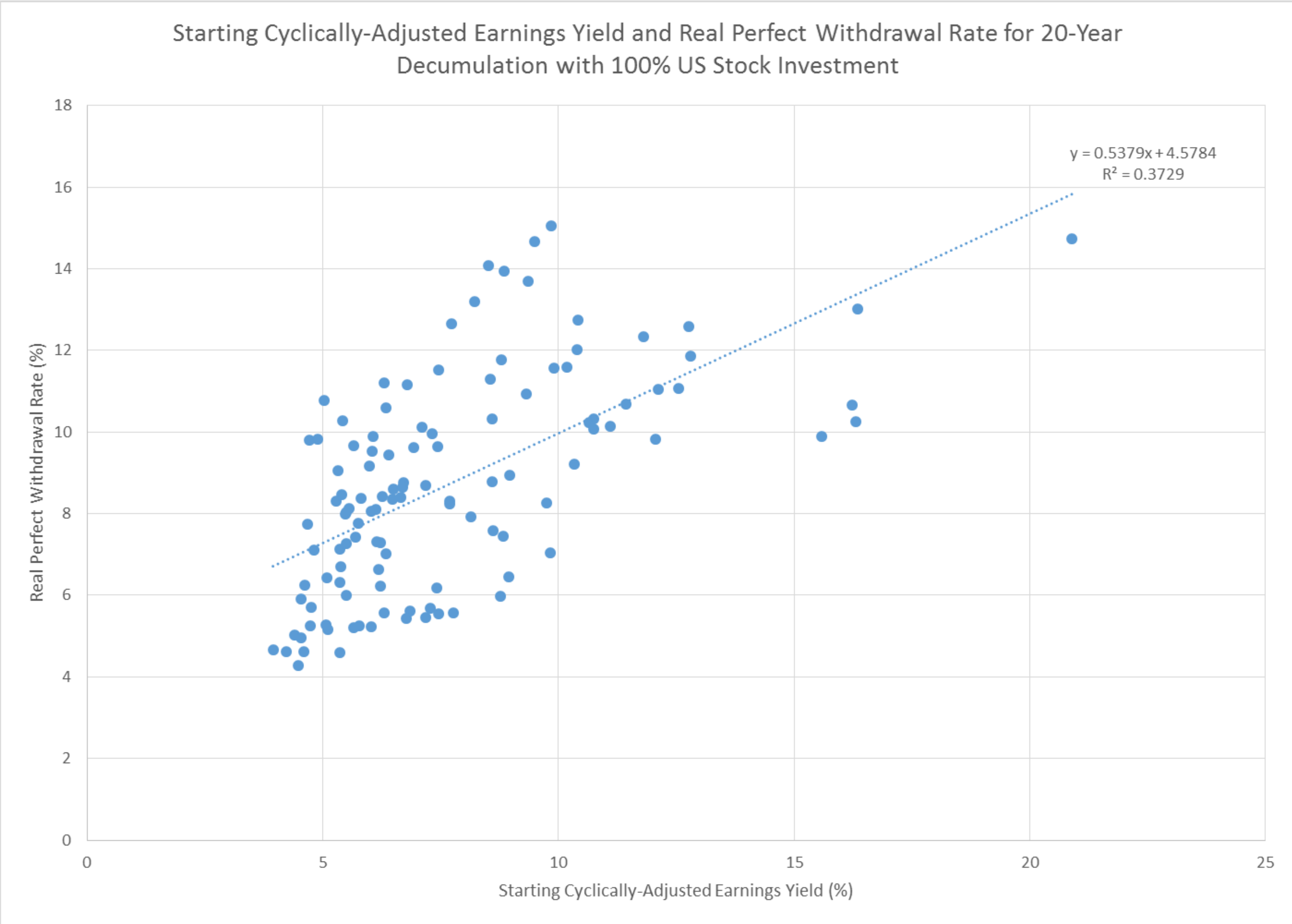
Drawdown of S&P and Trend Following 1872-2014
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Maximum Drawdown (%)
1870
1890
1910
1930
1950
1970
1990
2010
S&P
Trend Following

Figure 6.



Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment - S&P 500 and CAPE Ratio
Perfect Withdrawal Rate of Initial Balance (%)
CAPE Ratio
15.0
12.5
10.0
7.5
5.0
2.5
0.0
30
25
20
15
10
5
0
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
S&P
CAPE

Figure 7.



Starting Cyclically-Adjusted Earnings Yield and Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment
y = 0.5379x + 4.5784
R² = 0.3729
Real Perfect Withdrawal Rate (%)
Starting Cyclically-Adjusted Earnings Yield (%)
0
2
4
6
8
10
12
14
16
18
0
5
10
15
20
25

Figure 8.

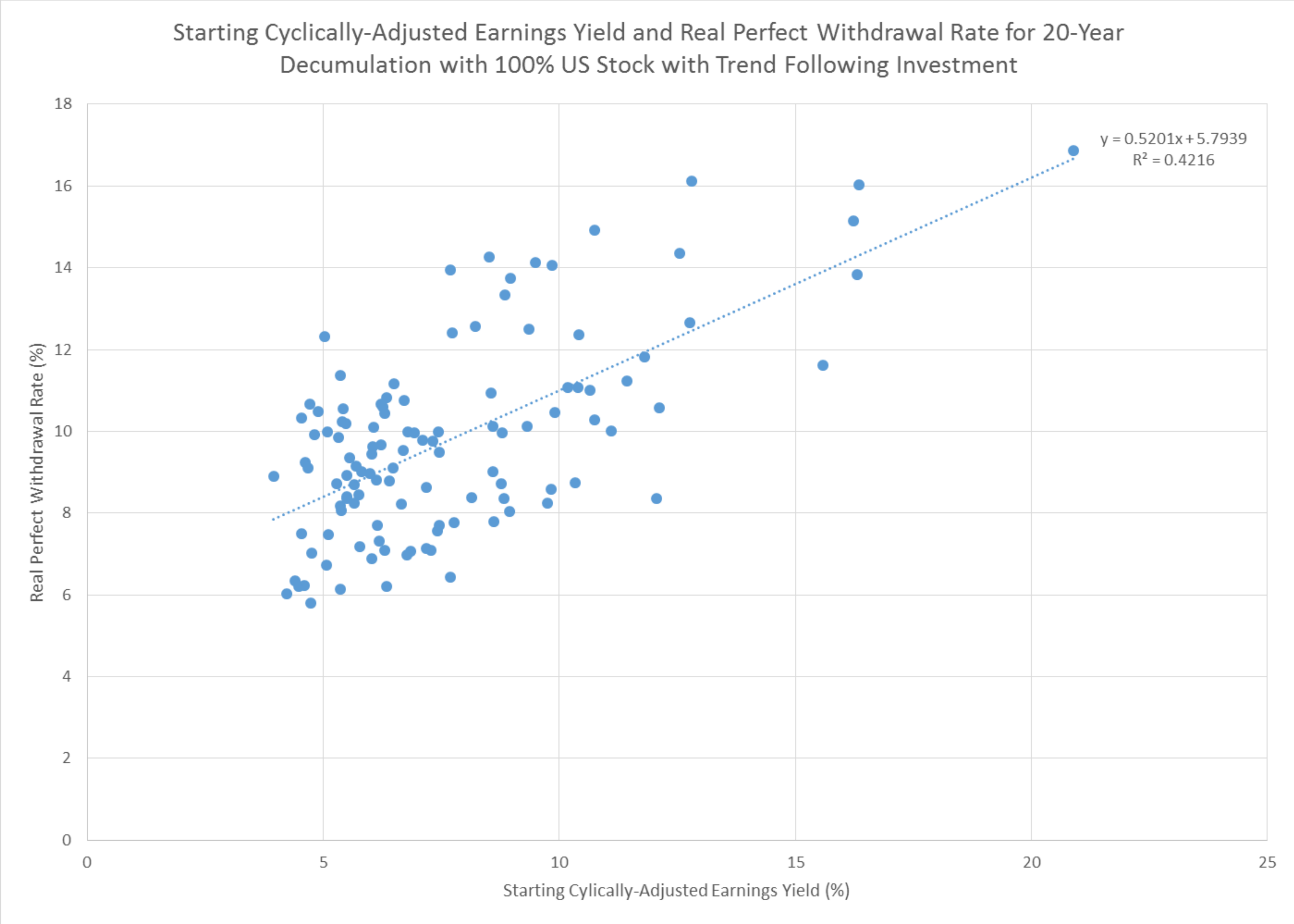
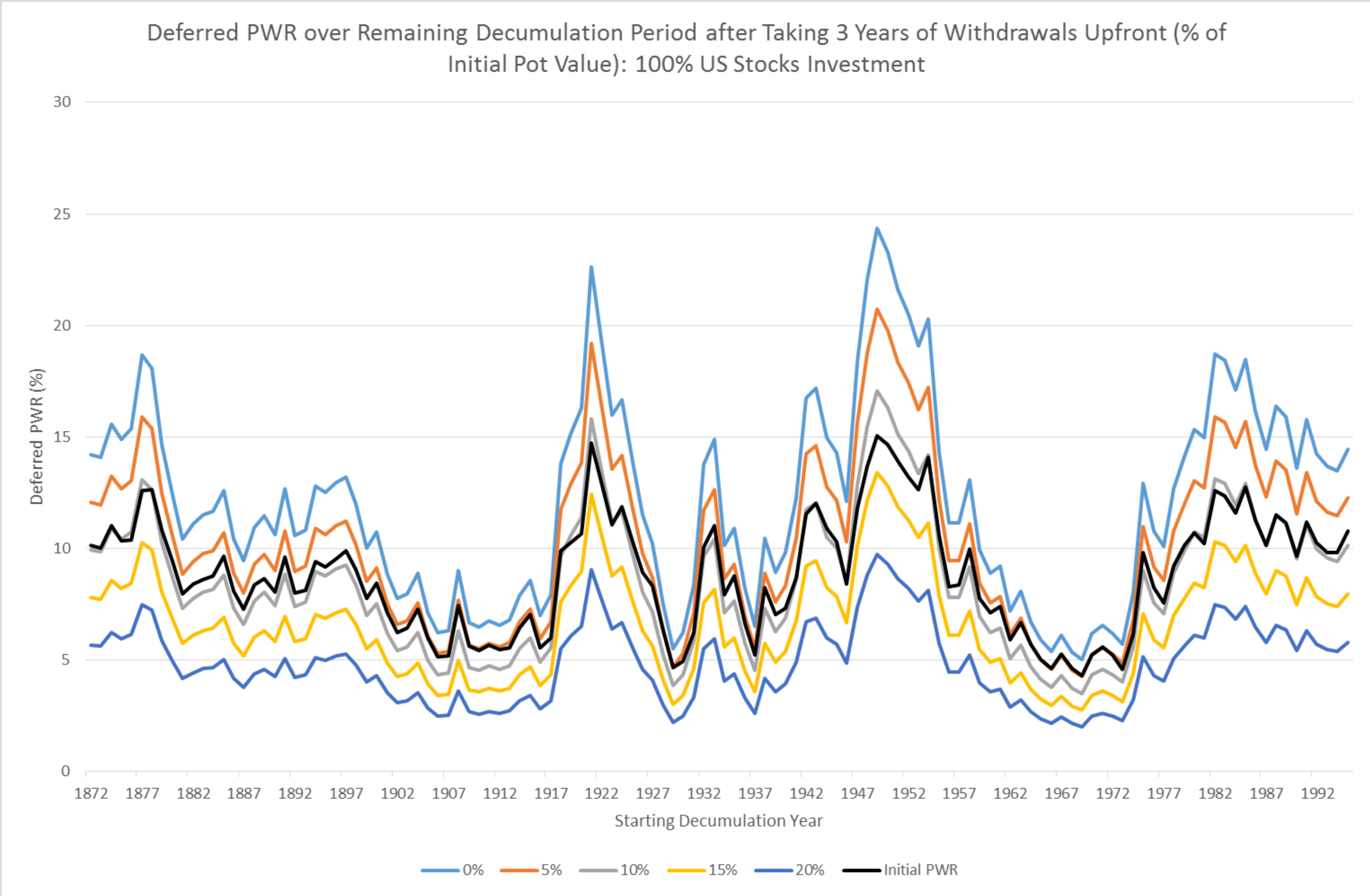

Starting Cyclically-Adjusted Earnings Yield and Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock with Trend Following Investment
y = 0.5201x + 5.7939
R² = 0.4216
Real Perfect Withdrawal Rate (%)
Starting Cylically-Adjusted Earnings Yield (%)
0
2
4
6
8
10
12
14
16
18
0
5
10
15
20
25

Figure 9.


Deferred PWR over Remaining Decumulation Period after Taking 3 Years of Withdrawals Upfront (% of Initial Pot Value): 100% US Stocks Investment
30
25
20
15
10
5
0
Deferred PWR (%)
1872
1877
1882
1887
1892
1897
1902
1907
1912
1917
1922
1927
1932
1937
1942
1947
1952
1957
1962
1967
1972
1977
1982
1987
1992
Starting Decumulation Year
0%
5%
10%
15%
20%
Initial PWR