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Pricing Basis Risk in Survivor Swaps

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Pricing basis risk in survivor swaps

This paper looks at basis risk in survivor swaps, which occurs when mortality of the reference population used to price the swap differs from the mortality of the population being hedged. The author finds that any basis risk present is usually small compared to the risk premium that any party hedging mortality should be willing to pay

BY PAUL SWEETING

THIS PAPER LOOKS AT basis risk in survivor swaps, instruments where a fixed payment is made by one party at some point in the future in exchange for a payment based on the longevity of a reference population at the same point in the future. Cox and Lin (2005) discuss the potential hedging of mortality and longevity risk by life assurance companies and pension schemes. They note that since changes in mortality experience have opposite effects on pensions and assurance business, these two parties should be able to hedge each other's risks.

There are, of course, additional statutory concerns when considering the mortality assumptions used in the valuation of liabilities. Furthermore, the risks borne by these two parties are potentially different. In particular:

- Different mortality tables might be used for valuing pension scheme liabilities to those used for valuing life assurance company liabilities.
- The realised mortality of the reference population on which swap payments are based may be different to the mortality assumptions used by either the pension scheme or the life assurance company, if not both.
- The reference age for the swap might be different from that used by the pension scheme and the life assurance company, and these might be different from each other.

These basis risks mean that rather than offering a hedge, an inappropriate survivor swap might offer little protection. This would mean that rather than trade with each other, pension schemes and life assurance companies might prefer to trade with speculators who will charge a risk premium in respect of the undiversifiable risk that is being taken on.

I do not seek to put an absolute price on mortality (or longevity) risk; rather I look at the relative sizes of the various risks in order to explore the theoretical criteria that need to hold for survivor swaps to be tradable between pension schemes, life assurance companies and speculators.

Survivor swaps

Survivor swaps have been discussed in detail by Dowd (2001), Blake et al (2006), Dowd et al (2006), and others. Both of the more recent papers consider two types of swap: a single-payment swap, where a single payment based on the surviving proportion of a reference population is exchanged for a payment representing the proportion expected at the outset to survive to the payment date. This instrument is a building block for what is described by Dowd et al (2006) as a vanilla survivor swap (VSS). With a VSS, a stream of mortality-related payments forms the floating leg of the swap, whilst the payments' expected values at the initial execution of the swap form the fixed or preset leg. In my analysis, I use a single-payment swap, generally with a one-year time horizon.

Assumptions

There are a number of additional assumptions that are required in this analysis. The most important is that all parties are risk averse, in particular that all parties have exponential utility functions and so exhibit constant absolute risk aversion. I also assume that all parties have the same level of risk aversion. Although allowing different levels of risk aversion means that prices for the survivor swaps can be obtained, my main concern is the range of circumstances under which trades might occur rather than the price that might be struck. Assuming that all parties have the same level of risk aversion allows this to be done, albeit with a restrictive risk aversion assumption.

In terms of calculating the expected change in mortality, I assume that all parties agree on the expected level of mortality in the future and the potential variation around this central estimate. The expected level of mortality in the future is not necessarily the same as the present level of mortality; indeed, it should be expected that mortality rates will fall. I make no assumptions as to how the future expected mortality rates are arrived at; I assume only that such expectations exist.

I assume that future mortality is normally distributed. This assumption has limitations, specifically that mortality assumptions are probabilities and the normal distribution allows for probabilities that are greater than unity or less than zero. However, such probabilities are low, and normality is a reasonable assumption for most ages. The importance of these assumptions is that they allow the risk premium to be expressed in terms of the standard deviation of mortality rates and a single risk aversion parameter.

I assume that all benefits are fixed and I ignore (or assume that the parties involved ignore) the risk that their mortality experience will differ from that of the reference population used to value their liabilities, if the reference population is not based on that party's own mortality experience. This means that pension schemes and life assurance companies using independent mortality tables would be prepared to accept a certain degree of basis risk at zero cost. This reflects the fact that the basis risk that I consider is the risk that the reference population used in the swap, will experience different mortality to that of the population used to value the liabilities. Such a risk might arise if, for example, the socio-economic distribution in the pension scheme or life assurance company is different from that of the reference population. Ignoring the basis risk between the pension scheme or life assurance company population and the reference table used might appear to be a strong assumption; however, it is appropriate since the reason that such a basis risk would exist is that the party hedging its risk would have insufficient information on the nature of its own demographic risk to derive mortality tables based on its own experience; any party with sufficient experience would use tailor-made tables and would experience no such basis risk (so this assumption would be redundant).

Furthermore, I ignore 'binomial' mortality risk. This is the risk that even if the mix of lives in a pension scheme or life assurance company closely mirrors that of the reference population, the mortality experience will differ due to bad luck. This risk reduces with the size of the population under investigation. I also ignore any cohort effects, as highlighted by Willets (2004).

The risk premium

To calculate the risk premium that a party wishing to hedge mortality or longevity will pay, I assume that all mortality is measured relative to a single standard reference population. This means that not only do all parties refer to the population mortality table when trading swaps, but that pension schemes and life assurance companies also use the population mortality tables to value their liabilities. Under this scenario, neither the pension scheme nor the life assurance company will perceive any basis risk between the liabilities and the swaps traded.

First, I look at the risk premium that a pension scheme would be willing to pay to hedge longevity risk. Let $\bar{p}_x$ be the expected proportion of pensioners surviving from age x to $x+1$, where x is the "reference age" for the swap; and let p_x be the actual proportion surviving over that period. Thus the fixed or preset leg of the swap is $\bar{p}_x$. The floating leg of this swap, which is the eventual payment allowing for actual changes to population mortality is p_x. Therefore the pension scheme receives a net payment of $p_x - \bar{p}_x$.

However, the risk is not diversifiable and if it is assumed that it cannot be hedged by the speculator, there must be an expected profit per unit notional payment for removing this risk, $\pi(p_x)$, which allows for uncertainty in the estimation of $\bar{p}_x$. This means that a speculator, receiving the fixed leg on a swap and paying a floating leg of p_x, would require a payment of $\bar{p}_x + \pi(p_x)$. Allowing for the risk premium, the net payment made to the pension scheme when the swap expires is now $p_x - \bar{p}_x - \pi(p_x)$.

Given that future mortality is assumed to be normally distributed, and given that exponential utility functions are being assumed, the risk premium is:

1 $$\pi(p_x) = \frac{1}{2} r \sigma^2 (p_x)$$

where r is the level of risk aversion and $\sigma^2(p_x)$ is the variance of future population mortality. An easy-to-follow derivation of this standard result is given in Eeckhoudt et al (2005). It is possible to estimate $\sigma^2(p_x)$ from past data as:

2 $$\bar{\sigma}^2 (p_x) = \frac{\sum (p_x - \bar{p}_x)^2}{n-1}$$

A similar approach can be used for a life assurance company wishing to hedge mortality, if the expected proportion of lives not surviving from age x to $x+1$ is defined as $\bar{q}_x$ with the actual proportion surviving being q_x. The risk premium here is:

3 $$\pi(q_x) = \frac{1}{2} r \sigma^2 (q_x)$$

However, since $q_x = 1 - p_x$, $\sigma^2(q_x) = \sigma^2(1-p_x) = \sigma^2(p_x)$, so $\pi(q_x) = \pi(p_x)$. This means that rather than receiving floating and paying fixed on a swap based on q_x, a life assurance company can receive fixed and pay floating on a swap based on p_x.

Basis risk

Trades between hedgers and speculators

I first look at a model that is as general as possible. I allow for the pension scheme and life assurance company to use different reference ages for their liabilities. I also allow for the reference age for the swap to be different from either of these two reference ages.

Another important assumption in this initial model is that the mortality table used to derive the swap price can be different from the model used by either the life assurance company or the pension scheme to value either of their liabilities. In practice, the most likely table to be used would be a population mortality table.

If a pension scheme values its liabilities using pension scheme mortality tables, then the price at which it will trade with a speculator is simple to derive if the speculator uses the same tables: it is simply $\bar{p}_x^A + \pi(p_x^A)$ for lives aged x, the superscripts referring to statistics calculated using mortality table A. Similarly, if a life assurance company values its liabilities using life assurance mortality tables, then the price at which it will trade with a speculator if the speculator uses the same tables is simply $\bar{p}_y^B - \pi(p_y^B)$ for lives aged y, the superscripts referring to table B. However, if the swap is traded using different reference ages and reference population mortality tables, then the calculations become a little more involved.

First, I look again at the pension scheme. If the floating leg of the swap is based on mortality tables with a reference age of z based on mortality table C, then the pension scheme will only be willing to pay a maximum rate of $\bar{p}_x^A + \pi(p_x^A) - \pi(p_x^A, p_z^C)$, where the final term represents the basis risk of population mortality improvements versus pension scheme mortality improvements.

In order to normalise the expected payment, the pension scheme would therefore enter into a swap where the scheme would receive a floating leg of $p_z^C h(p_x^A, p_z^C)$ in exchange for the payment of the fixed leg, where $h(p_x^A, p_z^C)$ is the optimal hedging ratio, equal to $\rho(p_x^A, p_z^C)\sigma(p_x^A) / \sigma(p_z^C)$ and where $\rho(p_x^A, p_z^C)$ is the correlation between unexpected changes in p_x^A and p_z^C. This hedging ratio, described by Ederington (1979), has some attractive properties. For reference populations which are perfectly correlated but with different volatilities, the size of swap would simply be a function of the ratio of the standard deviations: if the reference population of the swap is twice as volatile as that being hedged, then only half of the notional value of the swap is needed. The addition of correlation to the equation provides allowance for the fact that the hedge might not be perfect (or might be perfectly negative). Where the reference populations are unrelated – so the correlation is zero – the optimal solution is not to enter into the swap at all.

The premium receivable would therefore be analogous to (1), but the variance would be calculated from the difference between the pension scheme's experience, p_x^A, and the normalised experience from the reference population, $p_z^C h(p_x^A, p_z^C)$, as shown below:

4 $$\pi\left(p_x^A, p_z^C\right) = \frac{1}{2} r\sigma^2\left[p_x^A - p_z^C h\left(p_x^A, p_z^C\right)\right]$$

This means that the maximum total fixed leg payable becomes:

5 $$\bar{p}_x^A + \pi\left(p_x^A\right) - \pi\left(p_x^A, p_z^C\right)$$

Taking the risk premium of $\pi(p_x^A) - \pi(p_x^A, p_z^C)$, substituting for $\pi()$ in terms of $\sigma()$ and simplifying gives:

6 $$\pi\left(p_x^A\right) - \pi\left(p_x^A, p_z^C\right) = \frac{1}{2} r\rho^2\left(p_x^A, p_z^C\right)\sigma^2\left(p_x^A\right)$$

This tells us that the maximum risk premium that the pension scheme is willing to pay is simply the risk premium for a perfect hedge multiplied by the square of the correlation between the mortality experiences of the pension scheme and of the reference population.

If the pension scheme is trading with a speculator, then the minimum risk premium that the speculator will be willing to receive per unit of notional value is analogous to (1):

7 $$\pi\left(p_z^C\right) = \frac{1}{2} r\sigma^2\left(p_z^C\right)$$

However, as discussed above, the notional amount traded would be scaled by a factor of $h(p_x^A, p_z^C)$ to allow for the required level of coverage. Therefore, a trade will only occur if:

8 $$\pi\left(p_x^A\right) - \pi\left(p_x^A, p_z^C\right) \geq \pi\left(p_z^C\right) h\left(p_x^A, p_z^C\right)$$

In other words, the risk premium that the pension scheme is willing to pay must be at least as great as the premium required by the speculator. Given that the fixed leg of the swap traded would be $\bar{p}_z^C h(p_x^A, p_z^C)$ the inequality can be expressed as $h(p_x^A, p_z^C)[h(p_x^A, p_z^C)-1] \geq 0$ which can be simplified to:

9 $$\left|h\left(p_x^A, p_z^C\right)\right| \geq 1$$

It can be shown that the criterion required for a life assurance company to trade with a speculator is analogous to this.

One corollary of this result is that the criterion for trade to occur is independent of the level of risk aversion in the utility function if, as is assumed, the levels of risk aversion are the same. If levels of risk aversion differ, however, then the inequality derived above would reflect these differences.

Trades between hedgers of mortality and longevity

Another possibility is that the pension scheme and the life assurance company might want to trade directly with each other. The swap they would choose might be based on the reference age and/or reference population of one (or both) of the parties, or it might be based on a reference age between that of the two parties and a widely accepted mortality table. Since the former are specialised cases of the latter, I assume initially that reference age and tables are different for the two parties involved and the calculation of the swap contract.

The inequality that must be satisfied here is slightly more complicated than in the previous section due to the normalisation needed. One way of looking at the degree of normalisation required is to consider an expanded version of (8) and to rearrange it to give the following inequality:

10 $$h\left(p_y^B, p_z^C\right)\left[\pi\left(p_x^A\right) - \pi\left(p_x^A, p_z^C\right)\right] + h\left(p_x^A, p_z^C\right)\left[\pi\left(p_y^B\right) - \pi\left(p_y^B, p_z^C\right)\right] \geq 0$$

The term inside the first set of square parentheses represents the risk premium that the pension scheme would be willing to pay, less the basis risk premium it would require; the term inside the second set of square parentheses represents the risk premium that a life insurance company would be willing to pay, adjusted for the basis risk that it faced; and the two $h(\ )$ terms outside the square parentheses represent the adjustments to the notional values of the swaps traded, such that the levels of risk being hedged are consistent. Looking at the constituents of all these terms in the earlier expressions, this simplifies to:

11 $$h\left(p_x^A, p_z^C\right) h\left(p_y^B, p_z^C\right)\left[h\left(p_x^A, p_z^C\right) + h\left(p_y^B, p_z^C\right)\right] \geq 0$$

This expression suggests that hedging will only be impossible in very limited circumstances, in particular when exactly one of either $h(p_x^A, p_z^C)$ or $h(p_y^B, p_z^C)$ is negative; and the sum of $h(p_x^A, p_z^C)$ and $h(p_y^B, p_z^C)$ is positive.

What this tells us is that the higher the correlation of the pension scheme's population with the reference population used for the swap (relative to the correlation of the life assurance company's population and the swap reference population), and the more volatile the pension scheme's mortality relative to the life assurance company's mortality, then the more likely a trade is to take place.

Empirical Results

Trades between hedgers and speculators

The most basic calculation here is to consider the situation where all parties use the same mortality table, so expression (11) becomes $|h(p_x, p_z)| \geq 1$. An appropriate assumption is that this single reference table is the population mortality, and a good source of UK data is the Government Actuary's Department ("GAD") (2006a, 2006b) which, until 31 January 2006, produced population mortality tables and projections. The tables I use relate to United Kingdom males. The life tables produced cover triennia, so count all deaths in years t–1, t and t+1. Projections are annual and cover the one-year periods mid-year t to mid-year t+1, mid-year t+1 to mid-year t+2, up to mid-year $t+s$ to mid-year $t+s$+1.

There are a number of ways in which the tables and projections could be combined to calculate the price of risk. One way is to assume that the mortality tables apply to the middle year of the triennia (so that tables for years t–1 to t+1 apply to year t), and that the average of the projection in year t of mid-year t to mid-year $t+s$, and mid-year $t+s$ to mid-year $t+s$+1 gives the projection for the calendar year $t+s$.

For the calculations, p_x is the rate of survivorship calculated from the mortality tables relating to year t where t is 2001, 2002, 2003 or 2004, and $\bar{p}_x$ is the estimation in year t–1 of the value that p_x will take in year t. The results show that, broadly speaking, if the age of the reference population being hedged is greater than the age of the reference population used in the calculation of the swap, then hedging is possible. This appears counter-intuitive, but reflects the fact that for these combinations of ages, correlations are sufficiently high (though occasionally negative) for a hedge to be possible – according to historical data, at least.

Another reasonable scenario is for the reference population of the speculator to be a population mortality table, but for a pension scheme or a life assurance company to use industry-specific mortality tables. In this case, $|h(p_x^A, p_z^C)| \geq 1$ is again the appropriate inequality.

For pension scheme mortality, I use data from the Continuous Mortality Investigation ("CMI") (1990, 1999, 2005a, 2005b). In particular, I use the base tables PML80, PML92 and PML00. These give the mortality weighted by lives (rather than amounts) for male members of insured pension schemes. The data used for calculating these tables comes from the years 1979-82, 1991-94 and 1999-02 respectively. The data for pensioners under age 60 is less reliable than for older individuals due to the low number of early retirements and the reasons for retiring early. I therefore use pensions data only for ages 60 and over.

For life assurance mortality, I use the tables AM80, AM92 and AM00. These tables, also from the CMI and for the same period, give the mortality for males with whole-life and endowment assurance policies. For the population mortality, I again use the GAD tables, but this time covering the years 1980-82, 1992-94 and 2000-02. For these calculations I assume that the risk premium over several years is proportional to the risk premium for a single year and that there is an expectation of no improvement in longevity between these dates, so $1-\bar{p}_x$ for 1992 is equal to the actual mortality experienced in 1980.

Again, the results broadly demonstrate that if the age of the reference population being hedged is greater than the reference population of the swap, then the hedging is possible.

Trades between Hedgers of Mortality and Longevity

As discussed, the situation for hedgers of mortality and longevity to trade directly with each other is slightly different to that involving a speculator. In particular, the criterion that must be satisfied is (11), although since the same reference mortality tables are used for pricing the liabilities and the swaps the criterion becomes:

$$h(p_x, p_z)h(p_y, p_z)\left[h(p_x, p_z) + h(p_y, p_z)\right] \geq 0 \qquad (12)$$

For this analysis, I assume that all calculations are carried out with reference to the GAD tables. Looking at the results for a range of population reference ages z, and combinations of pensioner reference ages x, and life assurance company reference ages y, the results suggest, reassuringly, that hedging works best when the ages of the reference populations being hedged are close, and the reference age of the swap is close to that of both hedgers (the results for the swap reference age of 30 being anomalous).

If instead we consider the situation where different reference tables are used for valuation and for swaps, then the most likely situation is where the swaps are traded using population mortality tables, whilst the liabilities being hedged are industry specific. As before, pensioner liabilities x and life assurance company liabilities y are calculated with reference to CMIB mortality tables, whilst the swap contracts z are calculated relative to the GAD mortality tables. In this case, (11) applies unaltered. From this data, it appears that trades are possible with all but the youngest reference ages for the swaps (unless the reference age of one of the populations entering the trade is equally low).

The results in this section depend critically on the correlation figures. Correlations do tend to weaken as age differences increase (becoming negative in some cases), but this is less of an issue than the stability of these figures. The correlation of mortality rate changes at close ages can be expected to be quite strongly positive – for example, the factors that affect mortality for 70-year olds are likely to be similar to those that affect 80-year olds (treatments for hypertension, cardiovascular disease, age-related cancers and so on); however, these factors are less likely to be relevant for younger individuals who are more likely to be affected by lifestyle-related changes (road traffic accidents, the emergence of and treatments for AIDS, for example). In this dataset, there are some strong correlations across large age ranges, but these correlations could easily reverse making any hedge constructed on their account at best useless and at worst expensive.

Conclusion

The formulation here is simplistic. However, it appears to indicate that if the age being hedged is at least as high as the reference age used by the speculator, then a hedge is possible: the basis risk is lower than the risk premium required. Looking at trades between hedgers (most probably between pension schemes and life assurance companies), the picture is even clearer: providing the reference age used for the swap is not very low (less than 40 years of age), it is almost always better for a pension scheme and a life assurance company to trade with each other and to accept some basis risk than it is for them to pay the full risk premium to a speculator. In fact, cross-hedging in this way provides the optimal solution, justifying the empirical conclusions of Cox and Lin (2005), who discover that natural hedging across ages and mortality types does occur in practice within insurance companies between pension and annuity books.

The assumptions in this paper are strong and it is worth considering what would happen if they were weakened. The assumption that all parties have the same level of risk aversion is key, and this allows for a great deal of simplification in the hedging criteria. If levels of risk aversion do differ, then the higher a hedger's level of risk aversion is relative to that of a speculator, then the more likely a trade is to take place (or the greater the level of basis risk that could be borne).

The assumptions used for the forms of the preference functions and the development of future mortality (relative to expectations) are also key, in that they allow the risk premia to be expressed in relatively simple forms. It is by no means clear whether the results would hold if these forms were changed. It would be interesting to see further research in this area.

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