

# **Life-Cycle Finance in Theory and in Practice\***

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## **Abstract**

This paper draws upon the modern science of finance to address several important practical issues in personal finance. Chief among these is how much to save for retirement and how to invest those savings. The paper suggests ways that advances in the theory of finance combined with innovations in financial contracting technology might be used to improve social welfare by designing and producing a new generation of user-friendly life-cycle products for consumers.

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## **CONTENTS**

Introduction .....	1
The Theory of Life-Cycle Finance.....	3
Beyond Mutual Funds .....	5
New Measures of Risk and Reward .....	7
Life-Cycle Financial Products of the Future .....	7
Escalating Life Annuities .....	8
Bundled Risk Annuities .....	10
Changing Delivery Systems .....	10
References .....	12

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## ***Introduction***

Life-cycle finance, especially saving and investing for retirement, is today a matter of intense concern to millions, perhaps billions, of people around the world. Over the past three decades many respected finance theorists and behavioral scientists have studied how people *should* and *actually do* make such decisions. Theorists have produced optimization models that capture important features of reality, such as changing investment opportunities, unpredictable labor income, habit formation, and transaction costs. And at the same time, scientific studies of actual financial behavior have revealed that people consistently make certain mistakes because of lack of knowledge, faulty logic, cognitive dissonance, and biased statistics.

The new science of finance has had a profound impact on the practice of institutional risk management.<sup>1</sup> Sophisticated enterprise-wide risk-management systems are widely employed today by financial service firms and a growing number of non-financial firms.<sup>2</sup> By comparison, applications of this new science to the important life-cycle issues faced by households have been very limited. Online financial planning “tools” and “optimizers” lag far behind the best theory. Contemporary theory employs multi-period hedging techniques and contingent claims analysis, but the quantitative models routinely used by professional financial planners appear to be *ad hoc* blends of trial-and-error Monte Carlo forecasting and Markowitz’s static mean-variance model of efficient portfolio diversification.

This paper seeks to suggest ways to better align the practice of life-cycle finance with the latest scientific knowledge. Among the important insights of modern financial science are:

- A person’s welfare depends not only on her end-of-period wealth, but on the consumption of goods and leisure over her entire lifetime.

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<sup>1</sup> See Bodie (1999), Merton (1998), and Meulbroek (2002).

<sup>2</sup> See DeLoach (2000).

- Multi-period hedging (rather than “time diversification”) is the way to manage market risk over time.
- Portfolio managers can and should make greater use of the information embedded in the prices of derivatives such as swaps and options: interest rates and implied volatilities.
- The value, riskiness, and flexibility of a person’s labor earnings are of first-order importance in optimal portfolio selection at each stage of the life cycle.
- Habit formation can give rise to a demand for guarantees against a decline in investment income.
- Because of transaction costs, agency problems, and limited knowledge on the part of consumers, dynamic asset allocation will and should become an activity performed by financial intermediaries, rather than by their retail customers.

Table 1 summarizes and compares the old paradigm of life-cycle finance and the new one proposed herein. The rest of the paper will explain the table.

<b>Feature</b>	<b>Old Paradigm</b>	<b>New Paradigm</b>
Measure of Welfare	Wealth	Lifetime consumption of goods and leisure
Time frame	Single period (Stocks seem safe in long run)	Many periods (Stocks are risky in short and long run)
Risk management techniques	<ul style="list-style-type: none"> <li>• Precautionary saving</li> <li>• Diversification</li> </ul>	<ul style="list-style-type: none"> <li>• Precautionary saving</li> <li>• Diversification</li> <li>• Hedging</li> <li>• Insuring</li> </ul>
Retail investment products	<ul style="list-style-type: none"> <li>• Mutual funds</li> </ul>	<ul style="list-style-type: none"> <li>• Structured standard of living contracts</li> <li>• Targeted accounts (e.g., tuition-linked CDs)</li> </ul>
Quantitative model	Mean-variance efficiency and Monte Carlo simulation	Dynamic Programming and Contingent Claims Analysis
Capital market expectations	<ul style="list-style-type: none"> <li>• Estimated from historical statistics</li> </ul>	<ul style="list-style-type: none"> <li>• Inferred from current prices of financial instruments (swap curves and implied volatilities)</li> </ul>

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## **The Theory of Life-Cycle Finance**

I begin with a brief review of the theory underlying this paper. The seminal work is the “state-preference” theory of optimal resource allocation under uncertainty of Arrow and Debreu. The theory begins there, and at a high level of abstraction it also ends there. In that hypothetical world of complete markets for all contingencies, every individual chooses the combination of elementary time-state claims that maximizes expected utility.

As we know, many of those contingent claims do not exist in reality. However, Merton’s theory of continuous time finance provides a link from the Arrow-Debreu world to the real world through the technology of dynamic replication.<sup>3</sup>

Merton’s continuous time model is much more general than the older Markowitz model of portfolio choice that is at the core of most of today’s online asset allocation tools. The Markowitz model assumes that individuals make decisions in a static single-period framework. In Merton’s framework there are several distinct time horizons. The *planning horizon* is the total length of time for which one plans. Thus, for a 25 year old who expects to live to age 85, the retirement planning horizon would be 60 years. As one ages, the planning horizon typically gets shorter and shorter.

The *decision horizon* is the length of time between decisions to revise the portfolio. The length of the decision horizon is controlled by the individual within certain limits. Some people review their portfolios at regular intervals -- once a month (when they pay their bills), or once a year (when they file income tax forms). People of modest means with most of their wealth invested in bank accounts might review their portfolios very infrequently and at irregular intervals determined by some “triggering” event such as getting married or divorced, having a child, or receiving a bequest. A sudden rise or fall in the price of an asset a person owns might also trigger a review of the portfolio. People with substantial investments in stocks and bonds might review their portfolios every day or even more frequently.

The shortest possible decision horizon is the *trading horizon*, defined as the *minimum* time interval over which investors can revise their portfolios. The length of the trading

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<sup>3</sup> I refer to Merton’s continuous-time theory of optimal lifetime consumption and portfolio choice (1969, 1971, 1973, 1975) and his theory of financial intermediation through dynamic replication (1989, 1992, 1998).

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horizon is not under the control of the individual. Whether the trading horizon is a week, a day, an hour, or a minute is determined by the structure of the markets in the economy (for example, when the securities exchanges are open or whether organized off-exchange markets exist).

To add realism to Merton's model, Bodie, Merton and Samuelson (1992) have added a third choice variable -- the amount of work people choose to do. In our model, individuals start out with an initial endowment of financial wealth and earning power from labor (their human capital). The market values of both components of wealth — financial and human capital — change continuously and stochastically. The wage rate (the return on human capital) is perfectly positively correlated with the market return on traded assets. Consumption, wealth, and rates of return are all denominated in units of the consumption good. At each point of time, individuals determine the amount of their consumption, the proportion of their financial wealth to invest in risky assets (versus the safe asset), and the fraction of their maximum possible labor income that they will “spend” on leisure so as to maximize their discounted lifetime expected utility.

The model's results indicate that the fraction of an individual's financial wealth optimally invested in equity should “normally” decline with age for two reasons. The first stems from the fact that human capital is usually less risky than equity and that the value of human capital usually declines as a proportion of an individual's total wealth as one ages. For example, in an individual's early years of work, her wealth is often dominated by relatively safe human capital, so that a large share of her financial wealth should be in risky assets in order to get sufficient risk in her total wealth.

Second, at any given age, the greater the flexibility an individual has to alter her labor supply, the greater the amount she will invest in risky assets. Individuals may be able to offset changes in the value of their financial wealth by changing the amount they work. They may have the opportunity to work longer hours, take on extra jobs, or delay retirement. If younger workers have more opportunity to alter their labor supply than older workers, the share of assets held as risky equity should decline with age.

However the opposite result is also possible. For people with risky human capital, such as entrepreneurs or stock analysts, it may be optimal to start out early in life with no

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stock market exposure in one's investment portfolio and to increase that exposure as one ages.

Other continuous-time life-cycle models have incorporated the important effects of habit formation.<sup>4</sup> Habit formation provides a strong rationale for financial products which guarantee that future consumption will not fall below a level established by prior consumption as a minimum acceptable standard of living.<sup>5</sup>

Finally, transaction costs, agency costs, and cognitive limitations provide important theoretical justifications for financial intermediaries to supply user-friendly guaranteed retail investment products that have only a small number of well-understood options. A guarantee of a minimum rate of return is a good substitute for a course in statistics.

### ***Beyond Mutual Funds***

Most guides to personal finance recommend beginning the process of financial planning by defining goals — e.g., a comfortable retirement or paying for a child's education. This would seem to point towards a consumer demand for targeted investment products that assure achieving that goal. An example would be a college tuition account that hedges future tuition expenditures. Indeed, a bank in New Jersey has been offering this type of product since 1987.<sup>6</sup> Yet currently, the money invested in special accounts earmarked for tuition payments is mostly in mutual funds that hold portfolio mixes of stocks, bonds, and cash.<sup>7</sup> Perhaps people do not understand the risks to which they are exposed.

For many people the most important goal of financial planning is an adequate retirement income. In the past, one institutional response has been defined-benefit pension plans. In a typical defined-benefit plan for salaried employees, those who work for the organization sponsoring the plan their whole career receive a guaranteed life annuity that replaces 70-80% of final salary. The employee "pays" for this annuity by working for the organization for a certain minimum number of years. The plan

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<sup>4</sup> For a listing of papers see Detemple, Garcia, and Rindisbacher, (2002).

<sup>5</sup> See Constantinides (1990), Detemple and Zapatero (1991), Dybvig (1995).

<sup>6</sup> College Savings Bank <http://collegesavings.com/>

<sup>7</sup> In the U.S., these are called 529 accounts. Named after the federal Internal Revenue Code Section that gave these accounts special tax status, 529 Plans were specifically created for college savings. Investment firms like Fidelity have created special "age-based" portfolios for these plans. The younger the plan beneficiary, the higher the fraction invested in equities.

participant does not worry about the risk of a shortfall. That is the concern of the sponsor and in the U.S., the Pension Benefit Guarantee Corporation.<sup>8</sup>

Traditional defined-benefit pension plans have been on the decline in the U.S., eclipsed by cash-balance plans and defined-contribution plans. From a social-welfare perspective, this development might actually be a step backward. Risk is being transferred to those who are least qualified to manage it.<sup>9</sup> For the average participant in an employer-sponsored plan, this switch might cause a decline in welfare, even when offset by other benefits of greater monetary value.

The tendency in the last several years has been to offer participants in self-directed retirement plans more and more investment options. Economists generally believe that people are made better off when offered more choices, as long as they can always choose what they had before.<sup>10</sup> But when people do not have the knowledge to make choices that are in their own best interests, increasing the number of choices does not necessarily make them better off. In fact, it may make them more vulnerable to exploitation by opportunistic salespeople or by well-intentioned but unqualified professionals.

An analogy with medical care might help to clarify my point. Most of us look to physicians and other medical professionals to guide our choices about health maintenance practices and treatments for illnesses. We would not be made better off if the number of alternatives increased without our understanding enough about them to make rational choices. Like surgery, asset allocation is a complex procedure, requiring much knowledge and years of training. No one would imagine that you or I could perform surgery to remove our own appendix after reading an explanation in a brochure published by a surgical equipment company. Yet we seem to expect people to choose an appropriate mix of stocks, bonds, and cash after reading a brochure published by an investment company. Some of them are likely to make serious mistakes.

Inspection of the educational materials distributed to consumers by financial service firms confirms these fears. Consumers are led to believe that stocks are not risky in the

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<sup>8</sup> This guarantee is capped by the PBGC. Above the cap, the employee is at risk of default by the plan sponsor. For plans terminated in 2001, the cap was \$40,704.60 per year. It is adjusted annually.

<sup>9</sup> The AFL-CIO writes: "Defined benefit plans remain the best and soundest vehicles for building and safeguarding retirement income and security."  
<http://www.aflcio.org/publ/estatemnts/feb2002/governance.htm>

<sup>10</sup> But even economists acknowledge that there are exceptions, for example, when people have problems with self-control. Offering a shot of whiskey to a recovering alcoholic does not increase his welfare.



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long run.<sup>11</sup> Online asset allocation tools are heavily biased towards equity investment. Rarely is mention made of safe inflation-protected investments like I bonds or TIPS.

### ***New Measures of Risk and Reward***

A major side benefit of the emergence of markets for new financial instruments such as swaps and options is the new information it provides to investors.<sup>12</sup> Important examples are the term structure of interest rates that can be derived from swap curves and the term structure of implied volatilities from option prices. This information is rarely (if ever) used in today's online asset-allocation tools.

### ***Life-Cycle Financial Products of the Future***

The modern theory of contingent claims analysis provides the framework for the production and pricing of new and improved life-cycle contracts.<sup>13</sup> In Figure 1 I have identified several financial products that offer interesting new opportunities for people to better achieve financial security. Some of these products are currently marketed around the world, while others have yet to be brought to market.

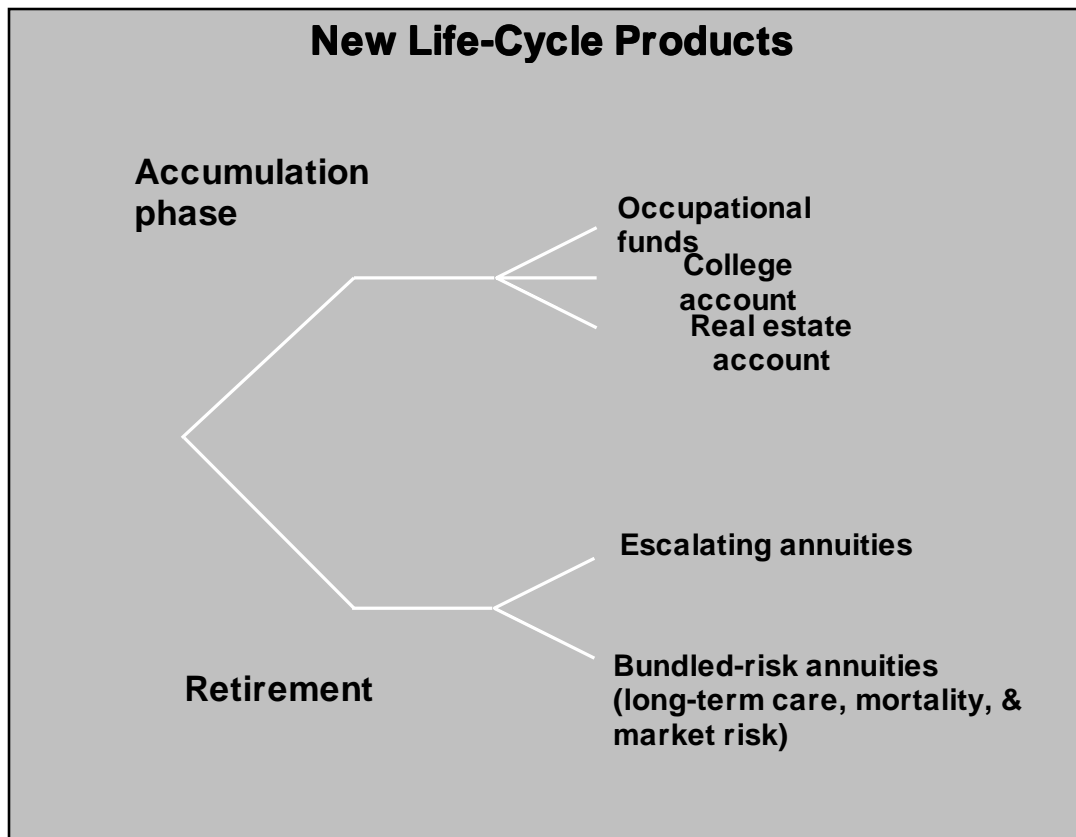
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<sup>11</sup> See Bodie (1995).

<sup>12</sup> On the informational role of financial market prices, see Bodie and Merton (1995).

<sup>13</sup> See Merton (1992) for a detailed analysis of the connection between contingent claims analysis and optimal life-cycle consumption and portfolio policies.

Figure 1



### Escalating Life Annuities

Let us illustrate using the example of a class of assets that I call “escalating annuities.” Traditional annuities in the United States, including those provided by defined-benefit plans have a glaring defect – they are not protected against inflation. Today it is possible for financial intermediaries to efficiently produce annuities that are protected fully or partially against inflation by hedging the liability with TIPS.<sup>14</sup> Moreover, they can also be combined with upside participation in the performance of various stock market indexes. As an example, consider an escalating life annuity with a minimum benefit linked to the cost of living. Payments increase with inflation and with the performance of a market index, and increases are locked in for life.

Escalating annuities conform to the new paradigm laid out in Table 1. They are designed to provide a guaranteed minimum standard of living defined in terms of a flow

<sup>14</sup> See Bodie (1990, 1997).

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of lifetime consumption (rather than a stock of wealth). They allow retirees to gradually increase their consumption if the stock market performs well without jeopardizing the standard of living to which they have become accustomed. Note that this is very different from a variable annuity benefit, which can either go up or down over time depending on market performance.

To make the example more concrete, let us assume that a typical customer reaches age 65 with \$1 million in her self-directed retirement account. She wants to retire and live off her income from Social Security (say \$15,000 per year) and the income generated by her \$1 million retirement account. How would a hypothetical escalating annuity work?

One simple design would be to allow the annuitant to choose the fraction of her \$1 million that would go into the guaranteed real annuity. Assume she chooses 90%, and that this establishes a guaranteed real floor of \$55,000 per year.<sup>15</sup> Together with her Social Security income, this gives her a real income floor of \$70,000 per year. The other \$100,000 in her retirement account would be invested in equities or equity derivatives to produce growth in real income. Each year part of this risky fund would be used to purchase additional guaranteed real annuity income.

The upside leverage of the escalating annuity could be increased by investing the \$100,000 at risk in a series of equity call options maturing in each of the next 10 years. If on the annual expiration date, the call is in the money, then the proceeds are used to increase the guaranteed income floor. If it is out of the money, the floor remains unchanged for another year.

Currently, exchange-traded options have maturities as long as three years. Firms that sell structured equity participation securities such as MITTS have issued notes with maturities of 10 years. It is not hard to imagine that innovative firms might issue even longer-dated index call options over the counter.

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<sup>15</sup> This is an approximation based on quoted rates on Lincoln National Life's Inflation-Proofer annuity.

### **Bundled Risk Annuities**

Research shows that retired people do not voluntarily annuitize much of their wealth. One reason might be that they feel they need to hold onto assets in case they have to finance nursing home care. The problem is that annuities, once bought, tend to be illiquid, so that buyers cannot readily access the needed cash to pay for nursing home bills. In point of fact, longer life expectancies have coincided with increased health care costs near the end of peoples' lives, and the specter of needing two to three years of long-term care (LTC) figures prominently in many discussions of retirement planning.

An integrated instrument could help resolve this problem by combining a life annuity with long-term care insurance. Combining the coverage mitigates the adverse selection that would occur in the demand for each of the two products on a stand-alone basis.

### ***Changing Delivery Systems***

As products and services for addressing the financial risks of retirement are changing, so too are the varieties of institutions available to provide support to the elderly. Today, many diverse retirement-income systems coexist around the world, each relying in varying proportions on one or more of the following institutional forms:

- Support from family or community;
- Pension plans sponsored by employers and/or labor unions;
- Social insurance programs run by governments;
- Personal savings in the form of real and financial assets— equity in one's home or business, savings accounts, insurance contracts, mutual funds, etc.

Many experts agree, however, that the mix of these institutional forms will change significantly in the next few years. This is particularly true for industrialized countries such as the United States, the United Kingdom, Australia, Western Europe, and Japan, where the rapid aging of the population reflects both that people are living longer and having many fewer children. In these nations, people will find they can rely less on family and government support than in the past, instead turning to financial markets and related institutions by saving and investing for their own retirement. Even in emerging markets, new demographic and economic realities have prompted the beginning of

widespread retirement system reforms, as seen in the pension reform movements of Latin America and Eastern Europe, and more recently, in Asia.

In response to global population aging and financial deregulation trends, governments and financial firms are seeking to create new institutions and services that might afford better protection against the financial consequences of old-age illness, disability, and longevity, and to insulate people against both inflation and asset price fluctuations. New opportunities will become available for older persons to continue employment, perhaps on a part-time basis, and to convert their assets, particularly housing wealth, into spendable income.

For better or for worse, these developments mean that people are being given more individual choice over their own asset accumulation and drawdown processes. As these new financial instruments transfer more responsibility and choice to workers and retirees, it will be a challenge to frame risk-reward tradeoffs and cast financial decision-making in a format that ordinary people can understand and implement.

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## References

- Bodie, Zvi, (1990), "Inflation Insurance," *The Journal of Risk and Insurance*, Vol. LVII, No. 4, (December), 634-645.
- \_\_\_\_\_, (1995), "On the Risk of Stocks in the Long Run," *Financial Analysts Journal*, (May/June).
- \_\_\_\_\_, (1997), "Inflation-Protected Retirement Plans," in *Managing Public Debt: Index-Linked Bonds in Theory and Practice*, Edward Elgar Publishers.
- \_\_\_\_\_, (1999), "Investment Management and Technology: Past, Present, and Future," in *Brookings-Wharton Papers on Financial Services*, Brookings Institution Press.
- Bodie, Zvi, and Dwight B. Crane, (1997), "Personal Investing: Advice, Theory, and Evidence," *Financial Analysts Journal*, (November/December).
- \_\_\_\_\_, (1999), "The Design and Production of New Retirement Savings Products," *Journal of Portfolio Management*, (January/February).
- Bodie, Zvi, Hammond, P. B., and Mitchell, O. S. (2001), "New Approaches to Analyzing and Managing Retirement Risks," *Benefits Quarterly*, Vol. 17, No. 4: 72-83.
- Bodie, Zvi, and Merton, R.C., (1995), "The Informational Role of the Financial System: The Case of Implied Volatility," Chapter 6 in *The Global Financial System: A Functional Perspective*, Harvard Business School Press.
- Bodie, Zvi, Merton, R.C. and Samuelson, W., (1992), "Labor Supply Flexibility and Portfolio Choice in a Life-Cycle Model," *Journal of Economic Dynamics and Control*, 16: 427-449.
- Brennan, M. and E. Schwartz (1976), "The Pricing of Equity-Linked Life Insurance Policies with an Asset Value Guarantee," *Journal of Financial Economics*, 3 (June): 195-214.
- Constantinides, G. M., (1990), "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98: 519-543.
- DeLoach, (2000), *Enterprise-Wide Risk Management*, London: Financial Times-Prentice Hall.
- Detemple, Jerome, B., Garcia, R., and Rindisbacher, M., (2002), "A Monte Carlo Method for Optimal Portfolios," forthcoming in *Journal of Finance*.
- Detemple, Jerome, B., and Zapatero, F., (1991), "Asset Prices in an Exchange Economy with Habit Formation," *Econometrica*, 59:1633-1657.
- Dybvig, P. H., (1995), "Dusenberry's Ratcheting of Consumption and Investment Given Intolerance for any Decline in Standard of Living," *Review of Economic Studies*, 62: 287-313.
- Merton, Robert C., (1969), "Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case," *Review of Economics and Statistics*, 51 (August): 247-57. Reprinted in Merton (1992, Ch. 4).
- \_\_\_\_\_, (1971), "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3 (December): 373-413. Reprinted in Merton (1992, Ch. 5).
- \_\_\_\_\_, (1975), "Theory of Finance from the Perspective of Continuous Time," *Journal of Financial and Quantitative Analysis*, 10 (November): 659-74.

- \_\_\_\_\_, (1989), "On the Application of the Continuous-Time Theory of Finance to Financial Intermediation and Insurance," *The Geneva Papers on Risk and Insurance*, Vol. 14, No. 52 (July) 225-262.
- \_\_\_\_\_, (1992), *Continuous-Time Finance*, Revised edition, Oxford: Basil Blackwell.
- \_\_\_\_\_, (1998), "Applications of Option-Pricing Theory: Twenty-Five Years Later," *American Economic Review*, (June): 323-349.
- Meulbroek, Lisa, (2002), "A Senior Manager's Guild to Integrated Risk Management," *Journal of Applied Corporate Finance*, Volume 14, No. 4, (Winter): pp. 56-70.