DISCUSSION PAPER PI-1802

Actuarially fair accrual rates and deferral of the UK state pension.

John Dagpunar.

February 2018

ISSN 1367-580X

The Pensions Institute
Cass Business School
City University London
106 Bunhill Row
London EC1Y 8TZ
UNITED KINGDOM

http://www.pensions-institute.org/

Actuarially fair accrual rates and deferral of the UK state pension

3 February 2018

John Dagpunar, Mathematical Sciences, University of Southampton, SO17 1BJ, UK

Address for correspondence: j.dagpunar@soton.ac.uk

Abstract: *Persons who have achieved state pension age (SPA) are entitled to defer taking the pension and instead receive an extra pension on termination of deferral. The Department of Work and Pensions (DWP) asked the UK Government Actuary to advise on an actuarially fair scheme, which is interpreted to mean one that gives on average no pre-tax advantage over a lifetime to the deferrer and consequently remains cost neutral to the Exchequer. After a review of the literature on deferral or early take-up of state pensions in the UK and other countries, this paper argues that the current scheme based upon a uniform accrual rate cannot meet the objectives of fairness. Instead, we propose a scheme where the accrual rate varies dynamically according to number of years deferred, gender, state pension age, number of years deferred under the existing uniform rates*, and *real rate of pension uprating. The model, developed in continuous time, is extended to the more complex scenario of a deferrer with partner entitled to inherit deferrer's extra pension or lump sum. Accrual rate curves are plotted for various scenarios and compared with the current uniform rates of 5.8% and 10.4% that apply to those who attained state pension age after 5 April 2016 or before 6 April 2016 respectively. The 10.4% scheme is shown to be actuarially unfair for a wide range of real pension uprating rates, at cost to the Exchequer. The methodology should be applicable to public pensions in other countries.*

Keywords: *UK State pension, deferral, actuarial fairness, actuarial neutrality*

1. Introduction and background to State Pension deferral and Actuarial fairness

This paper is motivated by a report (Government Actuary's Department, 2014) to the Department of Work and Pensions (DWP) on the determination of an actuarially fair value for β, the accrual rate for earning extra pension for those who choose to defer taking state pension beyond state pension age (SPA). If such a person defers for a period x then on terminating deferral the person is entitled to take his/her un-deferred periodic pension plus an *extra pension* equal to a proportion βx of the un-deferred pension. For those who achieve SPA before 6 April 2016 and therefore take the state retirement pension (RP), $\beta = 0.104$ per year deferred, while for others who take the new state pension (SP), $\beta = 0.058$ per annum deferred.

In the former (RP) case, on termination of deferral a person will choose to take either the periodic extra pension, or a lump sum equal to the total pension foregone during deferral, with in the latter case for those who have deferred for at least 12 months, interest added at a rate of 2% per annum above bank base rate. For SP persons the lump sum option is not available. For both RP and SP persons, death during deferral results in neither a periodic pension nor lump sum although the deceased's estate receives up to three months of pension foregone. A crucial difference between RP and SP pensioners is that in the former case a spouse or civil partner of the deceased will, for death during deferral, choose to inherit either the periodic extra pension or lump sum with interest, and for death after deferral will inherit the extra pension if that was the choice made at the end of

deferral. Such inheritance will have to wait until the partner reaches SPA (irrespective of whether or not that occurs before 6 April 2016), and then only if at that point in time he/she has neither remarried nor taken on a new civil partnership. If the deceased had deferred for less than a year then the surviving partner would have to inherit the periodic extra pension. For many RP deferrers the *basic pension* component will account for much if not all of the un-deferred pension. The inheriting partner can inherit 100% of the extra state pension or lump sum that the deferrer has earned on the basic pension. The corresponding proportions for other components of the un-deferred (RP) pension are 50% for Graduated Retirement, 50% for State Second Pension, between 50% and 100% for SERPS, and 50% of inherited extra pension from a recognised legal partner.

In addition to regular reviews of the appropriateness of current state pension ages, the details of the deferral scheme have also come under review prompting the Government Actuary's report. He says that '*the concept of actuarially fair*[1] *is* subjective. He interprets it to mean '*… at state pension age, the benefits available have broadly the same value whether the person chooses to defer or not*...' and later '*… the benefits available have broadly the same value in terms of cost to the Exchequer … whether the person chooses to defer or not.*' As the report highlights, rates which are truly actuarially fair will vary according to gender, state pension age, length of deferral, rate of pension uprating, to which list one might also add the discount rate to be applied to future payments, and in the case of RP persons their marriage/civil partnership status.

The fact is that it can be difficult to deliver an actuarially fair scheme when the extra pension is awarded according to a linear function βx, as some deferral periods will be more advantageous than others to the deferrer and therefore more costly to the Exchequer. It is of course true that one would not want to change the rate retrospectively as that might well compromise historical benefits accrued to date. However, a more dynamic setting of accrual rates in future, determined by gender, SPA, number of years deferred to date, and the real rate at which pensions are uprated, could be prescribed. The objective would be to ensure that the expected net present value of benefits for an individual remains constant from the point at which actuarial fairness is introduced, irrespective of how much longer he/she defers. Naturally, that also means that the scheme is cost neutral to the Exchequer. One might ask: what is the point of deferral with actuarial fairness if it has a neutral effect upon pre-tax net present value? One reason is that it can incentivize such persons to be economically productive without raising their marginal income tax rate. Arguably, this has benefits for both the individual and society.

The purpose of the present paper is to develop a model in continuous time which leads directly to actuarially fair rates by equating to zero the marginal benefit of extending the deferral period. It is applied to the UK situation. After a summary of the literature in section 2 we develop in section 3 a model for deferrer without spouse or civil partner. Section 4 shows specimen values when the pension uprating rate is identical to the discount rate. Section 5 gives results when they are not equal. Section 6 shows how the derived rates will have to be different for those who are already deferring at the point at which actuarial fairness is introduced. The rates would be different because a deferrer's position cannot be allowed to worsen after the introduction of actuarial fairness. Section 7 addresses the more challenging case of a deferrer with partner and inheritance options. Section 8 summarises and concludes.

[1] Some writers use 'actuarially fair', others 'actuarial neutrality'. 'Actuarially fair' has also been used to describe a different objective of equating Net Present Value of lifetime contributions to lifetime benefits.

2. Review of Literature

Useful background to the UK state pension system can be found in Bozio et al. (2010) and Thurley (2016). With changing demography including increased life expectancy, the system needs to be kept under regular review. The sustainability has been examined by Blake and Mayhew (2006) while Moizer et al. (2017) specifically look at the long term effect on the National Insurance Fund. They use a Systems Dynamics model over a 40-year horizon.

Several authors have looked at the deferral problem. In the context of the UK system, rules for deferral are described in Thurley (2010, 2017). In the UK, a break-even analysis that takes into account tax treatment is given by Farrar et al. (2012), while Stubbs and Adetunji (2016) and Kanabar and Simmons (2016) consider a deferrer without partner and without modelling of the survival risk. Dagpunar (2015) calculates optimal deferral periods with explicit modelling of the survival risk and this is extended to those with marriage or civil partnerships in Dagpunar (2017).

Mirer (1998) looks at the US case where there are accruals for deferral and decrements for early take-up. Coile et al. (2002) develop two models for the US case, one maximizing expected net present value of pension payments, and the other maximizing expected utility. They use simulation in a discrete time setting for various scenarios of gender, discount rate, mortality rates**.** Duggan and Soares (2002), again working in discrete time in the US Social Security system, calculate actuarial adjustment factors for each year's deferral and then compute the difference in benefits compared with the use of the statutory adjustment factors. They mention how authors have differing views on the setting of a discount factor. A parallel piece of work in Canada is detailed in Canada Pension Plan Actuarial adjustment factors (2003), set up to examine the appropriateness of the statutory uniform accrual rate of 6% p.a. for delays from age 65-70, and a penalty rate also of 6% for early retirement from 60-65. It was found to be too generous for early pension take-up and to penalize those deferring. Queisser and Whitehouse (2006) develop a discrete time formula for adjustment factors to be used for actuarial neutrality (a term they, along with many authors, use for actuarial fairness as understood here) and they derive some results based upon OECD average mortality rates. Medijainen (2011) uses these adjustment factors to derive rates for the Estonian system. He deduced that the 10.8% accrual rate for late deferrals, with no upper limit, is too generous to the deferrer.

Rasdal (2013) is concerned with reasons for deferral and the probabilities thereof as a function of an individual's or couple's attributes. There is the question as to deferrer's perception of his/her life expectancy. She highlights the difference between a couple's subjective and objective life expectancy given their attributes, the subjectivity of the discount factor, and the selection/moral hazard issue implicit in the implicit annuity take-up. Shoven and Slavov (2014) return to the US case. There, full retirement age is 66 for births in 1943-54. They show delay is actuarially advantageous for many. The method used is simulation. Pension can be claimed as early as 62, receiving 75% of the primary insurance amount (PIA). For delay beyond 66, there is a uniform accrual rate of 8% p.a. Spousal benefits can be claimed as early as 62 but only after the primary earner has claimed. Calculations are performed for different ethnic groups and health categories. Healthy individuals were assigned 75% of the population mortality rates, the less healthy 200%. The complicated rules for couples justify the use of simulation rather than a purely analytical model. They concluded that primary earners in two earner couples should always delay to 70. Heiland and Yin (2014) look at

early and late take-up of US Social Security benefits, computing actuarially fair accrual rates, and extend this to the case of married or civil-partnered deferrers. This does not have the additional complexity of the inherited lump sum option that is part of UK RP system. Meneu et al. (2016) look at intergenerational unfairness (contributions vis-à-vis benefits). Oksanen (2005) concentrates on adjusting parameters of the pension system to achieve both actuarial fairness (contributions versus benefits) and actuarial neutrality with respect to deferral and pension payments. He gives a numerical illustration to defined benefit schemes in the UK with some emphasis on inter-generational fairness. Rose (2015) concludes that delaying take-up in the US Social Security system can give substantial gains to married couples due to an inheritance provision. He also deals with the penalty for early retirement. As regards appropriate discount rate, he suggests that since social security is an inflation-linked obligation, the appropriate discount rate is that available on 20-year indexed US treasury bonds. As others have also observed he concludes that the present regime is very favourable towards deferral, particularly for the primary insured in a married couple. He calculates rate of return and concludes this is better than on Treasury index-linked instruments.

3. A model for unattached (single) deferrer, both RP and SP

We propose a modification whereby the uniform accrual rate β is replaced by a time dependent one $\beta(x)$ where x is the time deferred to date. Let $\mathrm{B}(x)$ denote the cumulative accrual multiple for a deferral of x years. Then $\mathrm{B}(x)=\int_0^x \beta(u)du$. Let a denote state pension age, $\alpha(t)$ the rate at which pensions are uprated (in the case of RP we assume that all components are uprated at the same rate as the basic pension) and $\mu(t)$ the discount rate at age $a+t$. Then $\lambda(t)=\mu(t)-\alpha(t)$ is the real rate of decrease in periodic pension at age $a+t$. We might model this as an Ornstein-Uhlenbeck process as in the Vasicek (1977) model for interest rates. A drawback of that model is sometimes said to be that it can lead to negative interest rates, whereas here we do require a model that does not sign-restrict $\lambda(t)$. Accordingly, $d\lambda=c(d-\lambda)dt+\sigma dW(t)$ where $\{W(t):t\geq 0\}$ is a Wiener process. Let $S(t)$ denote the probability of surviving to at least age t and $m(t)$ the mean residual life (life expectancy) at age t. Now suppose that an individual plans to stop deferral at age $a+x+dx$ rather than at age $a+x$. The present value (PV), referred to age $a+x$, of pension sacrificed in that time increment dx is $[1+\mathrm{B}(x)]dx$, the monetary value expressed in units of the un-deferred pension that would have been payable at age $a+x$. With probability $[1-r(a+x)dx]$, the increase in conditional expected NPV payments over the residual life is $d\mathrm{B}(x)\mathrm{E}\left[\int_x^\infty \frac{S(a+v)\exp\{-\int_x^v \lambda(u|\Im_x)du\}}{S(a+x)}dv\right]$, and with probability $r(a+x)dx$ it is zero. Now let $V(x)dx$ denote the conditional increase in expected NPV for delaying termination of deferral by this further increment dx. Then bringing the above together

$$V(x)dx=-[1+\mathrm{B}(x)]dx+d\mathrm{B}(x)\int_x^\infty \frac{S(a+v)\mathrm{E}[\exp\{-\int_x^v \lambda(u|\Im_x)du\}]}{S(a+x)}dv \tag{1}$$

We now *define* actuarial fairness at age $a+x$ to be attained by any function $\mathrm{B}(.)$ satisfying $V(x)=0$ [2] that is

$$-\left[1+\mathrm{B}(x)\right]+d\mathrm{B}(x)\int_x^\infty \frac{S(a+v)\mathrm{E}[\exp\{-\int_x^v \lambda(u|\Im_x)du\}]}{S(a+x)}dv=0 \tag{2}$$

The actuarially fair accrual rate is

$$\beta(x)=\frac{d\mathrm{B}}{dx}=\frac{1+\mathrm{B}(x)}{\int_x^\infty \frac{S(a+v)\mathrm{E}[\exp\{-\int_x^v \lambda(u|\Im_x)du\}]}{S(a+x)}dv} \tag{3}$$

Mamon (2004), for example, gives a closed form expression for the expectation term as

$$\mathrm{E}[\exp\{-\int_x^v \lambda(u|\Im_x)du\}]=\exp\{-\lambda(x)A(x,v)+D(x,v)\} \tag{4}$$

where

$$A(x,v)=\frac{1-e^{-c(v-x)}}{c} \tag{5}$$

and

$$D(x,v)=\left(d-\frac{\sigma^2}{2c^2}\right)\left\{A(x,v)-(v-x)\right\}-\frac{\sigma^2 A(x,v)^2}{4c} \tag{6}$$

The parameters could be calibrated using historical pension uprating and discount rates. This is left for further study.

In this paper we take a simplified model where at age $a+x$ it is assumed that $\lambda(u|\Im_x)=\lambda(x)$ for all $u\geq x$. It follows that

$$V(x)=-\left[1+\mathrm{B}(x)\right]dx+d\mathrm{B}(x)\int_x^\infty \frac{S(a+v)\exp\{-\lambda(x)(v-x)\}}{S(a+x)}dv \tag{7}$$ [3]

Setting $V(x)=0$

$$-\left[1+\mathrm{B}(x)\right]dx+d\mathrm{B}(x)\int_x^\infty \frac{S(a+v)\exp\{-\lambda(x)(v-x)\}}{S(a+x)}dv=0 \tag{8}$$

and the actuarially fair accrual rate is

$$\beta(x)=\frac{1+\mathrm{B}(x)}{\int_x^\infty \frac{S(a+v)\exp\{-\lambda(x)(v-x)\}}{S(a+x)}dv} \tag{9}$$

---

[2] This ensures that the act of extending deferral by an amount dx gives zero change to the deferrer's expected NPV, meaning that we neither add to nor subtract from the expected benefits or dis-benefits that the deferrer has accrued in $(a,a+x)$, and consequently there is no change in expected cost to the Exchequer.

[3] It follows that a deferrer's expected NPV of benefits will improve by continuing deferral only if $\beta(x)>\frac{1+\mathrm{B}(x)}{\int_x^\infty \frac{S(a+v)e^{-\lambda(x)(v-x)}}{S(a+x)}dv}$. In particular, if $\mathrm{B}(x)=\beta x$ and $\lambda(x)=0$ the condition becomes $m(a+x)>x+\beta^{-1}$

Results (8) and (9) can be used to update $\beta(x)$ continuously to reflect time varying $\lambda(x)$. In practice it might be done on a quarterly or yearly basis. Now suppose that $\mathrm{B}(x_0)$ encapsulates accruals in $(a, a+x_0)$ that are not necessarily actuarially fair. One scenario is that a person has deferred in this interval during which time there has been a uniform accrual rate or λ has not been updated to reflect the real economic context. However, we now require actuarial fairness for any $x > x_0$. In that case the solution to (8) subject to $\mathrm{B}(x_0) = \mathrm{B}_0$ say, is

$$\mathrm{B}(x) = \frac{[1+\mathrm{B}_0]\int_{x_0}^{\infty} S(a+u)e^{-\lambda(x_0)u}du}{\int_{x}^{\infty} S(a+u)e^{-\lambda(x_0)u}du} - 1 \qquad (x > x_0) \quad [4] \tag{10}$$

If we assume that the discount rate equals the pension uprating rate then $\lambda(x) = 0$ and so for all $x > x_0$ results (8-10) become

$$-[1+\mathrm{B}(x)] + \frac{d\mathrm{B}}{dx} m(a+x) = 0 \tag{11}$$

$$\beta(x) = \frac{1+\mathrm{B}(x)}{m(a+x)} \tag{12}$$

and

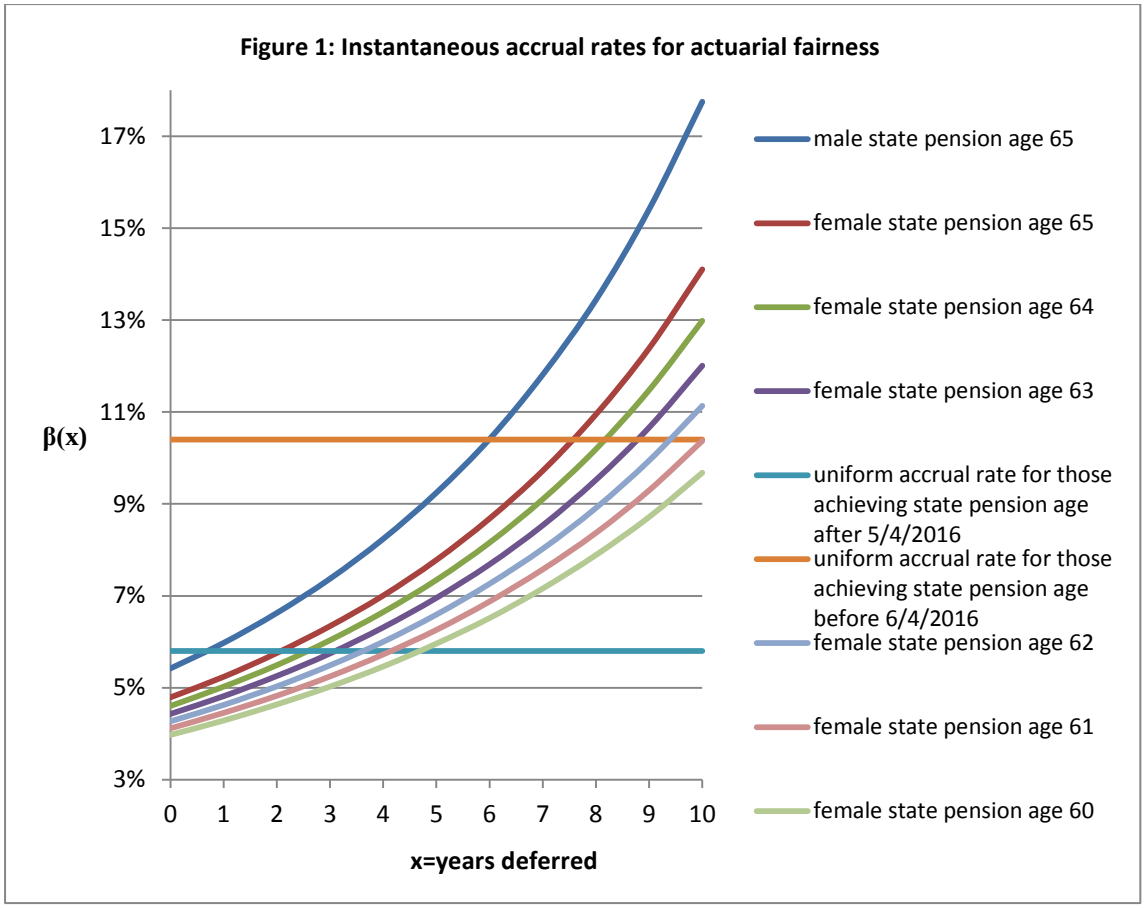
$$\mathrm{B}(x) = \frac{[1+\mathrm{B}_0]m(a+x_0)S(a+x_0)}{m(a+x)S(a+x)} - 1 \tag{13}$$

4. Sample instantaneous accrual rates when uprating rate is same as discount rate

We use Office of National Statistics (ONS) life tables[5] to calculate and display in figure 1 the actuarially fair accrual rates for males of state pension age 65 and females of state pension age 65 through to 60, for the case where a government introduces actuarial fairness before a person reaches SPA, that is where $x_0 = 0$. We take $\lambda(x) = \lambda = 0$; this is later relaxed in section 5. It should be recognised that the choice of discount rate is somewhat subjective. It could be chosen as the rate of return on a risk-free investment (currently considerably less than pension uprating rates), or on an investment alternative to deferral that has a similar risk profile to the survival risk, or as the Government Actuary has used, a rate that is 3% higher than the Consumer Price Index (CPI) rate to reflect a preference for 'consumption now rather than later'. A choice of $\lambda = 0$ tracks the middle scenario and is appropriate if one believes that the uprating and investment context will ensure that a pension broadly retains its purchasing value. This is perhaps not unreasonable if one takes the view that pension uprating may continue to roughly track the Consumer Price Index (CPI). For comparison, figure 1 also displays the current uniform accrual rates of 10.4 % (RP) and 5.8% (SP).

[4] $1+\mathrm{B}(x)$ is essentially the adjustment factor used by actuaries who usually develop the analysis in discrete time.

[5] https://www.ons.gov.uk/file?uri=/peoplepopulationandcommunity/birthsdeathsandmarriages/lifeexpectancies/datasets/nationallifetablesunitedkingdomreferencetables/current/nltuk1315reg.xls



The most obvious feature is that the RP 10.4% is delivering considerable benefits to the deferrer at a cost to the Exchequer. For SP males the current uniform 5.8% is delivering approximate actuarial fairness for deferral periods less than about 1.5 years (the point at which the area under the actuarially fair curve equates to the area under the uniform 5.8% line) but with deferral periods in excess of that it is delivering somewhat inferior benefits compared with not deferring. The situation for females is slightly better due to longer life expectancy. The departure from actuarial fairness can be quantified by plotting the expected equivalent pension years $EPY(x)$ against deferral period where

$$EPY(x) = \frac{S(a+x)}{S(a)}[1+\beta x]m(a+x) \tag{14}$$

Figures 2 and 3 show this for $\beta = 0.058$ and $\beta = 0.104$ respectively. Note that for women achieving SPA after 5 April 2016 (SP), their SPA must be at least 63 years.

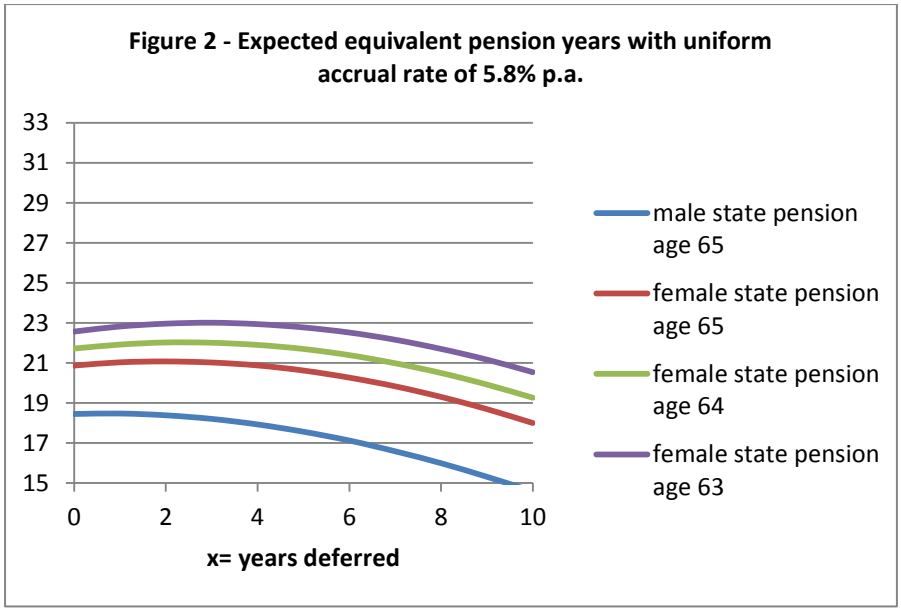
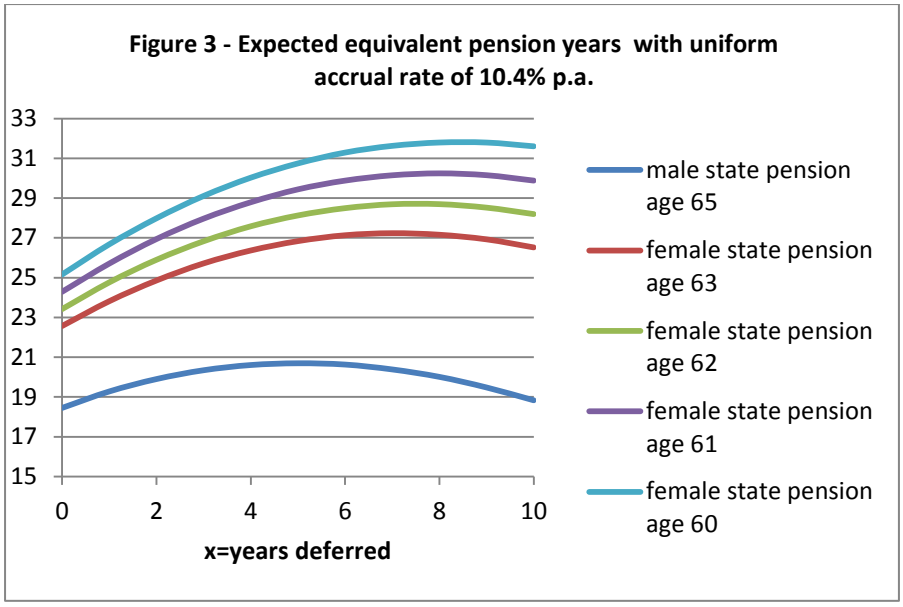


Figure 2 shows that for SP females any deferral less than about 4 years achieves roughly actuarial fairness. For males, the same is true for deferrals under 1.5 years. Longer deferrals give worse outcomes to the deferrer and cost savings to the Exchequer. The situation is entirely different for RP persons as shown in figure 3. For males, any deferral less than 10 years is advantageous to the deferrer and a cost to the Exchequer, with an optimum period of 5 years giving a modest expected increase equivalent to just over 2 years of un-deferred pension. For females, deferral offers even greater rewards due to longer life expectancy and lower SPA. It is clear that the current 10.4 % is actuarially unfair at significant cost to the Exchequer. Thus, the current 10.4% accrual rate fails the most basic tests of not imposing an extra burden on the Exchequer and of ensuring that the pensioner is (on average) in neither a better nor worse position than not deferring. The redistributive effects might be considered regressive, arguably benefiting those who find themselves in a financial position to defer at the expense of those who do not.[6] As Duggan and Soares (2002) say 'benefit adjustments that deviate systematically from actuarial equivalence create some (perhaps unintended) redistribution of benefits'.

[6] The same might be said of tax relief and National Insurance concessions on pension contributions for those subject to higher rate income tax. HM Treasury states 'In 2015/16 income tax and employer National Insurance Contributions relief cost around £50 billion, with around two thirds going to higher rate tax payers' – personal communication, the Rt. Hon. Greg Clark, MP, July 2017.



The foregoing analysis assumes no 'selection effect' where one may conjecture that people who choose to defer have a somewhat higher life expectancy[7] and it also ignores the three-month pension paid to an estate after death during deferral; the impact of the latter is thought to be small compared with that due to uncertainty about future pension uprating and discount rates.

5. Sample results for different uprating rates

As mentioned in the previous section, it is not unreasonable to assume that $\lambda(x) = 0$. Equally, arguments in favour of setting λ to be positive include one advanced in the Government Actuary's report, namely a preference for consumption now rather than later. A contrary view is that the cost of living including social care costs is likely to increase with age, supporting a negative value of λ. Given the reasonableness of each of these diverse scenarios it is appropriate to explore how the actuarially fair rates will respond, and we show in table 1 the situation for a single male of SPA 65, using results (9) and (10) where $\lambda(x) = \lambda$ for all x and where $x_0 = 0$.

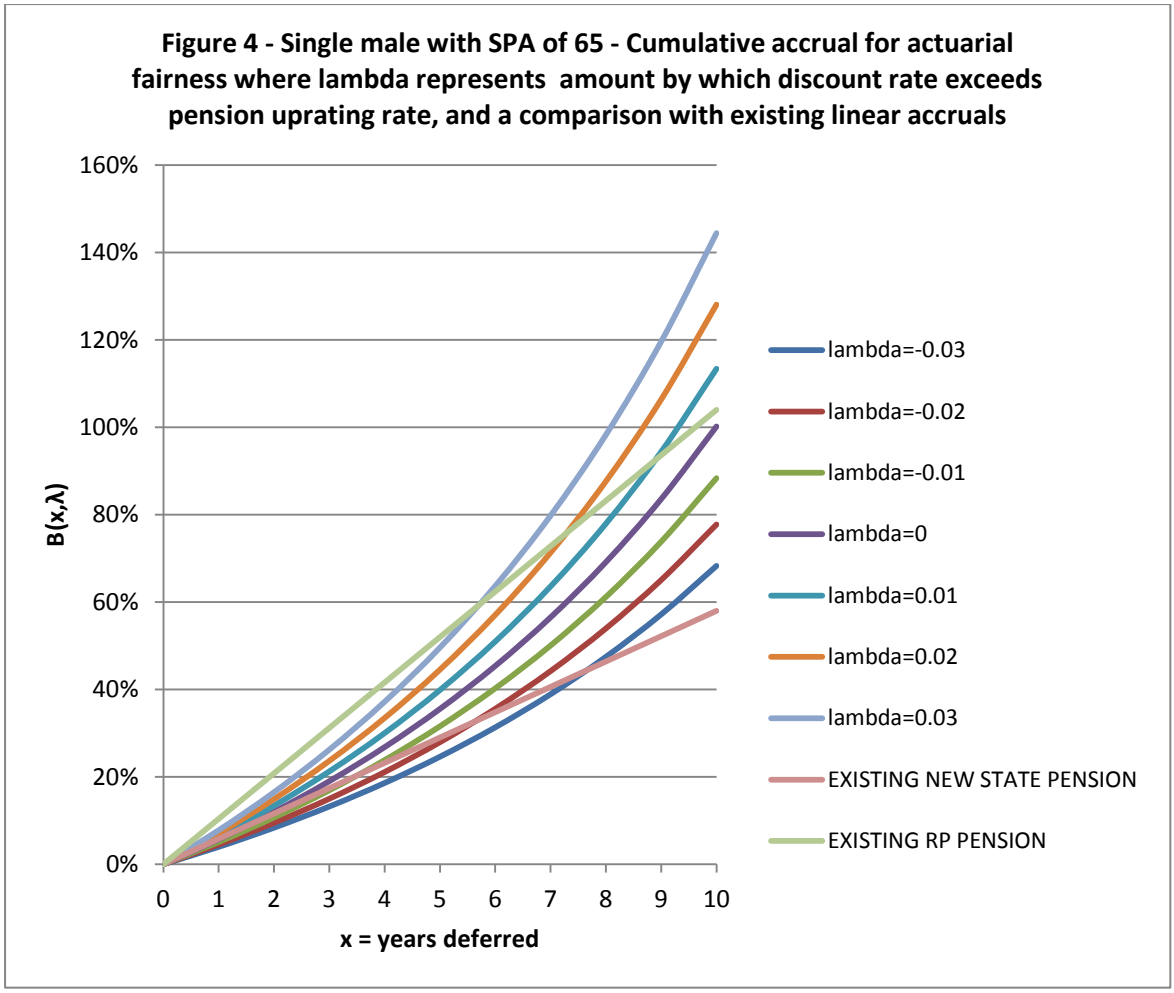
[7] We have assumed that $\{S(x)\}$ for deferrers is the same as for the general population. As the Government Actuary points out there may be a selection effect. This can be dealt with by enhancing life expectancies in the manner of Dagpunar (2015) or by decreasing the mortality rates by 15% as in the Government Actuary's report. However, to do either will inevitably make persons subject to standard population mortality rates worse off than not deferring, thereby introducing unfairness.

Table 1 - Single male with SPA of 65 years: Specimen percentage values of the actuarially fair instantaneous accrual rate $\beta(x)$ for various values of λ, the amount by which the discount rate exceeds the pension uprating rate.

$x \backslash \lambda$	-0.03	-0.02	-0.01	0	0.01	0.02	0.03
0	3.8%	4.3%	4.8%	5.4%	6.0%	6.7%	7.4%
1	4.2%	4.7%	5.3%	6.0%	6.7%	7.4%	8.2%
2	4.6%	5.2%	5.9%	6.6%	7.4%	8.3%	9.2%
3	5.1%	5.8%	6.6%	7.4%	8.3%	9.3%	10.3%
4	5.7%	6.5%	7.3%	8.3%	9.3%	10.4%	11.6%
5	6.4%	7.2%	8.2%	9.3%	10.4%	11.8%	13.2%
6	7.1%	8.1%	9.2%	10.4%	11.8%	13.3%	15.0%
7	8.0%	9.2%	10.4%	11.8%	13.4%	15.2%	17.2%
8	9.1%	10.4%	11.8%	13.5%	15.4%	17.5%	19.8%
9	10.4%	11.9%	13.5%	15.5%	17.7%	20.1%	23.0%
10	11.9%	13.6%	15.6%	17.8%	20.4%	23.4%	26.8%
11	13.7%	15.7%	18.0%	20.7%	23.8%	27.4%	13.7%

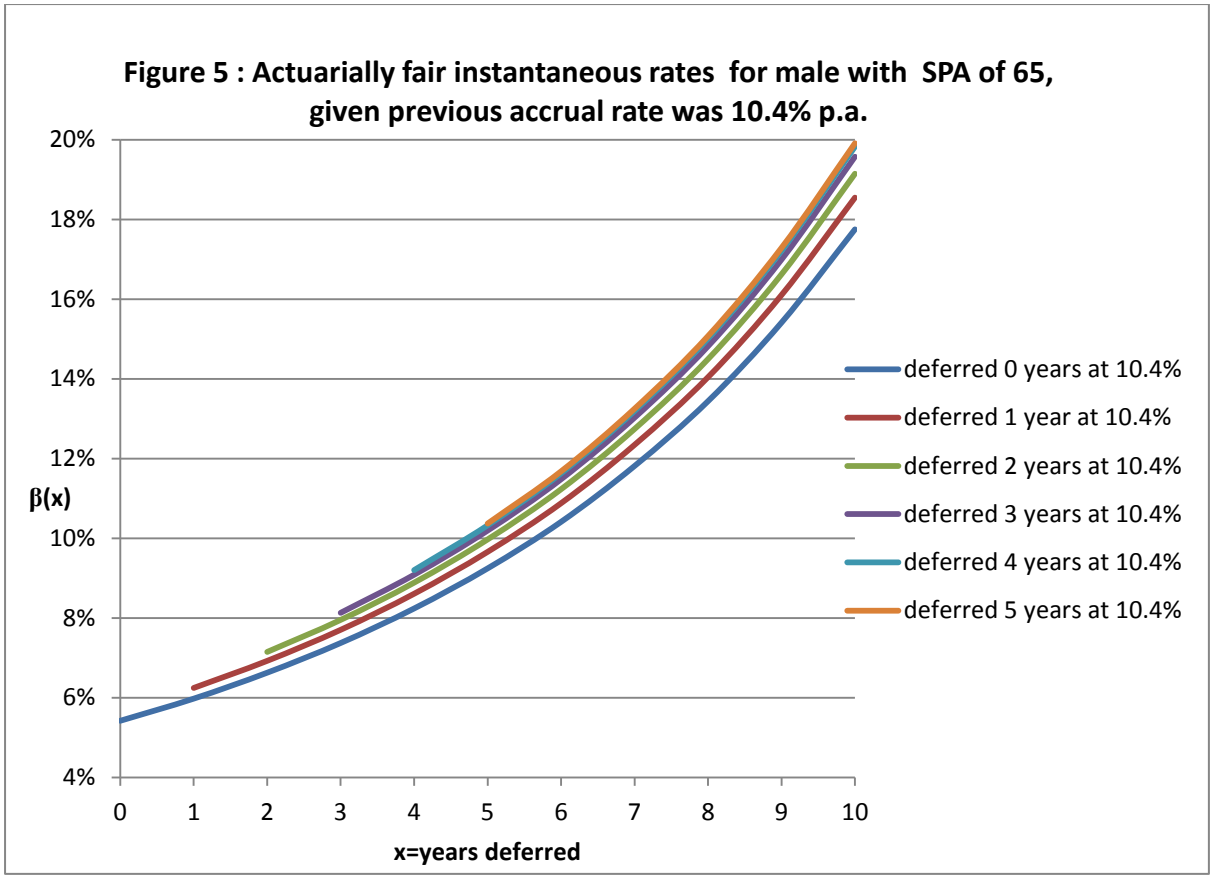
For positive values of λ the real value of a pension is decreasing as time passes and therefore $\beta(x)$ is increasing in λ for fixed x. At the time of writing (February 2018) the triple lock value is the CPI rate of 3.0% p.a. Suppose the discount rate is chosen to be 2% p.a. Then $\lambda = \mu - \alpha = 0.02 - 0.03 = -0.01$. If this were to be maintained throughout deferral then reference to the table shows that the rate during the first year should be approximately $0.5(4.8+5.3) = 5.05\%$, during the second year $0.5(5.3+5.9) = 5.6\%$ and so forth.

Figure 4 shows the cumulative accrual to date $\mathrm{B}(x)$ and it is apparent that the choice of λ can have an appreciable effect. It is also evident that since the majority of (x, λ) scenarios lie below the existing linear RP case but above the SP one, that in general RP is delivering to a 65 year old male more than a fair deal, and if anything SP is delivering less.



6. For those currently under deferral

In figure 5 we show $\beta(x)$ for a RP male with SPA of 65 years when $\lambda = 0$ for $x_0 = 0,1,2,3,4,5$. For example, the curve for $x_0 = 2$ is used if a man reached state pension age before 6 April 2016 and had already deferred for two years when a government introduces actuarial fairness. During these two years the accrual rate would have been 10.4% p.a. and then drops to 7.15%, rising to 7.95% after 3 years, 9.08% after 4 years, 9.97% after 5 years, and so on. Had the government introduced actuarial fairness before he reached state pension age then he would receive the equivalent of $m(65) = 18.45$ years of un-deferred pension no matter how long he deferred. As it happens, after deferring 1 year under an accrual rate of 10.4% he would receive the equivalent of 19.28 years, and after 2 years, 19.90 years. At that point the new lower rates apply and he receives 19.90 years no matter how much longer he defers. He does enjoy a better outcome than someone who is subject to actuarially fair rates throughout deferral, but that is unavoidable given the principle of not worsening the position of someone who already has a history of deferral on introduction of an actuarially fair scheme.



7. For those having a partner

We consider a deferrer (A) with SPA of a and his/her partner (B) having age b at the time at which A reaches age a, assuming that the pension uprating rate equals the discount rate. Since this section assumes that B can inherit, it means that A is entitled to a state retirement pension (RP), that is A achieved SPA before 6 April 2016. To illustrate the approach we will consider the easier to analyse case where $\lambda(x) = \lambda = 0$ and where b is at least equal to B's state pension age. This means that if A dies first then B will immediately be eligible for inheriting benefits as described in section 1, unless A had previously taken the lump sum. We also assume that the interest rate applied to lump sum payments equals the discount rate.

Let $m_A(x)$ and $m_B(x)$ denote their respective mean residual lives at ages x and let $M(x_A, x_B)$ denote the expected time to the second death of A and B given that they have respective ages x_A and x_B. Let $S_A(x)$ and $S_B(x)$ be the respective survivor functions for A and B and $r_A(x)$ and $r_B(x)$ their respective mortality rates. As before, we express all monetary values in units of the un-deferred periodic pension per annum that A would receive at age a. If A has just stopped deferring after x years where both deferrer and survivor are still alive, we assume that a rational A will take the extra pension in preference to the lump sum[8]. Regardless of who is the last survivor, A will receive an expected NPV of $m_A(a+x)$. The expected residual life of the partnership is

[8] Under Actuarial Fairness, surviving the deferral period has implicit value favouring extra pension over lump sum, unless either A or B has suffered a sufficiently large downgrade from the population life expectancy.

$M(a+x,b+x)$ and therefore the expected value of the extra pension that it will earn is $\mathrm{B}(x)M(a+x,b+x)$. Accordingly, the expected NPV of pension payments over the residual lifetime of the partnership as

$$\mathrm{B}(x)M(a+x,b+x)+m_A(a+x)\,^{9} \tag{15}$$

It is possible that either A or B dies before the end of the planned deferral period x is reached. If A predeceases B at age $a+u$ (where $u<x$), then B can choose to inherit either a lump sum of u or a periodic extra pension of $\mathrm{B}(u)$ per year. If B predeceases A at age $b+u$ (where $u<x$), then A chooses either to take a lump sum of u plus the periodic un-deferred pension, or take both extra pension of $\mathrm{B}(u)$ p.a. and periodic un-deferred pension, or to continue to defer. Now suppose that A plans to stop deferral at age $a+x+dx$ rather than at age $a+x$. The pension sacrificed in that time increment dx is $[1+\mathrm{B}(x)]dx$. With probability $[1-r_A(a+x)dx][1-r_B(b+x)dx]$ the increase in conditional expected NPV over the residual life of the partnership is $M(a+x,b+x)d\mathrm{B}(x)$. With probability $r_A(a+x)dx[1-r_B(b+x)dx]$ the conditional expected additional PV for B (compared with termination at $a+x$) is $\max[x-\mathrm{B}(x)m_B(b+x),0]$[10]. With probability $r_B(b+x)dx[1-r_A(a+x)dx]$ the conditional expected increase in PV for A is $\max[x-\mathrm{B}(x)m_A(a+x),0]$. Note that we have eliminated the possibility of A continuing to defer on B's death[11]. This is a consequence of the fact that $m_A(a+x)<M(a+x,b+x)$. Now let $V(x)dx$ denote the conditional increase in expected NPV to the partnership for delaying termination of deferral by this increment dx. Then bringing the above together

$$\begin{aligned} V(x) = {} & -1-\mathrm{B}(x)+M(a+x,b+x)d\mathrm{B}(x)+r_A(a+x)\max[x-\mathrm{B}(x)m_B(b+x),0] \\ & +r_B(b+x)\max[x-\mathrm{B}(x)m_A(a+x),0] \end{aligned} \tag{16}$$

Now suppose that A has deferred for a period x_0 at the point at which actuarially fair rates are to be brought in. Then this is achieved by setting $V(x)=0$ [12] [13] for all $x>x_0$ subject to $\mathrm{B}(x_0)=\beta x_0$ where β is the uniform accrual rate that has previously been applied. Then for all $x>x_0$

---

[9] $M(a+x,b+x)$ is computed using

$$\frac{dM(a+x,b+x)}{dx}=r_A(a+x)\{M(a+x,b+x)-m_B(b+x)\}+r_B(b+x)\{M(a+x,b+x)-m_A(a+x)\}-1$$

It is assumed that the respective times until death of A and B are independently distributed.

[10] To see this, note that if A terminates deferral at $a+x$ then A will take the extra pension which, if A dies in the next increment dx, leads to a future expected benefit to B of $\mathrm{B}(x)m_B(b+x)$. If A continues deferral beyond $a+x$ and dies within the increment dx then B is free to choose between taking either the lump sum x or $\mathrm{B}(x)m_B(b+x)$. In the first case the reward for delaying termination is $x-\mathrm{B}(x)m_B(b+x)$ and in the second it is zero.

[11] Under Actuarial Fairness, while both A and B are alive, A is indifferent between terminating or continuing with deferral. However, on B's prior death, the expected residual life of the partnership is reduced from $M(a+x,b+x)$ to $m_A(a+x)$, meaning that continuation of deferral would be sub-optimal behaviour for A.

[12] An alternative approach is to equate NPVs of different deferral periods. While this works well for the case of a single deferrer, it is more onerous for the inherited case considered here, and the marginality argument is preferred leading more easily to the derived condition.

[13] We can now prove the previous assertion that on B's prior death a rational A would not continue deferral. From (7), the conditional rate of increase of expected NPV for A following B's death is $V(x)=-1-\mathrm{B}(x)+\beta(x)m_A(a+x)<-1-\mathrm{B}(x)+\beta(x)M(a+x,b+x)<0$. The second inequality arises because under actuarial fairness, the terms containing $r_A(a+x)$ and $r_B(b+x)$ in an augmented version of (16) that includes the possibility of continued deferral by A, are still both non-negative. Since $V(x)<0$, A would not continue to defer.

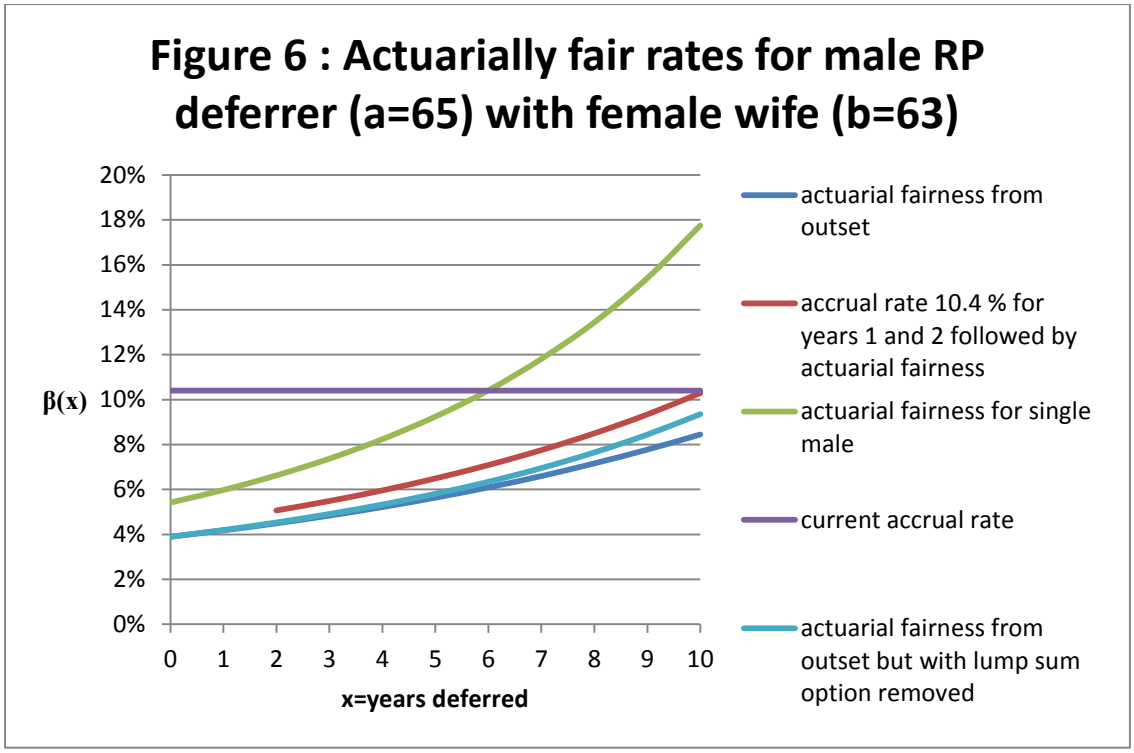
$$\beta(x) = \frac{d\mathrm{B}}{dx} = \frac{1+\mathrm{B}(x) - r_A(a+x)\max[x-\mathrm{B}(x)m_B(b+x),0] - r_B(b+x)\max[x-\mathrm{B}(x)m_A(a+x),0]}{M(a+x,b+x)} \tag{17}$$

It is worth noting that if the lump sum option were not available then the relevant equation would simply be

$$\beta(x) = \frac{1+\mathrm{B}(x)}{M(a+x,b+x)} \tag{18}$$

and the resulting marginal rates would now be higher to compensate for removal of that option, which would in any case only be invoked on A's prior death if $m_B(x) < x/\mathrm{B}(x)$ and on B's if $m_A(x) < x/\mathrm{B}(x)$.

By way of example we take the situation where A is a man who achieves SPA of 65 before 6 April 2016 and whose wife is 63 at that time and who was already taking her own state pension at that point. Then we solve (17) using Euler's method with step size of 1 year, subject to $\mathrm{B}(2) = 0.104\text{x}2 = 0.208$ for the case of someone who started deferring two years before the introduction of actuarially fair rates of accrual, and for comparison only, a man who would have experienced actuarially fair rates (with and without partner) from the outset, had that been possible. The resulting instantaneous accrual rates are shown in figure 6. As expected, the marginal rates are well below those for a single SPA male of 65, which increases still further the extent to which the current 10.4% departs from actuarial fairness. Removal of the lump sum option is seen to have no significant impact on rates for deferral periods of less than 6 years.



8. Summary and conclusions

A review of the literature on deferral or early take-up reveals that much of it relates to US or European situations. Most of the models are in discrete time, sometimes using simulation where the rules of the scheme are more complex. In the present paper a continuous time formulation is preferred which leads to results that emphasise the trade-offs, that are quite intuitive, and easily implemented.

The models lead to first order differential equations which have a closed form solution in some cases and can be solved numerically in others. The theory has been applied to both the old retirement state pension (RP) and the new state pension (SP) using 2103-15 ONS life tables. The graphs show how the accrual rates must necessarily rise with years deferred and we demonstrate how these can vary quite considerably according to gender, state pension age, any years deferred to date under existing schedules, and discount rate net of pension uprating rate. In the case of those attaining SPA before 6 April 2016, it is shown that under most scenarios the accrual scheme is significantly over-generous to the deferrer at cost to the Exchequer.[14] We have shown how on the introduction of an actuarially fair scheme one can preserve any benefits previously accrued under existing schedules. In the case of an RP deferrer with marriage or civil partner, the option of taking a lump sum on prior death of deferrer or partner introduces extra complexity into the analysis, and this is facilitated by using the same type of marginal analysis previously demonstrated for the unattached deferrer.

To achieve true actuarial fairness, it is important to recognise the changing pattern of real pension uprating rates. In that respect the assignment of accrual rates should be dynamic and

[14] It might be argued that dealing with the RP scheme is low priority since it might be perceived of as the 'old state pension'. However, the opposite is true. Since the introduction of SP is recent there are many more RP than SP pensioners. Further, as the SP scheme is currently roughly actuarially fair, the cost to the Exchequer of actuarial unfairness, will for many years be almost entirely attributable to the RP anomaly.

stochastic and so it would be of practical interest to follow up with a full empirical study of the stochastic extension described in section 3.

The methodology can also be applied to informing policy for early take-up of UK state pension should that ever be politically realistic. It could also be adapted for private sector pension schemes.

We have mentioned the issue of selection/moral hazard, and a not dissimilar problem is the dependence of lifetimes in a partnership, which could be examined through copula distributions.

References

Blake, D. and Mayhew, L. (2006) On the sustainability of the UK State Pension System in the light of Population Ageing and Declining Fertility. *The Economic Journal*, 116, 286-305

Bozio, A., Crawford, R., and Tetlow, G. (2010) The History of state pensions in the UK: 1948 to 2010. *Institute for Fiscal Studies*, Briefing Note BN 105

Canada Pension Plan Actuarial adjustment factors (2003) Actuarial Study No. 2, *Office of the Chief Actuary*, March 2003. Available from http://publications.gc.ca/collections/collection_2012/bsif-osfi/IN5-1-2-2003-eng.pdf

Coile, C., Diamond, P., Gruber J., and Jousten, A. (2002) Delays in Claiming Social Security Benefits. *Journal of Public Economics*, 84, 357-385

Dagpunar, J.S. (2015) Deferring a state pension - is it worthwhile? *Significance*, 12, 2, pp. 30-35

Dagpunar, J. S. (2017) Optimum Strategies for UK state pension deferral with particular reference to partner's right to inherit benefits, Discussion paper PI-1703*, the Pensions Institute, Cass Business Schoo*l, City University, London. Available from https://www.pensions-institute.org/papers.html

Duggan, J. and Soares, C. (2002) Actuarial nonequivalence in early and delayed Social Security Benefit Claims. *Public Finance Review*, 30, 3, 188-207

Farrar, S., Moizer, J., and Hyde, M. (2012) The value of incentives to defer the UK state pension. *Pensions*, 17, 1, 46-62

Government Actuary's Department (2014) Report by the Government Actuary on the actuarially fair rate of increments for those reaching State Pension age on or after 6 April 2016 and choosing to defer their State Pension beyond State Pension age. Available from http://data.parliament.uk/DepositedPapers/Files/DEP2014-1112/GAD_State_Pension_Deferral_Report.pdf

Heiland, F., and Yin, N. (2014) Have we finally achieved actuarial fairness of social security retirement benefits and will it last*? Michigan Retirement Research Center, University of Michigan*, working paper 2014-307, available from http://www.mrrc.isr.umich.edu/publications/papers/pdf/wp307.pdf

Kanabar, R. and Simmons, P. (2016) To defer or not to defer? UK state pension and work decisions in a lifecycle model. *Applied Economics*, 48, 58, 5699-5716

Mamon, R. (2004) Three ways to solve for bond prices in the Vasicek model. *Journal of Applied Mathematics and Decision Sciences*, 8, 1, 1-14

Medijainen, M. (2011) Incentives to retire by old-age pension policy in Estonia. *Estonian Discussions on Economic Policy*, 19, 101-120

Meneu, R., Devesa, E., Devesa, M., Domingguez, I., and Encinas, B. (2016) Adjustment mechanisms and intergenerational actuarial neutrality in pension reforms. *International Social Security Review*, 69, 87-107

Moizer, J., Farrar, S., and Hyde, M. (2017) UK State Pension deferral incentives and sustainability. *Applied Economics*, published online 6 November 2017

Mirer, T.W. (1998) The optimal time to file for social security benefits. *Public Finance Review*, 26, 6, 611-636

Oksanen, H. (2005) Actuarial neutrality across generations applied to public pensions under population ageing: effects on government finances and national saving. *European Commission Directorate-General for Economic and Financial Affairs, Economic Papers*, Number 230, July 2005

Queisser, M. and Whitehouse, W. (2006) Neutral or Fair? : Actuarial Concepts and Pension-System Design. *OECD Social, Employment and Migration Working Papers*, No.40. OECD Publishing, Paris. Available from http://dx.doi.org/10.1787/351382456457

Rasdal, K. (2013) Norwegian couples and the take-up of pension benefits. MPhil thesis, Department of Economics, University of Oslo

Rose, C. (2015) The return on investment for delaying social security beyond age 62. *Journal of Financial Planning*, 28, 4, 50-58

Shoven, J. and Slavov, S. (2014) Does it pay to delay social security? *Journal of Pension Economics and Finance*, 13, 2, 121-144

Stubbs, J. and Adetunji, J. (2016) UK pension changes in 2015: some mathematical considerations. *The Mathematical Gazette*, 100, pp. 193-202

Thurley, D. (2010) Deferred retirement increments. *House of Commons Library* Standard Note, SN/BT-2868, London: House of Commons

Thurley, D. (2016) State Pension uprating - 2010 onwards. *House of Commons Library*, Briefing Paper SN-05649, London : House Of Commons

Thurley, D. (2017) State Pension Deferral. *House of Commons Library*, Briefing Paper, CBP-02868, London : House of Commons

Vasicek, O. An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5, 177-188