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# LONGEVITY RISK AND CAPITAL MARKETS: THE 2009–2010 UPDATE

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This Special Issue of the *North American Actuarial Journal* contains 10 contributions to the academic literature, all dealing with longevity risk and capital markets. Draft versions of the papers were presented at *Longevity Five: The Fifth International Longevity Risk and Capital Markets Solutions Conference*, which was held in New York City on September 25–26, 2009. It was hosted by J. P. Morgan and St. John's University and organized by the Pensions Institute at Cass Business School, London, and the Edmondson-Miller Chair at Illinois State University.

Longevity risk and related capital market solutions have grown increasingly important in recent years, both in academic research and in the markets we refer to as the new Life Markets, that is, the capital markets that trade longevity-linked assets and liabilities. Mortality improvements around the world are putting more and more pressure on governments, pension funds, life insurance companies, as well as individuals to deal with the longevity risk they face. At the same time, capital markets can, in principle, provide vehicles to hedge longevity risk effectively and transfer the risk from those unwilling or unable to handle it to those willing to speculate in such risk for increased returns or who have a counterpoising risk that longevity risk can hedge, for example, life insurance. Market investors may be interested in mortality and longevity derivatives since they can provide an essentially zero-beta as-

set to diversify their portfolios. Many new investment products have been created both by the insurance/reinsurance industry and by the capital markets. Mortality catastrophe bonds are an example of a successful insurance-linked security. Some new innovative capital market solutions for transferring longevity risk include longevity (or survivor) bonds, longevity (or survivor) swaps, and mortality (or q-) forward contracts. The aim of the *International Longevity Risk and Capital Markets Solutions Conferences* is to bring together academics and practitioners from all over the world to discuss and analyze these exciting new developments.

The conferences have followed closely the developments in the market. The first conference (*Longevity One*) was held at the Cass Business School in London in February 2005. This conference was prompted by the announcement of the Swiss Re mortality catastrophe bond in December 2003 and the European Investment Bank/BNP Paribas/PartnerRe longevity bond in November 2004.

The second conference was held in April 2006 in Chicago and hosted by the Katie School at Illinois State University.<sup>1</sup> Since *Longevity One*, there had been further issues of mortality catastrophe bonds, as well as the release of the Credit Suisse Longevity Index. Life settlement securitizations were also beginning to take place in the United States. In the United Kingdom, new life companies backed by global investment banks and private equity firms were set up for the express purpose of buying out the defined benefit pension liabilities of U.K. corporations. Goldman Sachs announced it was setting up such a buyout company itself (Rothesay Life) because the issue of pension liabilities was beginning to impede its mergers and acquisitions activities. It decided that the best way of dealing with pension liabili-

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<sup>1</sup> The conference proceedings for *Longevity Two* were published in the December 2006 issue of the *Journal of Risk and Insurance*.

ties was to remove them altogether from the balance sheets of takeover targets. So, firm evidence was now at hand that a new global market in longevity risk transference had been established. However, as with many other economic activities, not all progress follows a smooth path. The EIB/BNP/PartnerRe longevity bond did not attract sufficient investor interest and was withdrawn in late 2005. Much was learned, however, from this failed issue about the conditions and requirements needed to launch a successful capital market instrument.

The third conference was held in Taipei, Taiwan, on July 20–21, 2007, hosted by National Chengchi University.<sup>2</sup> It was decided to hold *Longevity Three* in the Far East, not only to reflect the growing importance of Asia in the global economy, but also in recognition of the fact that population aging and longevity risk are problems that affect all parts of the world and that what we need is a global approach to solving these problems.<sup>3</sup> Since the Chicago conference, many new developments had taken place, including the release of the LifeMetrics Indices covering England and Wales, the United States, the Netherlands, and Germany in March 2007 by J. P. Morgan, the Pensions Institute, and Towers Watson ([www.lifemetrics.com](http://www.lifemetrics.com)); and the world's first publicly announced longevity swap between Swiss Re and the U.K. life office Friends' Provident in April 2007 (although this was structured as an insurance contract or indemnification rather than a capital market transaction).

Since the Taiwan conference, further developments were seen in the capital markets. In December 2007, Goldman Sachs launched a monthly index suitable for trading life settlements.<sup>4</sup> The index, QxX.LS, was based on a pool of 46,290 anonymized U.S. lives over the age of 65 from a database of life policy sellers assessed by the medical underwriter AVS. In 2008, Institutional Life Services (ILS) and Institutional Life

Administration (ILA), a life settlements trading platform and clearing house, were launched by Goldman Sachs, Genworth Financial, and National Financial Partners. ILS/ILA was designed to modernize dealing in life settlements and meet the needs of consumers (by ensuring permanent anonymity of the insured) and of the capital markets (by providing a central clearing house for onward distribution of life settlement assets, whether individually or in structured form).<sup>5</sup>

Xpect Age and Cohort Indices were launched in March 2008 by Deutsche Börse. These indices cover, respectively, life expectancy at different ages and survival rates for given cohorts of lives in Germany, the Netherlands, and England and Wales.

The world's first capital market derivative transaction, a q-forward contract<sup>6</sup> between J. P. Morgan and the U.K. pension fund buy-out company Lucida, took place in January 2008. The world's first capital market longevity swap was executed in July 2008. Canada Life hedged £500m of its U.K.-based annuity book (purchased from the defunct U.K. life insurer Equitable Life). This was a 40-year swap customized to the insurer's longevity exposure to 125,000 annuitants. The longevity risk was fully transferred to investors, which included hedge funds and insurance-linked securities funds. J. P. Morgan acted as the intermediary and assumes counterparty credit risk. There have been nine publicly announced longevity swaps in the United Kingdom since the beginning of 2008, covering five insurance companies' annuity books, three private sector pension funds, and one local authority pension fund. The largest to date, covering £3 billion of pension liabilities, was the longevity swap for the BMW (U.K.) Operations Pension Scheme, arranged by Deutsche Bank and Paternoster in February 2010, and involving numerous reinsurers, including Hannover Re, Pacific Life Re, and Partner Re. The most recent swap to date, announced in February 2011, was between the Pall (U.K.) Pension Fund and J. P. Morgan: This was innovative in being the world's first swap to hedge the longevity risk of nonretired pension plan members. In February

<sup>2</sup> The conference proceedings for *Longevity Three* were published in the Fall 2008 issue of the *Asia-Pacific Journal of Risk and Insurance*.

<sup>3</sup> Asia has the world's largest and fastest growing aging population (United Nations 2007).

<sup>4</sup> Life settlements are life insurance policies traded in the secondary market. In April 2007 the Institutional Life Markets Association was inaugurated in New York as the dedicated institutional trade body for the life settlements industry.

<sup>5</sup> In 2010, National Financial Partners became the sole owner of ILS/ILA.

<sup>6</sup> Coughlan et al. (2007).

2010, Mercer launched a pension buyout index for the United Kingdom to track the cost charged by insurance companies to buy out corporate pension liabilities: At the time of launch, the cost was some 44% higher than the accounting value of the liabilities, which highlighted the attraction of using cheaper alternatives, such as longevity swaps.

The fourth conference was held in Amsterdam on September 25–26, 2008, hosted by Netspar and the Pensions Institute.<sup>7</sup> In 2008, Credit Suisse initiated a longevity swap with Centurion Fund Managers, whereby Centurion acquired a portfolio of synthetic (i.e., simulated) life policies, based on a longevity index built by Credit Suisse. In 2009, survivor swaps began to be offered to the market based on Deutsche Börse's Xpect Cohort Indices.

On February 1, 2010, the Life and Longevity Markets Association (LLMA) was established in London by AXA, Deutsche Bank, J. P. Morgan, Legal & General, Pension Corporation, RBS, and Swiss Re. The original members were later joined by Morgan Stanley, UBS, Aviva and Munich Re. LLMA was formed to promote the development of a liquid market in longevity- and mortality-related risks. This market is related to the insurance-linked securities market and is similar to other markets with trend risks, for example, the market in inflation-linked securities and derivatives. LLMA aims to support the development of consistent standards, methodologies, and benchmarks to help build a liquid trading market needed to support the future demand for longevity protection by insurers and pension funds.

In December 2010, building on its successful mortality catastrophe bonds and taking into account the lessons learned from the EIB bond, Swiss Re launched a series of eight-year longevity-based insurance-linked securities (ILS) notes valued at \$50 million. To do this, it used a special purpose vehicle, Kortis Capital, based in the Cayman Islands. As with the mortality bonds, the longevity notes are designed to hedge Swiss Re's own exposure to longevity risk.

In January 2011, the Irish government issued bonds that allow the creation of sovereign annu-

ities. This followed a request from the Irish Association of Pension Funds and the Society of Actuaries in Ireland. If the bonds are purchased by Irish pension funds, this will have a beneficial effect on the way in which the Irish funding standard values pension liabilities.

At the same time as these practical developments in the capital markets were taking place, academics were continuing to make progress on theoretical developments, building on the original idea of using longevity bonds to hedge longevity risk in the capital markets (Blake and Burrows 2001). These included the following:

- Design and pricing of longevity bonds (e.g., Blake et al. 2006; Bauer 2006; Bauer and Ruß 2006; Denuit et al. 2007; Barbarin 2008; Bauer et al. 2010; Chen and Cummins 2010; Kogure and Kurachi 2010)
- Design and pricing of longevity-linked derivatives, such as survivor swaps (e.g., Dowd et al. 2006), survivor forwards and swaptions (e.g., Dawson et al. 2010), q-forwards (e.g., Brockett et al. 2010), and mortality options (e.g., Milievsky and Promislow 2001)
- Securitization and hedging (e.g., Cowley and Cummins 2005; Lin and Cox 2005; Dahl 2004; Dahl and Møller 2006; Friedberg and Webb 2007; Cox and Lin 2007; Denuit 2009; Wang et al. 2009; Biffis and Blake 2010; Wills and Sherris 2010; Tsai et al. 2010)
- Mortality modeling and mortality term structure modeling<sup>8</sup> (e.g., Brouhns et al. 2002; Cairns et al. 2006, 2008, 2009, 2011; Renshaw and Haberman 2006; Dowd et al. 2010; Blake et al. 2008; Girosi and King 2008; Hari et al. 2008; Biffis et al. 2010; Jarner and Kryger 2009; Pitacco et al. 2009; Plat 2009; Brockett et al. 2010; Cox et al. 2010; Yang et al. 2010)
- Improvements in the analysis and design of longevity-linked retail products (e.g., Gong and Webb 2010; Stevens et al. 2010).

It was also becoming clear that policy makers needed to have a greater understanding of the developments in the new Life Markets because governments now have an important role to play in helping these markets grow, namely, by issuing

<sup>7</sup> The conference proceedings for *Longevity Four* were published in the February 2010 issue of *Insurance: Mathematics and Economics*.

<sup>8</sup> The mortality term structure is the two-dimensional surface showing projected mortality rates at different ages for different future years.

longevity bonds. As argued in Blake et al. (2010), government-issued longevity bonds would allow longevity risk to be shared efficiently and fairly between generations. In exchange for paying a longevity risk premium, the current generation of retirees could look to future generations to hedge their aggregate longevity risk. There would also be wider social benefits. Longevity bonds would lead to a more secure pension savings market—both defined contribution and defined benefit—together with a more efficient annuity market, resulting in less means-tested benefits and a higher tax take arising from the higher taxable annuity income. The new Life Markets could get help to increase market participation through the establishment of reliable longevity indices and key price points on the mortality term structure and could build on this term structure with liquid longevity derivatives. Increasing global support was seen for government-issued longevity bonds (e.g., the U.K. Pension Commission 2005, p. 229; International Monetary Fund 2006; Antolin and Blommestein 2007; World Economic Forum 2009).

As mentioned before, not all paths to progress are smooth. In recent years this has been particularly true in currently the largest market dealing with microlongevity risk, namely, life settlements.<sup>9</sup> The life settlements market has been dogged by systematic underestimates of policyholders' life expectancies by certain medical underwriters, issues concerning premium financing, fraud, and ethical issues associated with "profiting" from individuals dying and policies maturing. In December 2009 Goldman Sachs announced it was closing down its QxX.LS index, partly because of the reputational issues associated with life settlements, but mainly because of insufficient commercial activity in the index. Although the ethical issues are no different in substance from those relating to the macrolongevity

market (see, e.g., Blake and Harrison 2008), the microlongevity market needs to learn important lessons from the macrolongevity market. The macrolongevity market has been very successful at promoting good basic research on the analysis of the stochastic mortality forecasting models it uses and putting these models into the public domain and has also been much more transparent with the data it uses. This suggests a way forward for the life settlements micromarket.

As with the previous conferences, *Longevity Five* consisted of both academic papers and more practical and policy-oriented presentations. The conference location in New York was motivated by the fact that U.S. pension plans in the aggregate have the most significant exposure to longevity risk of pension plans anywhere in the world. The conference was addressed, among others, by the following keynote speakers:

- James Poterba, MIT, and President of the National Bureau of Economic Research: Defined Contribution Plans, Mortality Risk, and the Demand for Annuities
- Tom Boardman, Prudential UK: Why Governments Should Issue Longevity Bonds
- John Iacovino, Fasano Associates: Longevity Extension—Dissecting Mortality Improvements over the Last Century
- Guy Coughlan, Managing Director and Global Head of LifeMetrics and Pension Solutions, J. P. Morgan: Population Basis Risk and Hedge Effectiveness
- Ari Jacobs and Martin Bird, Hewitt Associates: Pensioner Longevity Data Analysis and Applications
- Anthony Webb, Boston College: Valuing the Longevity Insurance Acquired by Delayed Claiming of Social Security
- John Fitzpatrick, Pension Corporation: Aggregating Longevity Risk for the Capital Markets
- Scott Willkomm, Coventry: Micro-Longevity as an Alternative Asset Class
- Richard MacMinn, Illinois State University: The Annuity Puzzle
- Joe Coughlin, Age Lab, MIT: Retiring Retirement—Implications of Longer Worklife on Work, Pensions and Capital Markets

The academic papers that were selected by us as the editors of this Special Issue went through a refereeing process subject to the usual high

<sup>9</sup> The market for microlongevity risk trades assets involving a small number of lives. In the case of life settlements, for example, the products involve individual lives and hence are subject to a significant degree of idiosyncratic mortality risk. This contrasts with the market for macrolongevity risk, which deals with pension plans and annuity books and hence involves a large number of lives: Here idiosyncratic mortality risk is much less important than aggregate mortality risk, which is essentially the trend risk of getting life expectancy projections wrong.

standards of the *North American Actuarial Journal*. They cover the following themes: longevity risk hedges, the role of product design in mitigating the longevity risk facing annuity providers, the valuation of annuities and longevity bonds, and mortality modeling. We briefly discuss each of the 10 papers selected.

In “Longevity Hedging 101: A Framework for Longevity Basis Risk Analysis and Hedge Effectiveness,” Guy D. Coughlan, Marwa Khalaf-Allah, Yijing Ye, Sumit Kumar, Andrew J. G. Cairns, David Blake, and Kevin Dowd show that basis risk is an important consideration when hedging longevity risk with instruments based on longevity indices, because the longevity experience of the hedged exposure may differ from that of the index. As a result, any decision to execute an index-based hedge requires a framework for (1) developing an informed understanding of the basis risk, (2) appropriately calibrating the hedging instrument, and (3) evaluating hedge effectiveness. The authors describe such a framework and apply it to two case studies: one for the United Kingdom (which compares the population of assured lives from the Continuous Mortality Investigation with the England and Wales national population) and one for the United States (which compares the population of California with the U.S. national population). The framework is founded on an analysis of historical experience data, together with an appreciation of the contextual relationship between the two related populations in social, economic, and demographic terms. Despite the different demographic profiles, each case study provides evidence of stable long-term relationships between the mortality experiences of the two populations. This suggests the important result that high levels of hedge effectiveness should be achievable with appropriately calibrated, static, index-based longevity hedges. Indeed, this is borne out in detailed calculations of hedge effectiveness for hypothetical pension portfolios where the basis risk is based on these case studies.

In “Measuring Basis Risk in Longevity Hedges,” Johnny Siu-Hang Li and Mary R. Hardy also examine the basis risk in index longevity hedges for pension funds. They argue that it is important not to ignore the dependence between the population underlying the hedging instrument and the population being hedged. They consider four

extensions to the Lee-Carter model that incorporate such dependence: (1) both populations are jointly driven by the same single time-varying index ( $k_t$ ), (2) the two populations are cointegrated, (3) the populations depend on a common age factor, and (4) an augmented common factor model in which a population-specific time-varying index is added to the common factor model with the property that it will tend toward a certain constant level over time. Using data from the female populations of Canada and the United States, the authors show the augmented common factor model is preferred in terms of both goodness-of-fit and ex post forecasting performance. This model is then used to quantify the basis risk in a longevity hedge of 65-year-old Canadian females structured using a portfolio of  $q$ -forward contracts predicated on U.S. population mortality. The hedge effectiveness is estimated at 56% on the basis of longevity value-at-risk and 81.61% on the basis of longevity risk reduction.

In “Hedging Longevity Risk When Interest Rates Are Uncertain,” Larry Y. Tzeng, Jennifer L. Wang, and Jeffrey T. Tsai propose an asset-liability management strategy to hedge the aggregate risk of annuity providers under the assumption that both the interest rate and mortality rate are stochastic. They assume that annuity providers can invest in a mix of longevity bonds, long-term coupon bonds, and short-term zero-coupon bonds to hedge longevity and interest rate risks. Subject to a required minimum profit level for equity holders in the annuity provider, they show that the optimal allocation strategy leads to the lowest risk under different yield curve and mortality rate assumptions. A longevity bond is shown to be an effective hedging vehicle that significantly reduces the aggregate risk facing annuity providers.

In “Mortality-Indexed Annuities: Managing Longevity Risk via Product Design,” Andreas Richter and Frederik Weber also recognize that longevity risk has become a major challenge for governments, individuals, and annuity providers in most countries. In its aggregate form, that is, the systematic risk of changes to general mortality patterns, it has the potential for causing large cumulative losses for insurers. Since obvious risk management tools, such as (re)insurance or hedging, are less suited for managing an annuity

provider's exposure to this risk, the authors propose a type of life annuity with benefits contingent on actual mortality experience. Similar adaptations to conventional product design exist with investment-linked annuities, and a role model for long-term contracts contingent on actual cost experience can be found in German private health insurance. By effectively sharing systematic longevity risk with policyholders, insurers may avoid cumulative losses. Policyholders also gain in comparison with a comparable conventional annuity product: Using a Monte Carlo simulation, the authors identify a significant upside potential for policyholders while downside risk is limited.

In "A Computationally Efficient Algorithm for Estimating the Distribution of Future Annuity Values under Interest-Rate and Longevity Risks," Kevin Dowd, David Blake, and Andrew J. G. Cairns propose an efficient methodology for quantifying the impact of interest-rate risk and longevity risk on the distribution of annuity values in the distant future. The algorithm simulates the state variables out to the end of the horizon period and then uses a Taylor series approximation to compute approximate annuity values at the end of that period, thereby avoiding a computationally expensive "simulation-within-simulation" problem. Illustrative results suggest that annuity values are likely to rise considerably but are also quite uncertain. These findings have unpleasant implications for both defined contribution pension plans and for defined benefit plan sponsors considering using annuities to hedge their exposure to these risks at some point in the future.

In "Human Survival at Older Ages and the Implications for Longevity Bond Pricing," Leslie Mayhew and David Smith focus on human survival at age 65, the starting age point for many pension products. Using a simple model, they link basic measures of life expectancy to the shape of the human survival function and consider its various forms. The model is then used as the basis for investigating actual survival in England and Wales. The authors find that life expectancy is increasing at a faster rate than at any time in history, with no evidence of this trend slowing or of any upper age limit. With interest growing in the use of longevity bonds as a way to transfer longevity risks from pension providers to the capital markets, the paper seeks to understand how lon-

gevity drift affects pension liabilities based on mortality rates at the point of annuitization versus what actually happens as a cohort ages. The main findings are that longevity bonds are an effective hedge against longevity risk; however, it is not only the oldest old that are driving risk, but also more 65-year-olds reaching less extreme ages, such as 80. In addition, they find that inflation risk and interest rates risk could be as important to annuity values as longevity risk itself.

Mortality dynamics are characterized by changes in mortality regimes. In "Mortality Regimes and Pricing," Andreas Milidonis, Yijia Lin, and Samuel H. Cox describe a Markov regime-switching model that incorporates mortality state switches into mortality dynamics. Using U.S. population mortality data 1901–2005, the authors show that regime-switching models can perform better than well-known models in the literature. Furthermore, they extend the Lee-Carter model in such a way that the time-series common risk factor to all cohorts has distinct mortality regimes with different means and volatilities. Finally, they show how to price mortality securities with this model.

Katja Hanewald in "Explaining Mortality Dynamics: The Role of Macroeconomic Fluctuations and Cause of Death Trends" uses data for six OECD countries over the period 1950–2006 to study the impact of macroeconomic fluctuations and cause of death trends on mortality dynamics in the Lee-Carter mortality forecasting model. The key results of this study are the following: (1) periods can be identified in which the Lee-Carter mortality index ( $k_t$ ) correlates significantly with macroeconomic fluctuations, (2) a few causes of death such as diseases of the circulatory system, influenza and pneumonia, and diabetes mellitus account for a large fraction of the variations in the mortality index, and (3) most cause-specific mortality rates show pronounced trends over the last few decades. These trends change the composition of deaths and alter how total mortality reacts to external factors such as macroeconomic fluctuations.

Life insurance companies deal with two fundamental types of risks when issuing annuity contracts: financial risk and demographic risk. In regard to the latter, recent work has focused on modeling the trend in mortality as a stochastic process. A popular method for modeling death

rates is the Lee-Carter model. This methodology has become widely used, and there have been various extensions and modifications proposed to obtain a broader interpretation and to capture the main features of the dynamics of mortality rates. In order to improve the measurement of uncertainty in survival probability estimates, in particular for older ages, Valeria D'Amato, Emilia Di Lorenzo, Steven Haberman, Maria Russolillo, and Marilena Sibillo in "The Poisson Log-Bilinear Lee-Carter Model: Applications of Efficient Bootstrap Methods to Annuity Analyses" propose an extension based on simulation procedures and on the bootstrap methodology. The paper aims to obtain more reliable and accurate mortality projections, based on the idea of obtaining an acceptable accuracy of the estimate by means of variance reducing techniques. In this way the forecasting procedure becomes more efficient. The longevity question constitutes a critical element in the solvency appraisal of pension annuities. The demographic models used for the cash flow distributions in a portfolio impact on the mathematical reserve and surplus calculations and affect the risk management choices for a pension plan. The paper extends the investigation of the impact of survival uncertainty for life annuity portfolios and for a guaranteed annuity option in the case where interest rates are stochastic. In a framework in which insurance companies need to use internal models for risk management purposes and for determining their Solvency Capital Requirement, the authors consider the surplus value, calculated as the ratio between the market value of the projected assets to that of the liabilities, as a meaningful measure of the company's financial position, expressing the degree to which the liabilities are covered by the assets.

Finally, in "A Gravity Model of Mortality Rates for Two Related Populations," Kevin Dowd, Andrew J. G. Cairns, David Blake, Guy D. Coughlan, and Marwa Khalaf-Allah show that the mortality rate dynamics between two related but different-sized populations can be modeled consistently using a new stochastic mortality model, which they call the gravity model. The larger population is modeled independently, and the smaller population is modeled in terms of spreads (or deviations) relative to the evolution of the former, with the spreads in the period and cohort effects between the larger and smaller populations depend-

ing on gravity or spread reversion parameters for the two effects. The larger the two gravity parameters, the more strongly the smaller population's mortality rates move in line with those of the larger population in the long run. This is important where it is believed that the mortality rates between related populations should not diverge over time on grounds of biological reasonableness. The model is illustrated using an extension of the Age-Period-Cohort (APC) model and mortality rate data for English and Welsh males representing a large population and the Continuous Mortality Investigation assured male lives representing a smaller related population.

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*Discussions on this paper can be submitted until October 1, 2011. The authors reserve the right to reply to any discussion. Please see the Submission Guidelines for Authors on the inside back cover for instructions on the submission of discussions.*