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Securitization, Structuring and Pricing of Longevity Risk

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Abstract

Pricing and risk management for longevity risk has increasingly become a major challenge for life insurers and pension funds around the world. Risk transfer to financial markets, with their major capacity for efficient risk pooling, is an area of significant development for a successful longevity product market. The structuring and pricing of longevity risk using modern securitization methods, common in financial markets, has yet to be successfully implemented for longevity risk management. There are many issues that remain unresolved in order to ensure the successful development of a longevity risk market. This paper considers the securitization of longevity risk focussing on the structuring and pricing of a longevity bond using techniques developed in the financial markets, particularly for mortgages and credit risk. A model based on Australian mortality data and calibrated to insurance risk linked market data is used to assess the structure and market consistent pricing of a longevity bond. Age dependence in the securitized risks is shown to be a critical factor in structuring and pricing longevity linked securitizations.

Keywords: Longevity risk, securitization
JEL Classification: G22, G23 G32, G13

1 Introduction

Longevity risk has become an increasingly important risk facing an increasing proportion of the world's population. Despite this there are limited products available for individuals to insure against and manage this risk. Life insurance companies offer life annuities but these markets are limited. Pension plans have increasingly offered defined contribution benefits with the risk of longevity remaining with individuals. Annuity providers' traditional methods for managing longevity risk have focused on participating policies, and financing through capital reserves. Reinsurers have been reluctant to accept the risk, some describing it as 'toxic' (Wadsworth, 2005). Financial markets have the potential to provide a risk pooling and risk management function for longevity risk. Securitisation has been well developed for a range of risks including credit risk. Longevity bonds and related derivative contracts allow the securitization of the risk inherent in annuity portfolios leading to a more vigorous retail market in longevity risk management products.

People are living longer yet more are retiring at younger ages. Labour participation rates for OECD males aged 60-64 have fallen from 70-90% in the 1970s to 20-50% today (Creighton et al, 2005). This will result in an increased reliance on income sources including life annuities and life time income guarantee products to fund longer retirement time periods. Demand for individual annuity products will also be influenced by the shift from defined benefit (DB) to defined contribution (DC) pension plans. Defined contribution (DC) plans do not currently provide longevity protection (Creighton et al, 2005; and Lin and Cox, 2005).

The Australian pension (superannuation) industry had AUD 1,177 billion in funds under management at December 31, 2007, two thirds of which were in defined contribution or hybrid DC/DB funds (APRA, 2007a). In contrast, the Australian lifetime annuity market was only AUD 3.9 billion in assets (APRA, 2007b). Purcal (2006) investigates the demand and supply constraints that have contributed to the size of this annuity market and concludes that on the supply side, longevity risk and the lack of long term debt instruments have prevented insurers actively pursuing annuity business. On the demand side there has also been a shift towards investment linked products because of their flexibility and potential for higher returns.

Securitization has become an important technique for transferring illiquid risks into financial markets allowing risk pooling and risk transfer for many illiquid retail products such as house mortgages, corporate loans and life insurance policies. Mortgage and other asset-backed securities have been the main focus of securitization but increasingly the insurance market has been developed initially with the sale of rights to emerging profits from life insurance business (Cowley and Cummins, 2005). The transfer of credit risk via collateralized debt obligations (CDOs) has been a more recent development. The securitization of pure insurance risks began in the mid 1990s through Insurance Linked

Securitization (ILS) and the catastrophe bond market. This has since grown to over USD 5.6 billion worth of bonds issued in 2006 (Lane and Beckwith, 2007). Insurance risk securitization allows the transfer of pure insurance risk to investors. Securitization also provides an efficient alternative to insurance risk-transfer methods such as reinsurance.

The initial mortality risk securitization was the Swiss Re Vita Capital issue in December 2003. This mortality bond was designed to reduce the exposure of Swiss Re to catastrophic mortality deterioration over its three year term (Blake et al 2006a). Lane and Beckwith (2005, 2006) outline a number of other recent mortality-linked issues by Swiss Re (Queensgate, 2005 and ALPS II, 2006) and Scottish Re (Orkney Holdings, 2005). These transactions securitize entire blocks of in-force business. Payments are indemnity based, linked to the actual experience of the cedent. These structures bundle underwriting, business, interest rate and mortality risks. Investors do not gain pure mortality exposures, a major contributor to the success of the Vita issues (Blake et al, 2006a).

The securitization of longevity risk was proposed using a 'survivor bond' (Blake and Burrows (2001), Cox et al, 2000; Dowd, 2003; and Blake, 2003). These bonds offer coupon payments linked to the survival rates of a reference population. Annuity providers can receive a payment stream that matches their liability profile, hedging longevity exposures. In November 2004 the European Investment Bank, advised by BNP Paribas, proposed the first survivor bond issue. Coupon payments were linked to a mortality index for English and Welsh 65 year old males and discounted at LIBOR minus 35 basis points, including a premium for the transfer of longevity risk. The exposure was underwritten by Partner Re through a series of longevity swaps. In late 2005 the bond was withdrawn for redesign. Blake et al (2006a) provide a thorough analysis of the major concerns including an insufficient term to maturity of 25 years, excessive basis risk, model and parameter risk, and the capital intensive structure.

Securitization, structuring and pricing of longevity risk for a multi-age annuity portfolio is considered in this paper extending Lin and Cox (2005) and Liao et al (2007). The impact of age dependence with a multiple age portfolio is analysed. A tranche structure similar to that used in the collateralized debt obligation (CDO) market is assessed. Pricing models for longevity bonds include application of the Wang (1996, 2000, 2002) transform to a mortality distribution and the Lee-Carter (1991) model. This approach has been subject to criticism (see Cairns et al, 2006a; and Bauer and Russ, 2006) and an approach that falls naturally within the framework of financial risk models holds more potential. Dahl (2004) has developed financial risk models for mortality risk modelling. Risk adjusted probability measures can then be calibrated to market data. Although there is no currently active market in longevity risk, there are a number of existing mortality-linked securities and a significant insurance linked security market that can provide price information for related risks.

The paper is structured as follows. Section 2 provides a background to longevity risk

products and their pricing. In Section 3, a tranched longevity bond structure designed for an annuity portfolio with multiple ages is developed. The data and methodology used to price the longevity bond structure is covered Section 4. Section 5 discusses the pricing and structuring and Section 6 concludes.

2   Longevity Bonds Background

Longevity bonds are designed to be issued by annuity providers, to transfer their longevity exposure to the capital markets. Lin and Cox (2005), and others including Blake et al (2006a), (2006b), (2006c) and Blake et al (2006d), analyse the bond structure. Coupon payments are contingent on a single-age reference index describing the number of annuitants initially aged x alive at time t, l_{x+t}. If this index, known as the loss measure, is greater than anticipated then the coupon paid to investors is reduced. The difference is paid to the bond issuer as compensation for higher annuity liabilities. These structures usually assume a single cohort and the impact of dependence between ages is not assessed. Only the coupons are at risk and the bond is designed to be issued in a single tranche.

Lane and Beckwith (2007) note that tranched issues are becoming increasingly popular in the insurance linked security market. In 2006 and 2007, new ILS issues were dominated by multi-tranche offerings. The tranche structures are based on CDOs and provide more tailored risk structures for capital markets investors. In the mortality bond market, the transactions since the Vita I issue have involved multiple tranches.

Chang and Shyu (2007) analyze the pricing of a tranched life insurance-linked security under mortality dependence. Their 'Collateralized Insurance Obligation' is a product designed to transfer the mortality risk of a life insurance portfolio. It has the same underlying concept as the Swiss Re Vita Capital bonds. Mortality dependence between lives is incorporated using a Clayton copula, an approach that is popular in credit risk securitization. Different mortality rates between ages are considered by a linear mortality rate function:

$$\mu_i(t) = \mu_i^0(t) + \mu_i^g(t) + \mu_i^a(t) + \mu_i^l(t) \log \left[\frac{I(t)}{B(t)} \right], \tag{1}$$

where the first term is the base mortality rate, and the others allow for gender, age and income respectively. The price of each tranche is determined under a risk neutral measure. Their analysis suggests that Lin and Cox's (2005) independence assumption overestimates the premium of the equity tranche, and underestimates the premiums of the mezzanine and senior tranches. A result consistent with the JP Morgan analogy of the 'correlation cat' for CDO's (Bluhm and Overbeck, 2006).

Liao et al (2007) further examine tranching in mortality linked securities with a product designed to transfer longevity risk. They let mortality follow a non-mean reverting

stochastic process for a single age as proposed by Luciano and Vigna (2005). If B is a level annuity payment per period, $X(t)$ is the actual number of survivors at time t and $\bar{X}(t)$ the expected number; the loss on an annuity portfolio at t is defined as

$$l(t) = \begin{cases} B(X(t) - \bar{X}(t)) & \text{if } X(t) \geq \bar{X}(t) \\ 0 & \text{if } X(t) \leq \bar{X}(t) \end{cases} \tag{2}$$

A difficulty in structuring longevity bonds is defining the percentage cumulative loss on the portfolio. In a CDO or mortality bond, this is naturally the percentage of the portfolio that has defaulted or died by a certain time, as defaults and deaths only occur once. In longevity securitization, the number alive can exceed expectations consistently over a number of years. Liao et al (2007) overcome this by defining the percentage cumulative loss based on the face value of the bond issued. They determine optimal tranche weights to match hypothetical market demands for expected loss exposures.

In order to price a longevity risk linked security, the underlying mortality risk process needs to be a risk adjusted pricing measure. In an incomplete market, such as the longevity risk market, equilibrium pricing theory can be used to derive a risk adjustment (Hull, 2003). Wang (1996, 2000, 2002) developed a framework for pricing risks that aimed to unify actuarial and financial theory based on the distortion operator

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) - \lambda], \tag{3}$$

where the parameter λ is the 'market price of risk'. The distortion can be applied to a cumulative density function $F(t)$, to yield a 'risk-adjusted' function $F^*(t) = g_\lambda(F(t))$. Both Lin and Cox (2005) and Liao et al (2007) employ this approach to determine a risk adjusted mortality measure. Market annuity data is used to calibrate λ. The major criticism of this approach is that it does not readily reflect different prices of risk across ages (Cairns et al, 2006a; and Bauer and Russ, 2006).

A number of ways of calibrating the risk adjusted probability measure have been proposed. Biffis (2005) suggests using basic insurance contracts traded in the secondary reinsurance market to imply a risk adjusted probability measure, but concedes that a deep market in such contracts does not exist. Blake et al (2006) incorporate the market price of risk as a parameter within their model and calibrate it to the BNP/EIB issue. Haberman (2006) assumes the market is risk neutral with respect to mortality risk mainly because of the inability to adequately assess the market risk premium. Lin and Cox (2005) use annuity data to calculate implied market probabilities for their model, an approach mirrored by Bauer and Russ (2006). As opposed to adjusting the underlying probability distribution, Lane (2000) has constructed an empirical model for pricing insurance linked securities where the spread on the security is a function of the expected losses of the issue (EL) and its expected excess return (EER). The EER is the 'risk loading' demanded by the market for accepting the exposure. To accommodate asymmetric loss distributions, Lane (2000) proposes using the probability of first loss (PFL) and the conditional expected loss

(CEL) given exceedance of a tranche attachment point. Equivalent concepts in the credit and insurance risk literature are the probability of default (frequency) and the loss given default (severity). ILS tranches are priced using the functional form:

$$EER = \gamma(PFL)^\alpha \times (CEL)^\beta. \tag{4}$$

Lane fits the model to insurance linked security (ILS) transactions to estimate γ, α and β. The model provides a reasonable fit for the market based EER. Lane and Beckwith (2005) use the model to evaluate the Swiss Re Vita Capital issue, and Lane and Beckwith (2006) review prices in the wind catastrophe bond market following the 2005 US hurricane season.

3   Structuring a Longevity Bond

The proposed structure for a longevity bond is based on that used for a collateralized debt obligation. Over the term of the bond the issuer pays a regular premium to the tranche investors. The tranche is 'triggered' by higher than anticipated longevity improvements. In this event, the investor forfeits a fraction of their prescribed capital to the issuer, as compensation for the issuer's incurred losses on an annuity portfolio. The payments are based on a specified mortality index. In the following period, the premium is paid on a notional tranche principal that has been reduced by the incurred loss. The proposed longevity bond is structured in a number of tranches. This allows the risk profile of the bonds to be tailored to investor demands.

The underlying annuity portfolio is assumed to comprise individual annuitants that are all affected by the same population mortality process. This gives rise to both systematic and non-systematic longevity risk. Systematic risk is the risk associated with changes in underlying (population) mortality rates. Non-systematic risk refers to the distribution of lives in a portfolio given a fixed mortality rate. Systematic risk is not diversifiable and thus does not decrease with the increasing size of a portfolio. Longevity bonds aim to provide an alternative means of managing the systematic risk.

The price of each tranche is found by equating the expected present values of its premiums and losses. This is done under an equivalent risk adjusted probability measure for the mortality process. The risk adjusted pricing measure is calibrated to market data from the insurance linked securities market and the empirical model proposed by Lane (2000) for pricing insurance-linked securities.

The longevity bond structure proposed involves a number of tranches. Payments will be defined based on a population mortality index. The portfolio to be securitized is an annuity portfolio with annuitants of different ages. Payments are aggregated for all the

lives in the portfolio and then allocated to tranches reflecting the seniority of the tranche. Figure 1 summarizes the cashflows of the longevity bond.
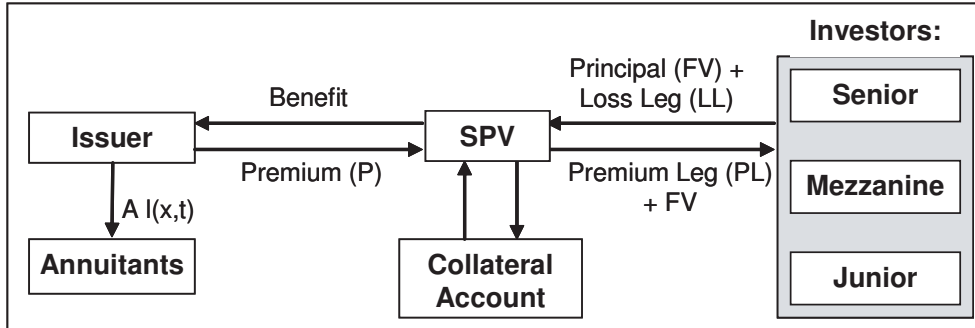

Figure 1: The proposed tranched longevity bond.

Tranche investors subscribe an initial principal amount FV (in total), that is held as collateral in a Special Purpose Vehicle (SPV). If no losses are incurred on the tranche, FV is returned to the investors at maturity. At the beginning of each period, investors receive a premium payment, P, from the issuer via the SPV. In exchange, a fraction of FV is transferred from the SPV to the issuer, in the event that longevity exceeds expectations. This provides compensation for higher than anticipated losses on the issuer's annuity portfolio. FV is not evenly reduced for all investors, rather the amount depends on the allocation of portfolio loss between the tranches. After a loss has occurred, premiums are then calculated as a percentage of the reduced notional principal.

3.1 Bond Cash Flows and Annuity Portfolio Losses

The payments on the longevity bond are contingent on the losses on the underlying annuity portfolio. The longevity bond has a term to maturity of T periods and a total face value of FV. The bond's cashflows are determined by a reference annuity portfolio of $n(0)$ lives of different ages at time $t = 0$. We assume that annuitant i, is paid a whole of life annuity of A_i per period, $i = 1, \ldots, n(0)$. The indicator $I_i(t) = 1_{\tau_i > t}$ jumps from 1 to 0 at the time of death τ_i of an annuitant. The loss on the portfolio at time t is defined as

$$L(t) = \sum_{n(x,0)} \left(I_i(t)A_i - E\left[I_i(t)A_i\right] \right)^+ . \tag{5}$$

which is the amount that the annuity payments at time t exceed the expected payments. Losses on the portfolio are not symmetric giving rise to a number of 'option-like' features.

For an initial age of life i at time 0 of x_i, the probability of the life living to age $(x_i + t)$

is:

$$_tp_{x_i} = E[I_i(t)] \tag{6}$$

$$= \exp\left[-\int_0^t \mu(x_i, s)ds\right]. \tag{7}$$

The portfolio of annuitants have different ages, so denote the number of lives initially alive aged x by $l(x, 0)$, with:

$$\sum_{\text{all x}} l(x, 0) = n(0). \tag{8}$$

The number of lives alive at time t, initially aged x, is denoted by $l(x, t)$. For a given population survival probability $_tp_x$, the distribution of the number alive at time t is binomial:

$$l(x, t) = \sum_{\text{all } i \text{ for age } x} I_i(t) \sim Binomial\Big(l(x, 0), {_tp_x}\Big)|_{_tp_x}. \tag{9}$$

As noted by Lin and Cox (2005), there are two sources of randomness in the portfolio loss at time t. The first is due to uncertain life times given the mortality rates. The second is due to the stochastic nature of the survival probabilities. The total variability in the portfolio is the unconditional variance of the compound binomial distribution:

$$Var[l(x, t)] = E[Var[l(x, t)|_tp_x]] + Var[E[l(x, t)|_tp_x]]. \tag{10}$$

For large portfolios of lives the main source of randomness will arise from changes in the mortality rates impacting all lives in the portfolio rather than the variability in the number of deaths at a particular age given the mortality rate. The randomness in $l(x, t)$ will mainly be due to the randomness in $_tp_x$. This assumption understates the total variability of a portfolio but most of the variability in $l(x, t)$ is from the randomness of $_tp_x$. Figure 2 shows both the total variance $Var[l(x, t)]$ and the population variance due to the random survival probability $_tp_x$, $Var[E[l(x, t)|_tp_x]]$. The two plots are almost identical, showing that the volatility in $l(x, t)$ is primarily explained by the underlying mortality process. Lin and Cox (2005) model a longevity bond based on longevity risk volatility from $l(x, t)$ given a fixed $_tp_x$, understating the longevity risk in a portfolio. Since the longevity bond aims to manage systematic longevity risk of the annuity portfolio, systematic risk is the focus of this paper.

The portfolio loss in Equation (5) can then be written as

$$L(t) = \left(A\sum_{\text{all x}} l(x, 0)_tp_x - E\Big[A\sum_{\text{all x}} l(x, 0)_tp_x\Big]\right)^+ \tag{11}$$

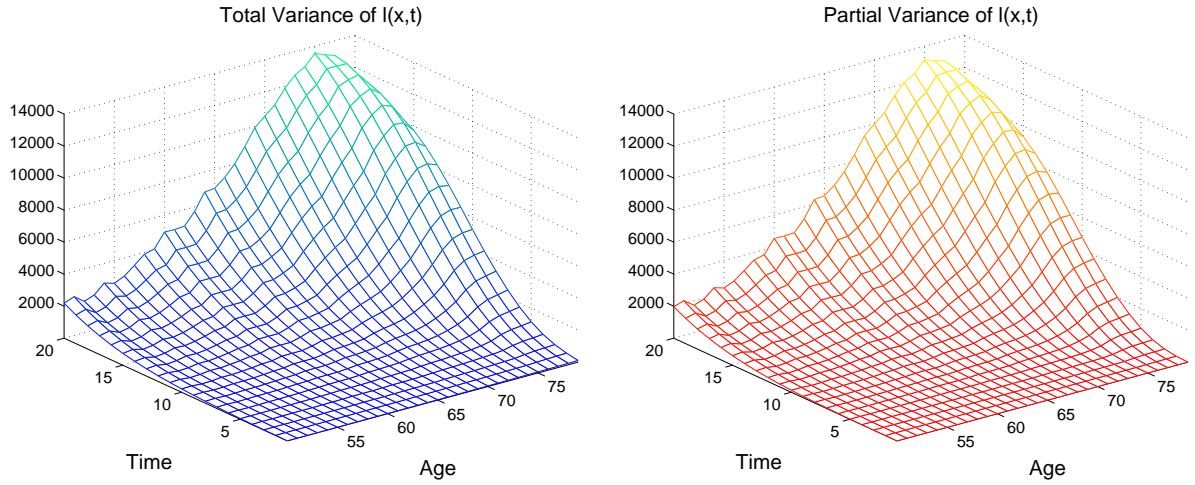$$= \left(A\sum_{\text{all x}} l(x, 0)_tp_x - A\sum_{\text{all x}} l(x, 0)_t\bar{p}_x\right)^+. \tag{12}$$

Figure 2: Total and partial variance of $l(x,t)$.

The randomness in the portfolio loss $L(t)$ is due to the random survival probability $_tp_x$, with expectation $_t\bar{p}_x$. The portfolio percentage cumulative loss can be written as:

$$CL(t) = \frac{\sum_{s=1}^{t} L(s)}{FV}. \tag{13}$$

$CL(t)$ describes the percentage of the bond's face value that has been exhausted by portfolio losses up to that time. The choice of face value FV affects this loss, and in turn the risk profile of the bond. Unlike CDOs and Vita-style mortality bonds, it is possible for $CL(t) > 1$, particularly for smaller values of FV. This arises when annuitants repeatedly exceed expectations of longevity whereas credit risky bonds and individual lives can default or die only once. Payments are restricted to cases where $CL(t) <= 1$.

3.2   Tranching by Percentage Cumulative Loss

Having defined the losses it is then necessary to develop a method of allocating portfolio losses between the J tranches. Each tranche is characterised by an attachment and detachment point denoted by $K_{A,j}$ and $K_{D,j}$ for $j = 1, \ldots, J$ respectively. The expected loss in each tranche determines the appropriate premium. The proposed tranche structure is based on the percentage cumulative loss of the portfolio with the longevity risk associated with the annuity portfolio tranched 'vertically', in a way that is similar to an Excess of Loss reinsurance contract. As losses are incurred on the underlying annuity portfolio, they are allocated to a tranche when the cumulative loss falls between its attachment and detachment points. These points are expressed as a percentage of the bond's face value FV,

and are chosen such that

$$
\begin{aligned}
K_{A,1} &= 0; \\
K_{D,j-1} &= K_{A,j}; \\
K_{A,j} &< K_{D,j}; \text{ and,} \\
K_{D,J} &= 1.
\end{aligned}
\tag{14}
$$

If cumulative losses exceed the detachment point of a tranche, it is retired and the losses are allocated to the next in order of seniority. The senior tranches will only attach if all subordinated ones have been retired. The cumulative loss on the jth tranche at time t is given by:

$$
CL_j(t) = \begin{cases}
0 & \text{if } L(t) < K_{A,j}; \\
CL(t) - K_{A,j} & \text{if } K_{A,j} <= L(t) < K_{D,j}; \\
K_{D,j} - K_{A,j} & \text{if } L(t) >= K_{D,j},
\end{cases}
\tag{15}
$$

where:

$$
CL(t) = \sum_{j=1}^{J} CL_j(t).
\tag{16}
$$

The expected percentage cumulative loss in tranche j at time t is:

$$
TCL_j(t) = \frac{E[CL_j(t)]}{K_{D,j} - K_{A,j}}.
\tag{17}
$$

Defining tranche payments by cumulative loss has a number of advantages. These can be seen by contrasting it with the approach used by Lin and Cox (2005), based on losses per period. In the latter case, the coupon paid to tranche investors only depends on the level of portfolio losses in each period. In a year of strong mortality improvement, coupons may be reduced to zero for a particular tranche. In the next year, if mortality rates return to expectation then coupons may be fully reinstated. The coupon payment stream thus becomes highly variable, reducing its attractiveness to investors. Initial capital costs would also be high, as the risk coverage per period is limited to the coupon size. Tranching by cumulative loss leads to a more predictable stream of cashflows. Once a tranche is exhausted, it will not be reinstated again. The tranches also provide greater coverage, as both principal and interest are at risk.

3.3 Pricing Structured Longevity Risk

The price of a longevity bond tranche P_j is defined as a percentage of the principal at risk. This percentage is paid to the investor each period as the tranche premium. The fair price P_j^* is set so that the expected present value of the premium and claim payment legs of the tranche are equal. Each leg is a function of the expected percentage cumulative loss on

the tranche at time t, $TCL_j(t)$. Assume that premium payments occur at the beginning of each time period $t = 0, \ldots, T$. The value of tranche j's premium leg is equal to the expected present value of all premium payments to the investor:

$$PL_j = \sum_{t=1}^{T} P_j B(0, t-1)[1 - TCL_j(t-1)]. \tag{18}$$

The term $B(0, t-1)$ is the present value of a risk-free zero coupon bond that pays \$1 at time $(t-1)$. $TCL_j(t-1)$ is the expected percentage cumulative loss on the tranche at the time the premium is paid so that $[1 - TCL_j(t-1)]$ determines the notional face value of the tranche on which the premium is calculated. At the beginning of the contract, the premium is paid on 100% of the notional face value. This reduces over time to zero when the tranche is exhausted, and the premium payments cease.

The value of the claim payment leg is the present value of the expected loss payments, which occur at the end of each period:

$$LL_j = \sum_{t=1}^{T} B(0, t)[TCL_j(t) - TCL_j(t-1)]. \tag{19}$$

The fair price of the tranche is then defined as the premium P_j^* such that:

$$PL_j(P_j^*) - LL_j(P_j^*) = 0. \tag{20}$$

giving

$$P_j^* = \frac{\sum_{t=1}^{T} B(0, t)[TCL_j(t) - TCL_j(t-1)]}{\sum_{t=1}^{T} B(0, t-1)[1 - TCL_j(t-1)]}. \tag{21}$$

P_j^* is determined using a risk-adjusted probability measure to incorporate a 'risk premium', in order to satisfy investor risk aversion. Wills and Sherris (2008) develop a mortality model with the dynamics of the mortality rate determined under a risk adjusted probability measure $\mathbb{Q}$ given by:

$$d\mu^{\mathbb{Q}}(x, t) = \left(a(x+t) + b + \sum_{i=1}^{N} \delta_{xi} \lambda_i(t) \right) \mu^{\mathbb{Q}}(x, t) dt + \sigma \mu^{\mathbb{Q}}(x, t) dW(x, t) \text{ for all } x. \tag{22}$$

where $\underline{\lambda}(t) = [\lambda_1(t), \ldots, \lambda_N(t)]'$ is a vector of 'risk adjustments'. The 'prices of risk', $\lambda_i(t)$, can be calibrated to reflect different investor risk premiums for cash flows with differing risk exposures especially to the age profile underlying the security cash flows. This provides more flexibility compared to approaches based on the Wang transform.

4   Implementation and Analysis

Table 1 provides details of the structure for the longevity bond used for analysis. The FV determines the amount of coverage provided by the longevity bond issue. Losses

are measured as a percentage of the bond's face value. The choice of FV, $n(0)$ and A were determined so that the longevity bond tranches have risk profiles commensurate with AAA, BBB- and equity (unrated) securities. A long term to maturity of the bond is required to manage exposure to long-term longevity improvements. The bond covers 30 ages in the portfolio over a 20 year term reflecting the longest dated Australian Government bonds. In 2005, the French and UK governments issued 50 year bonds, which facilitate the management of interest rate risk over a longer time horizon. Longer term bonds will facilitate the market for longer longevity risk linked securities.

Assumptions	
Bond Face Value:	$FV = \$750,000,000$.
Term to Maturity:	$T = 20$ years.
Payment Frequency:	Annually, for both premium and loss payments.
Number of Tranches:	$J = 3$.
Initial Age of Annuitants:	$x = 50, \ldots, 79$.
Initial No. of Annuitants:	$n(0) = 60,000$. We assume this is evenly distributed between the 30 ages, with $l(x,0) = 2,000 \forall x$.
Annuity Payments:	$A = \$50,000$ paid at the end of each year to each living annuitant.

Table 1: Longevity bond structure.

The attachment and detachment points of the jth tranche, $K_{A,j}$ and $K_{D,j}$, are defined in terms of the portfolio percentage cumulative loss and given in Table 2. The allocation

Tranche j	$K_{A,j}$	$K_{D,j}$
1	0%	15%
2	15%	30%
3	30%	100%

Table 2: Attachment and detachment points as a percentage of bond face value (FV).

of losses between tranches results in an unrated junior or equity tranche, a BBB- rated mezzanine tranche and a AAA rated senior tranche. The junior tranche is usually retained by the issuer, managing moral hazard by aligning the issuer's interests with those of the investor. Tranching also creates a range of risk profiles, expanding the potential pool of risk capital to provide the funding.

4.1 Pricing Tranched Longevity Risk

In an annuity portfolio the ages of the lives will vary and dependence between the ages is an important factor. In order to clearly demonstrate the impact of dependence, the longevity bond tranches are analyzed under three specifications of dependence between ages in the underlying mortality process. These are independence, dependence based on Principal Components Analysis (PCA) of Australian population mortality data and perfect dependence.

Data to calibrate and analyse the longevity bond were obtained from:

- Australian Population Mortality Data: ages 50-99, 1971-2004. Drawn from the Human Mortality Database, University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany).

- Australian Government Treasury Bill and Note Prices: maturity ranging from 12 months to 12 years. Drawn from the Bloomberg data service, 24/09/2007.

- Market Insurance-Linked Security Data: 2007 issues. Drawn from Lane (2007).

The stochastic mortality process adopted is that of Wills and Sherris (2008) who fitted a multivariate mortality rate model to Australian population mortality data using maximum likelihood estimation and PCA.

Zero coupon bond yields were determined using linear interpolation between quoted maturities. For periods exceeding the 12 years of available data, the yield curve is assumed to be flat at 6.246%. The zero coupon bond curve is given in Figure 3.

Under the risk adjusted pricing measure, $\mathbb{Q}$, the losses on each tranche are discounted at the risk-free rate. To price longevity-linked securities it is necessary to calibrate $\underline{\lambda}(t)$ to market data. Longevity risk is not liquidly traded so it is necessary to use available data for similar risk securities including mortality bonds. The prices of risk are determined so that the price of each tranche is consistent with that obtained using the Lane (2000) model based on market data. For all N ages the change of measure becomes

$$dW^{\mathbb{Q}}(x,t) = dW(x,t) + \underline{\lambda}(t)dt. \tag{23}$$

From Equation (22), the mortality dynamics for each initial age x under the risk adjusted measure $\mathbb{Q}$ is

$$d\mu^{\mathbb{Q}}(x,t) = \Big(a(x+t) + b - \sigma\lambda^*\Big)\mu^{\mathbb{Q}}(x,t)dt + \sigma\mu^{\mathbb{Q}}(x,t)dW(x,t). \tag{24}$$

Lane (2000) proposed an empirical model for pricing insurance-linked security tranches. This price is comprised of two elements: the expected losses per period (EL), and a risk
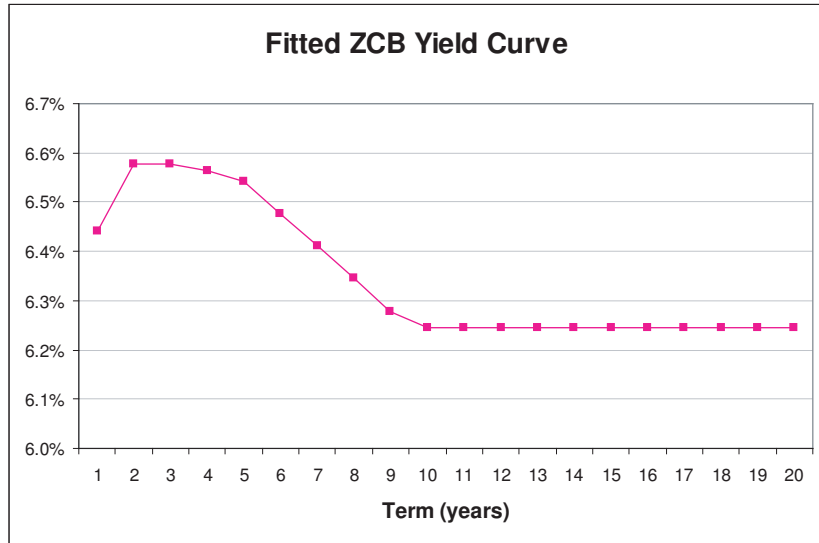
Figure 3: The assumed Zero Coupon Bond yield curve (Bloomberg 24/09/2007)

Parameter	2006-07 All	2006-07 Mortality
γ	0.9736	0.9980
α	0.6006	0.8965
β	0.6261	0.5034
X^2	1.84	0.04
χ^2_9 at 99%	44.64	2.09

Table 3: Fitted parameters and chi-square statistic X^2 for the Lane model, using 2006-2007 data.

adjustment known as the expected excess return (EER). Under this model the premium per period is

$$\hat{P^L} = EL + EER \tag{25}$$

A tranche's expected loss can be separated into its probability of attachment, and the loss given attachment. Lane denotes these by the probability of first loss PFL, and the conditional expected loss CEL respectively, such that $EL = PFL \times CEL$. These values are determined under a real world probability measure $\mathbb{P}$. The expected excess return is

$$EER = \gamma(PFL)^\alpha(CEL)^\beta. \tag{26}$$

Lane's model has been calibrated to the observed premiums of mortality-linked securities issued in 2006-2007. Data is drawn from Lane (2007), incorporating thirteen tranches

across the issues by the Vita III and Osiris Capital special purpose vehicles. For comparison, the model is also fit to all 72 insurance-linked security issues over the same period. Table 3 lists the parameters fitted using the method of non-linear least squares. The Pearson's Chi-Square statistic and 99% confidence level is provided as a summary of the fit of the model, illustrated in Figure 4, which is found to be satisfactory.
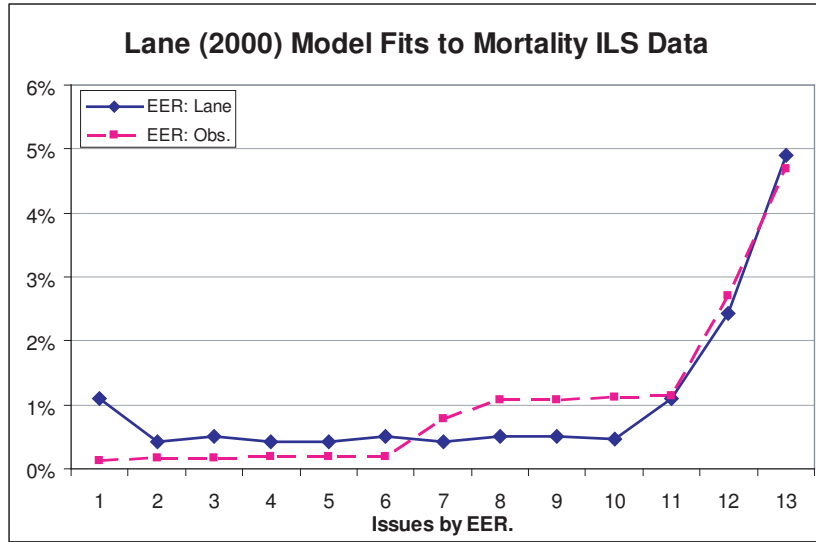

Figure 4: Observed and fitted EER found by calibrating the Lane model to 2006-2007 mortality bond issues.

The proposed model determines tranche losses using Monte-Carlo simulation. The probability of first loss for tranche j is estimated by:

$$P\hat{F}L_j = \frac{\text{number of simulations where the tranche is triggered}}{\text{total number of simulations}}. \quad (27)$$

where the expected cumulative percentage loss on tranche j at time t is given by $TCL_j(t)$. This is annualized for use in Lane's model to give:

$$\hat{EL}_j = \frac{TCL_j(T)}{T}. \quad (28)$$

The annualized cumulative expected loss is then

$$C\hat{EL}_j = \frac{\hat{EL}_j}{P\hat{F}L_j}. \quad (29)$$

The parameters listed in Table 3 and the estimated values of $\hat{EL}_j$, $P\hat{F}L_j$ and $C\hat{EL}_j$ yield an estimate of the risk-adjusted premium for tranche j, P_j^L. Prices of risk are calibrated to this tranche price by choosing λ_j^* to give the model premium:

$$P_j^{\lambda^*} = P_j^L. \quad (30)$$

The price of risk λ_j^* is a function of the risk characteristics of the cash flows as priced using the Lane model so that

$$\lambda_j^* = f(P\hat{F}L_j, C\hat{E}L_j, \gamma, \alpha, \beta). \tag{31}$$

4.2 Pricing and Monte Carlo Simulation

The approach used to determine tranche prices of risk is implemented using Monte Carlo simulation. The process is as follows.

Step 1. The values of $\mu(x,0)$ and its expectation $\bar{\mu}(x,0)$ are initially set to the latest observed values.

Step 2. The $\mathbb{P}$-distribution of $d\mu(x,t)$ at each time t is modeled using 100,000 simulations. This is driven by simulations of the multivariate process $dW(t)$, under three assumptions of dependence. $d\bar{\mu}(x,t)$ is determined with $dW(t) = [0,\ldots,0]'$.

Step 3. Simulations of $\mu(x,t)$ are derived

$$\mu(x,t) = \mu(x,0) + \sum_{s=0}^{t} d\mu(x,s).$$

Step 4. The portfolio loss $L(t)$ and percentage cumulative loss $CL(t)$ are determined for each simulation at each time. They are both functions of $\mu(x,t)$ and $\bar{\mu}(x,t)$.

Step 5. Losses are allocated between tranches based on their attachment and detachment points. This gives the expected percentage cumulative tranche loss $EM_j(t)$.

Step 6. $EM_j(t)$ is used to find the tranche fair premium P_j^*.

Step 7. Estimates of $\hat{E}L_j$, $P\hat{F}L_j$ and $C\hat{E}L_j$ are used to find the market price of the tranche, P_j^L.

Step 8. Steps 1-7 are repeated, to determine the price of risk adjustment for $d\mu(x,t)$. This gives a $\mathbb{Q}$-distribution of $d\mu(x,t)$ where λ^* is chosen such that the premium in Step 6 of this iteration, $P_j^{\lambda^*}$, equals the P_j^L calculated with the Lane model under the measure $\mathbb{P}$.

Step 9. The process is repeated for each tranche $j = 1, 2$ and 3.

5 Analysis of Bond Structure

For the Wills and Sherris (2008) mortality model, the expected number of lives alive under 20 year projections is given in Figure 5. Mortality is expected to continue to improve for all ages.
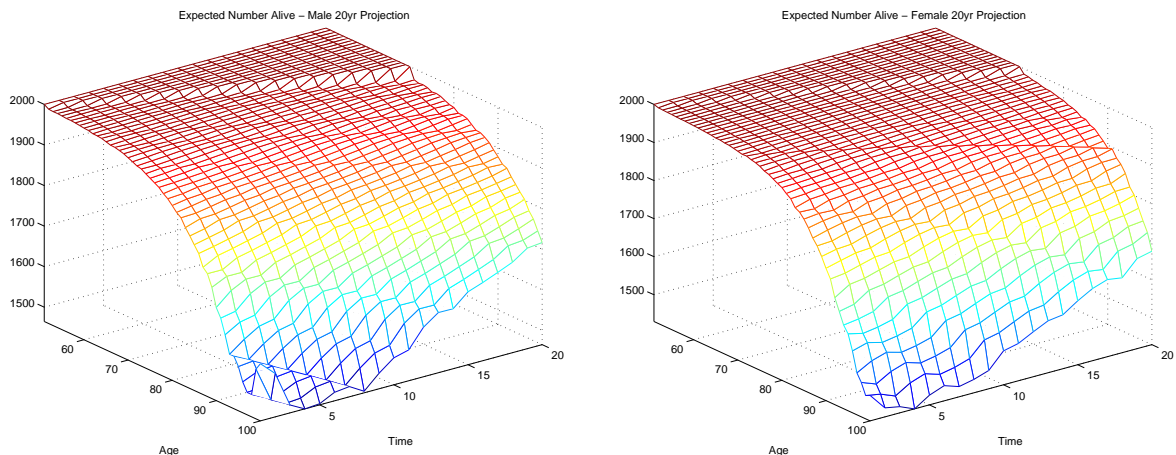

Figure 5: Expected number alive given $l(x, 0) = 2000$.

Expected portfolio cumulative loss, given as the percentage of the bond's face value that has been exhausted by losses up to each point in time and falling between [0,1], on the proposed 20 year longevity bond are given in Figure 6. Values are shown for both time and the initial age of the annuitant, providing a summary of portfolio losses before they are allocated between tranches.

A statistical summary of portfolio loss is given in Table 4. P^0 is the premium on an untranched longevity bond, calculated under real world mortality projections. $CL(T)$ is the portfolio expected cumulative loss at the bond's maturity, T, also calculated under the real world probability measure $\mathbb{P}$ expressed as a percent of the bond's face value. This is the fraction of invested capital that investors can expect to lose over the term of the issue. The final column gives the standard deviation of portfolio cumulative losses at time T. This is an indication of the volatility of the portfolio, though it does not account for the non-symmetric nature of losses.

The statistics in Table 4 are shown under the three age-dependence assumptions. Independence assumes that mortality improvements at any age are not impacted by changes at other ages. Dependence results in the mortality curve shifting proportionately across all ages. PCA refers to mortality rates simulated using principal components analysis, with a dependence structure taken from observed Australian data.
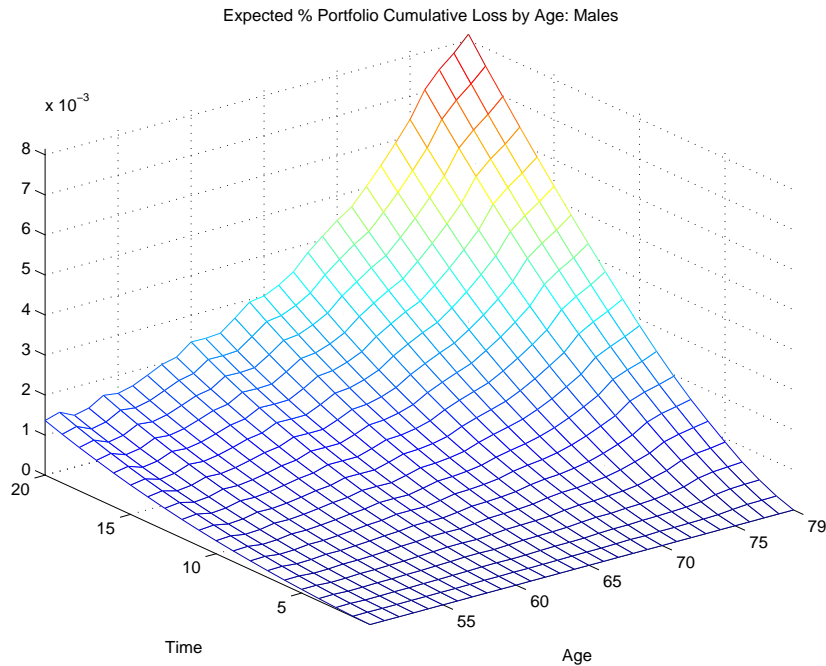
Figure 6: Portfolio % cumulative loss by initial age.

	P^0	$CL(T)$	std dev(T)
Ind	75	0.181	0.042
PCA	44	0.108	0.074
Dep	62	0.149	0.196

Table 4: Whole portfolio premiums (bps) for $\lambda = 0$, and loss statistics under 3 assumptions for mortality dependence.

Table 4 shows that the variability of the loss on the portfolio increases with age-dependence. This is to be expected, as under higher dependence, fluctuations in mortality at one age are exacerbated by the flow-on effects at all other ages. Less intuitive is the relationship between expected losses assuming age dependence. This is influenced by the one-sided nature of the loss function $L(t)$. If longevity improvements are less than expected, a 'negative loss' is not incurred on the annuity portfolio for the purpose of securitization. As a result, higher volatility would be expected to increase the expected loss on the portfolio. This is similar to the impact of volatility on the price of an option as discussed, for example, in Hull (2003). This is seen in the higher expected losses under perfect independence compared with the dependence case. Lower volatilities are balanced by higher dispersion in the mortality rate at ages above 95. Few lives reach these ages over the term of the bond, as the initial cohorts range from 50-79, so that their effect is limited. Although the range of outcomes is more dispersed than under independence, its average is lower.

In Figure 7, differences in portfolio cumulative loss under the three assumptions are further illustrated along with 95% confidence intervals for the losses over the 20 year term of the bond.

5.1 Tranching by Percentage Cumulative Loss

Cumulative losses on an annuity portfolio increase over time. In the proposed tranche structure, when cumulative loss exceeds the detachment point of a tranche, it is retired, and the next tranche in order of seniority bears the following losses. This continues until the final tranche has been exhausted, or the bond matures. As a result of this 'waterfall' effect, ordered tranches are created that have significantly different risk profiles, both from the original portfolio and amongst themselves. The first, or 'junior', tranches bear the majority of risk, whilst senior tranches only attach in the event of particularly adverse experience.

The operation of this tranching structure is illustrated in Figure 8, which shows 100 simulations of portfolio cumulative loss over 20 years. Each simulation is a step function that traces portfolio cumulative losses at discrete annual intervals. At each step, cumulative loss is divided between the three tranches, based on the thresholds illustrated in the diagram. The loss on each tranche is then calculated as a percentage of the tranche 'coverage' - or the difference between its attachment and detachment points. In the case of the middle tranche in our example, portfolio cumulative loss is divided by (0.3-0.15) to find the tranche percentage cumulative loss.

		P_j^0	$TCL_j(T)$	std dev(T)
	Ind	574	0.966	0.082
Tranche 1	PCA	306	0.626	0.282
	Dep	304	0.532	0.403
	Ind	66	0.238	0.233
Tranche 2	PCA	24	0.082	0.225
	Dep	84	0.228	0.384
	Ind	0	0.000	0.002
Tranche 3	PCA	1	0.003	0.025
	Dep	15	0.050	0.166

Table 5: Tranche premiums (bps) for $\lambda = 0$, and loss statistics under 3 assumptions for mortality dependence

A statistical summary of tranche losses is given in Table 5, and mirrors the portfolio results in Table 4 for each tranche. P_j^0 is the premium on each tranche, $TCL_j(T)$ is the
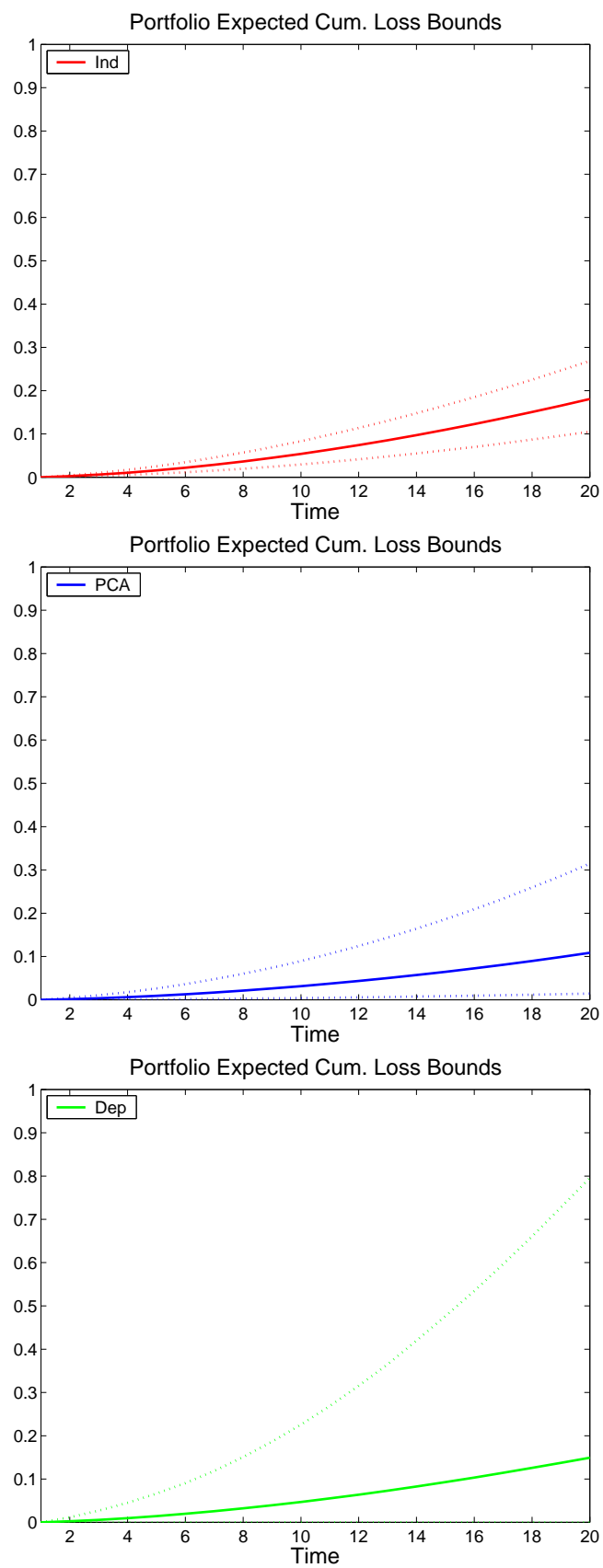
Figure 7: Portfolio expected cumulative loss, 95% confidence intervals.
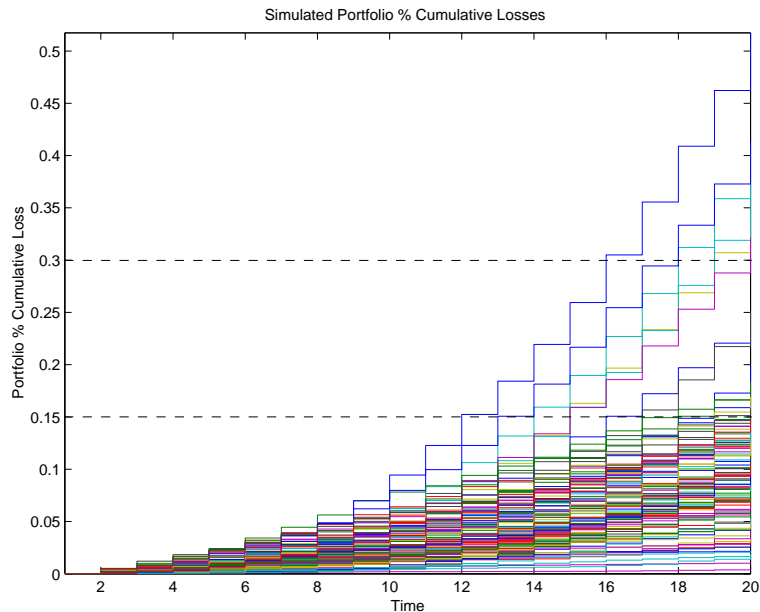
Figure 8: Simulations of total portfolio % cumulative loss (male). Tranche thresholds illustrate attachment and detachment points.

tranche expected cumulative loss at T, and the standard deviation of cumulative tranche losses at T is given in the final column. $TCL_j(T)$ is expressed as a percent of the face value of the tranche and, like $CL(t)$ in the portfolio case, is interpreted as the fraction of invested capital that investors can expect to lose over the term of the issue.

The statistics in Table 5 provide an insight into the impact of age dependence on each tranche. Assuming independence, losses in the equity tranche (Tranche 1) are overestimated, whilst those in the senior tranche are underestimated. The opposite is true for completely dependent ages. Senior tranche investors face expected losses that are 17 times greater under perfect dependence than they are under observed PCA dependence estimated from Australian mortality data.

In Figure 9, the difference in tranche risk profiles is further illustrated under the three dependence assumptions with 95% confidence intervals for the losses in each tranche shown, giving an indication of their dispersion about the mean. For each tranche, the variability of losses increases with age-dependence. Under higher dependence, mortality improvements at each age impact all ages, accumulating the impact of mortality improvement losses for an annuity portfolio. As a result, equity tranche investors expect fewer losses under dependence; because along with the higher probability of large losses, to which their exposure is capped, there is also a higher probability of low losses. For the senior tranche investor, expected loss increases with dependence, as there is a greater possibility that their tranche will attach.
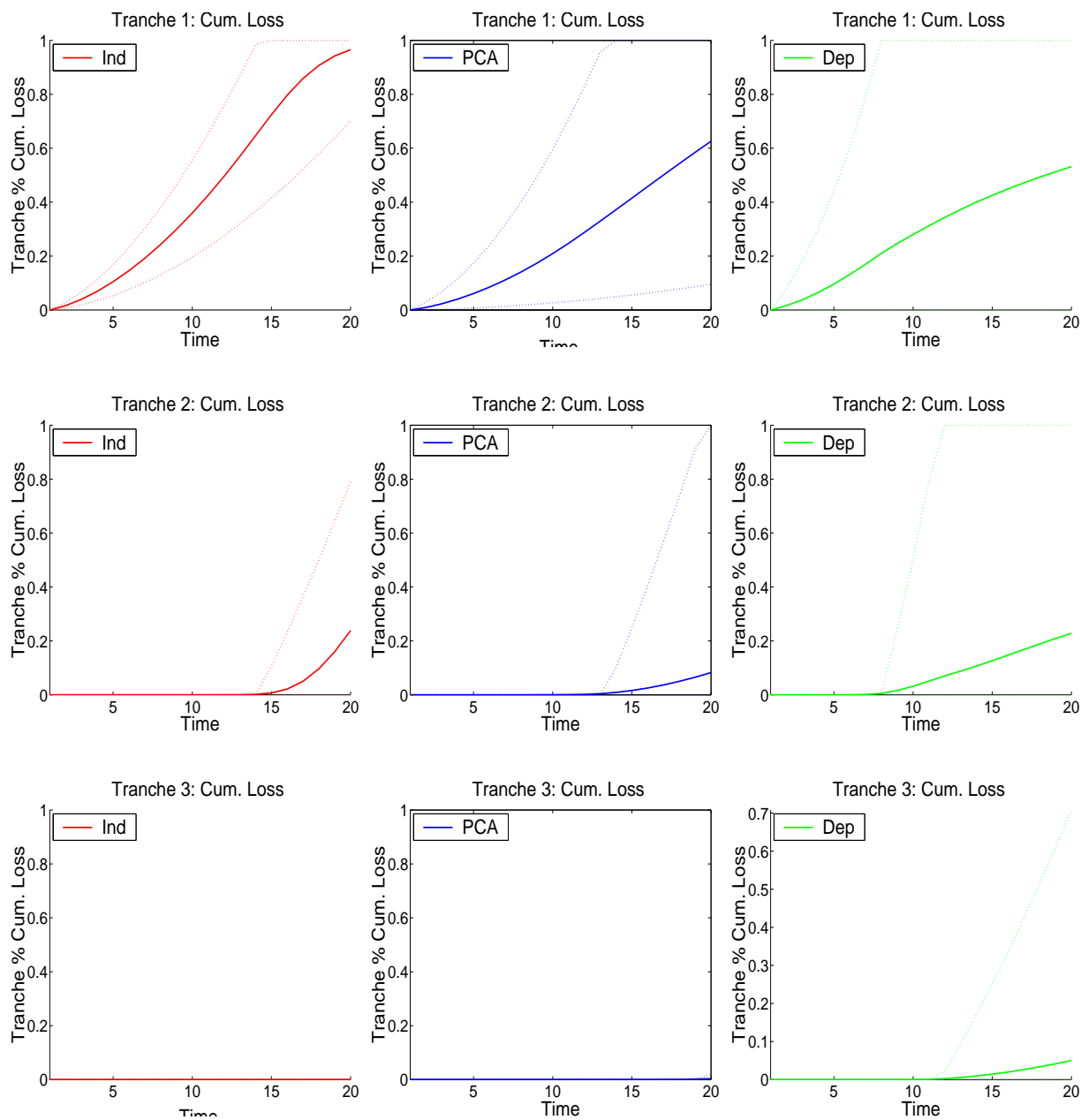
Figure 9: Expected cumulative tranche loss $TCL_j(t)$ (solid), with 95% bounds (dotted), under 3 assumptions for mortality dependence.

Tranche losses are not equally incurred across all cohorts in the annuity portfolio. Figures 10-12, show the expected cumulative tranche loss at each time disaggregated by initial age. This clearly shows that the largest contribution to tranche loss at each time is made by the older cohorts. The difference is particularly pronounced in more senior tranches due to the higher volatility in the mortality process at these ages.

These results clearly demonstrate how age-dependence has a significant impact on structuring and pricing multiple age longevity linked securities. As each tranche has a different exposure to the loss distribution, their characteristics vary greatly. The contribution to tranche losses by each cohort in a multi age portfolio are not equal, and vary by the tranche and by the age dependence in the underlying portfolio. Modelling the age structure and age dependence is a crucial factor in structuring and pricing a longevity securitization.

5.2   Pricing and Calibration of Market Price of Risk

Figure 13 presents a summary of the tranche premiums and associated risk adjustments λ_j^* in the longevity bond structure. They are calculated under three age-dependence assumptions. Premiums are calibrated to those generated by the Lane model as fitted to existing mortality bond issues to estimate the price of longevity risk.

The parameters used in this model are given in Table 3.

	Inputs			Premiums		
	PFL	CEL	Annual EL	Lane: All	Lane: Mort.	Calibrated
Tranche 1	1.000	0.048	0.048	1943	2653	2652
Tranche 2	0.758	0.016	0.012	732	1082	1085
Tranche 3	0.005	0.001	0.000	6	3	3

Table 6: Model inputs and premiums calculated using the Lane (2000) model, under perfect age independence.

	Inputs			Premiums		
	PFL	CEL	Annual EL	Lane: All	Lane: Mort.	Calibrated
Tranche 1	1.000	0.031	0.031	1426	2058	2058
Tranche 2	0.196	0.021	0.004	368	373	371
Tranche 3	0.030	0.005	0.000	46	32	31

Table 7: Model inputs and premiums calculated using the Lane (2000) model, under age co-dependence using PCA.

Two sets of premiums are calculated under the Lane model, based on two different data sets. The first (Lane: All), is based on the characteristics of all 72 insurance linked securities
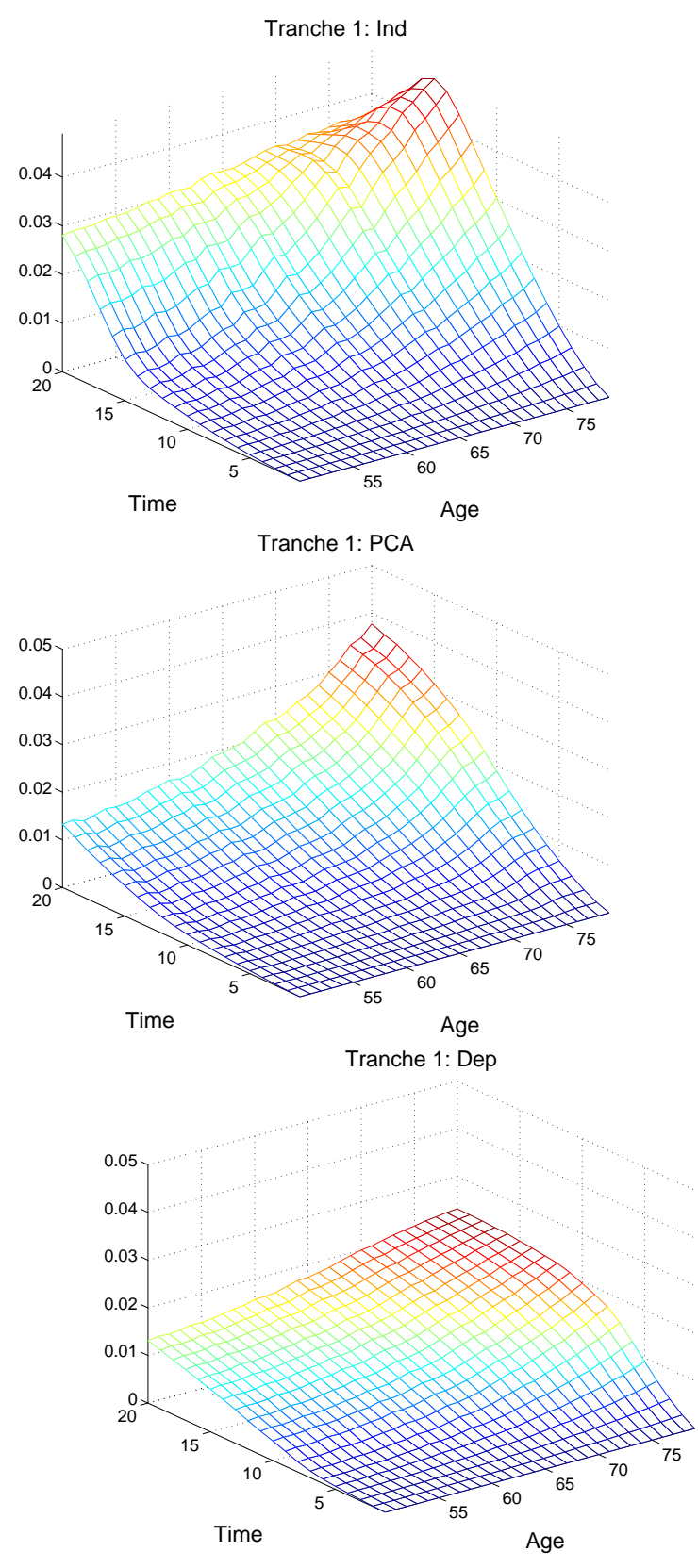
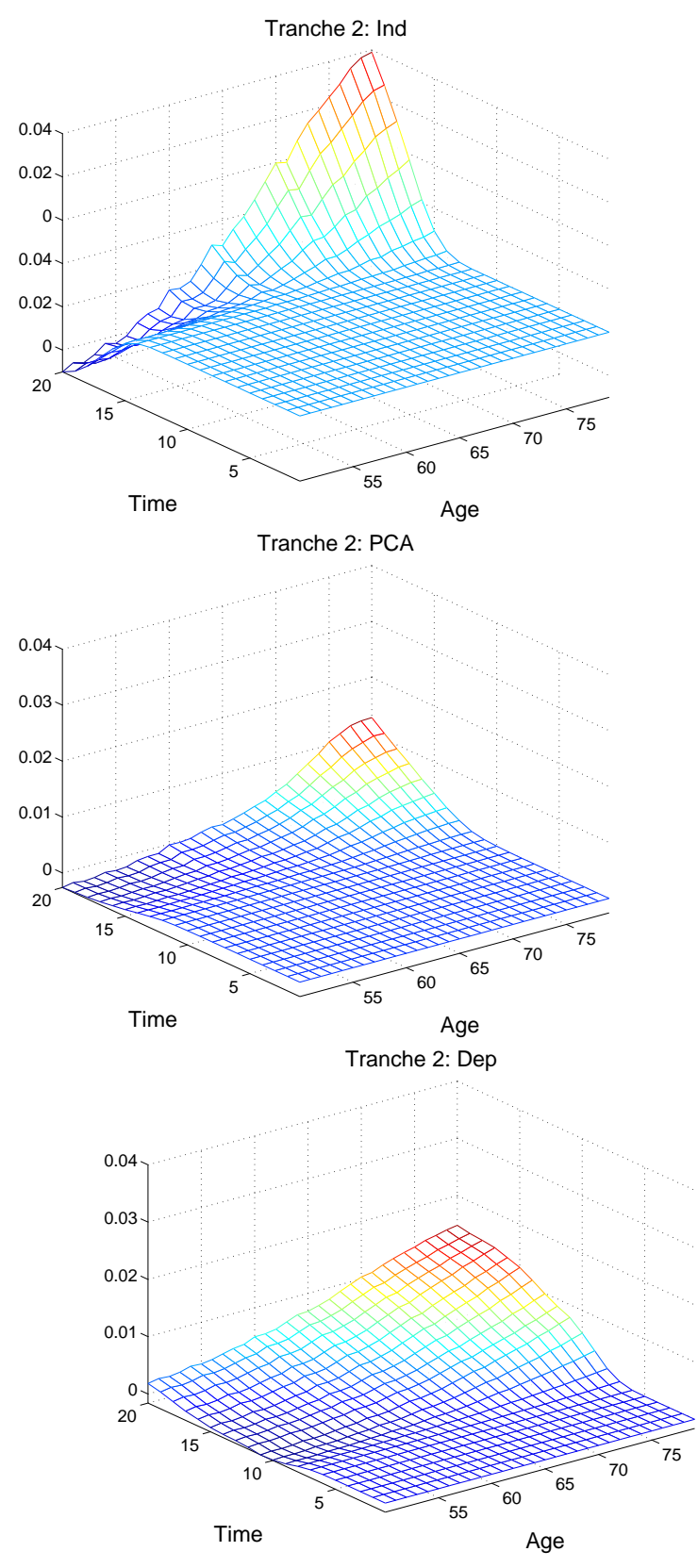**Figure 10:** Tranche 1: Expected cumulative losses by age.

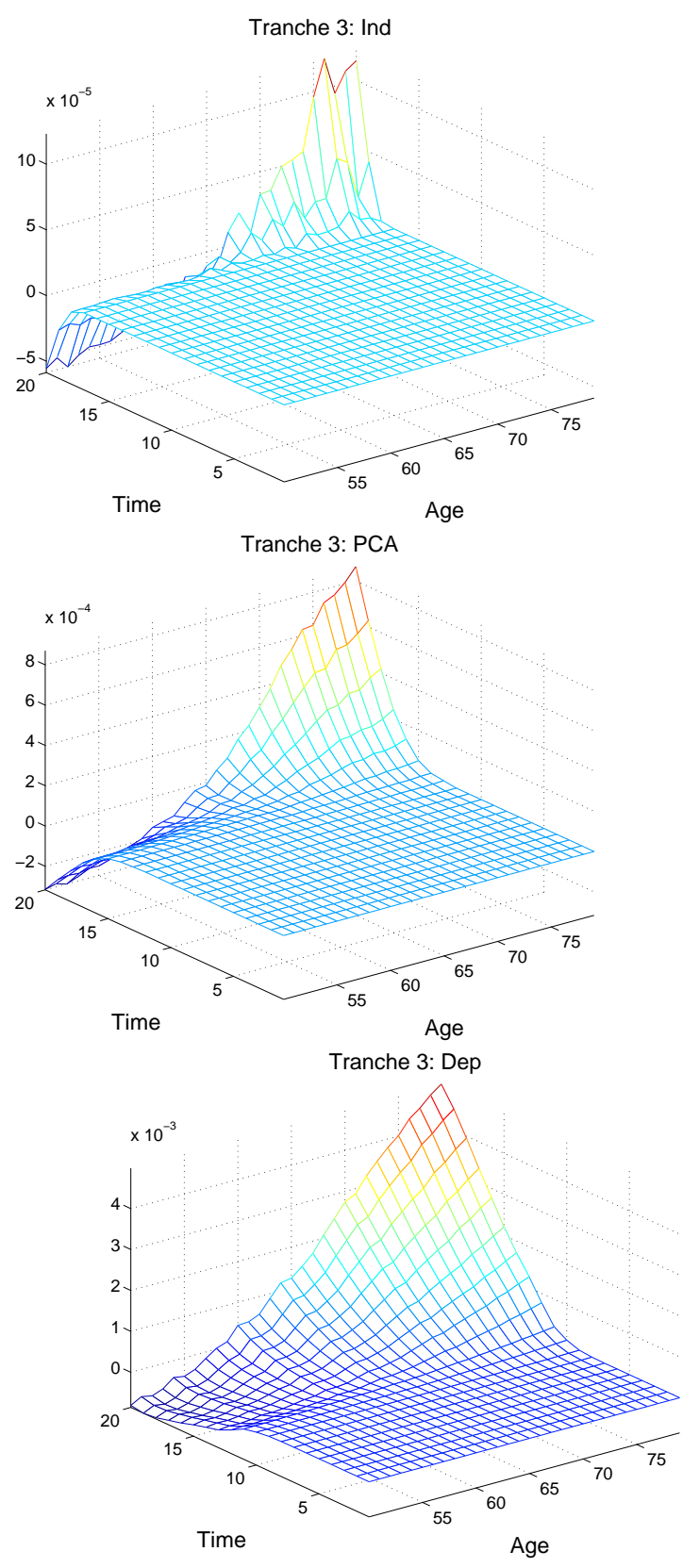**Figure 11:** Tranche 2: Expected cumulative losses by age.

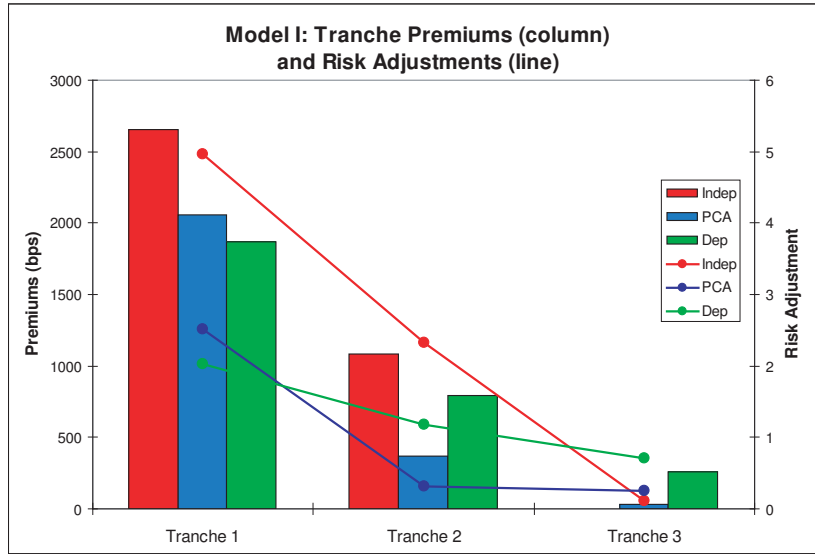**Figure 12:** Tranche 3: Expected cumulative losses by age (scales vary).

Figure 13: Tranche premiums and risk adjustments.

| | Inputs | | | Premiums | | |
	PFL	CEL	Annual EL	Lane: All	Lane: Mort.	Calibrated
Tranche 1	1.000	0.027	0.027	1271	1873	1870
Tranche 2	0.327	0.035	0.011	723	790	789
Tranche 3	0.153	0.016	0.003	266	260	261

Table 8: Model inputs and premiums calculated using the Lane (2000) model, under perfect age dependence.

issued in 2006 (drawn from data in Lane, 2007). This includes catastrophe bonds exposed to a number of risks. The second (Lane: Mort.) is fitted only to the thirteen tranches of the mortality-linked issues, Vita III and Osiris Capital, over the same period. The latter is selected as the basis for calibration since it provides a closer reflection of the market price for longevity risk.

The calibration to mortality bond data has higher yielding junior tranches, and lower premiums for the senior tranches. Tranche premiums are a function of expected loss, and its constituent parts PFL and CEL. The premiums in Figure 13 reflect tranche expected losses.

The parameter λ_j^* is the estimated 'price of risk'. It differs by tranche and depends on the underlying mortality risk. As λ_j^* is an adjustment to the real-world mortality process, it captures mortality risk preferences under the market calibrated measure $\mathbb{Q}$, as compared to the real world measure $\mathbb{P}$. In Equation (24), λ_j^* was defined as a reduction in mortality

	Premium	λ_j^*	Sensitivities				
			$\hat{PFL}_j$	$\hat{CEL}_j$	γ	α	β
Tranche 1	2652	4.97	-	1.38	1.91	-	-2.89
Tranche 2	1085	2.32	2.48	1.52	2.43	-0.60	-5.02
Tranche 3	3	0.11	0.27	0.16	0.30	-1.38	-0.97

Table 9: Tranche premiums, risk adjustments and sensitivities of λ_j^* under perfect age independence.

	Premium	λ_j^*	Sensitivities				
			$\hat{PFL}_j$	$\hat{CEL}_j$	γ	α	β
Tranche 1	2058	2.52	-	1.39	2.04	-	-3.52
Tranche 2	371	0.31	1.24	0.76	1.21	-1.76	-2.33
Tranche 3	31	0.25	0.29	0.17	0.31	-0.95	-0.79

Table 10: Tranche premiums, risk adjustments and sensitivities of λ_j^* under age co-dependence using PCA.

	Premium	λ_j^*	Sensitivities				
			$\hat{PFL}_j$	$\hat{CEL}_j$	γ	α	β
Tranche 1	1870	2.03	-	1.41	2.11	-	-3.81
Tranche 2	789	1.18	1.83	1.15	1.72	-1.72	-2.88
Tranche 3	261	0.70	0.76	0.46	0.76	-1.26	-1.55

Table 11: Tranche premiums, risk adjustments and sensitivities of λ_j^* under perfect age dependence.

drift. Higher values result in improved longevity assumptions under $\mathbb{Q}$, resulting in larger expected annuity portfolio losses and a larger risk loading in tranche premiums.

A summary of tranche premiums, risk adjustments and the sensitivities of λ_j^* to inputs for the Lane model are provided in Tables 9, 10 and 11. The value of λ_j^* decreases with tranche seniority under each assumption for age dependence. This is consistent with investors having a non-linear risk/return tradeoff and being willing to accept smaller risk adjustments in more senior tranches.

The sensitivities of λ_j^* to the Lane model inputs describe changes in λ_j^* as a multiple of the change in each underlying parameter, all else being equal. Sensitivities to PFL and α for Tranche 1 are zero since $PFL = 1$ for this tranche. The sensitivity of the risk adjustment to all underlying parameters decreases with tranche seniority. Parameters α and β are both exponents of values in [0,1], and have an inverse relationship with price,

and in turn λ_j^*. The risk adjustment is most sensitive to changes in β, followed by the probability of first loss in the tranche, and then γ.

Tranche premiums reflect expected losses when calibrated under the Lane model. The risk adjustment λ_j^* is the market price of mortality (longevity risk for each tranche. The size of this price of risk is consistent with the utility profiles of risk averse investors with non-linear risk/return tradeoffs. This price of risk can be used to calibrate mortality models for application to pricing other longevity related securities.

6 Conclusion

Longevity risk is an increasingly important risk facing individuals and financial institutions as they offer products to provide insurance and risk transfer for this risk. Longevity bonds have yet to be successfully structured and issued to capital markets. Longevity bonds are considered to have many advantages over other methods of securitizing longevity risk, such as the survivor bonds used in the failed BNP/EIB issue of 2004. As longevity bonds manage the risk of the originating annuity provider, basis risk can be minimized by tailoring the reference portfolio. This is akin to the method used in mortality bonds by Swiss Re, with considerable success. By basing losses on publicly-available mortality data, the costs of moral hazard and information asymmetry can also be managed.

Since its introduction, the Lee Carter mortality model has been widely used, and has become a benchmark for the stochastic modeling of mortality. However, its suitability for pricing longevity linked securities is restricted, due to limitations in incorporating a risk adjustment into the mortality distribution. Both Lin and Cox (2005), and Liao et al (2007) employ the Wang (1996, 2000, 2002) transform to price longevity bonds. This method has been subject to criticism (see Cairns et al, 2006a; and Bauer and Russ, 2006), because of the assumption of a constant market price of mortality risk across ages and time.

The pricing and structuring in this paper is based on a multivariate mortality model calibrated under a risk neutral pricing measure. The model is flexible enough to allow calibration of the risk adjustment to insurance linked security prices that can vary across ages and time. This allows for the incorporation of varying investor preferences for exposures at different points on the survival curve.

There is flexibility in the choice of the underlying loss measure and tranche specification for a longevity bond. A natural approach is to base this on the cumulative loss of an annuity portfolio. This compensates a bond issuer for losses on an annuity portfolio as they occur. Other alternative loss measures could be based on the value of an annuity portfolio's future obligations, allowing for all future expected longevity improvements to be captured by the bond payoffs, as proposed by Sherris and Wills (2007). This is a structure that could also

have potential interest and worthy of further analysis.

This paper provides a detailed analysis of the structuring and pricing of a longevity bond suitable for a multiple age annuity portfolio. Losses on a multiple age portfolio are not evenly incurred across all cohorts. Dependence between ages plays a significant role in accurately specifying the mortality process and its importance reflects in the impact this has on the pricing and tranche structure for a longevity risk bond. Age dependence must be considered in order to successfully structure and price longevity-linked securities.

7   Acknowledgement

References

[1] APRA, 2007a, December 2007 Quarterly Superannuation performance report (revised 8 April 2008), Australian Prudential Regulation Authority, Sydney.

[2] APRA, 2007b, December 2007 Life Office Market Report, Australian Prudential Regulation Authority, Sydney.

[3] Bauer, D., and Russ, J., 2006, Pricing Longevity Bonds using Implied Survival Probabilities. (Available at http://www.mortalityrisk.org/Papers/Models.html)

[4] Biffis, E., 2005, Affine Processes for Dynamic Mortality and Actuarial Valuations, *Insurance: Mathematics and Economics*, 37, 443-468.

[5] Blake, D., Cairns, A.J.G., and Dowd, K., 2006a, Living with Mortality: Longevity Bonds and Mortality-Linked Securities. (Available at http://www.pensions-institute.org/papers/html)

[6] Blake, D., Cairns, A.J.G., and Dowd, K., 2006b, Discussion held by the Faculty of Actuaries: "Living with Mortality Longevity Bonds and Other Mortality-Linked Securities". (Available at http://www.pensions-institute.org/papers/html)

[7] Blake, D., Cairns, A., Dowd, K., and MacMinn, R., 2006, Longevity Bonds: Financial Engineering, Valuation, and Hedging, *The Journal of Risk and Insurance*, 73(4), 647-672.

[8] Blum, C., and Overbeck, L., 2006, Structured Credit Portfolio Analysis, Baskets and CDOs, *Chapman and Hall/CRC*, London.

[9] Booth, H., Maindonald, J., and Smith, L., 2002, Age-time Interactions in Mortality Projection: Applying Lee-Carter to Australia, *Working Paper*, The Australian National University.

[10] Cairns, A.J.G., Blake, D., and Dowd, K., 2006, Pricing Death: Frameworks for the Valuation and Securitization of Mortality Risk, *ASTIN Bulletin*, 36, 79-120

[11] Chang, C., C., and Shyu, D., 2007, The Pricing of Securitization of Life Insurance under Mortality Dependence, *Presented at the Third International Longevity Risk and Capital Market Solutions Symposium*, Taipei.

[12] Creighton, A., Hongbo Jin, H., Piggott, J., and Valdez, E.A., 2005, Longevity Insurance: A Missing Market, *The Singapore Economic Review*, 50, 417-435.

[13] Dahl, M., 2004, Stochastic Mortality in Life Insurance: Market Reserves and Mortality-Linked Insurance Contracts, *Insurance: Mathematics and Economics*, 35, 113-136.

[14] Dahl, M., and Møller, T., 2005, Valuation and Hedging of Life Insurance Liabilities with Systematic Mortality Risk, In *Proceedings of the 15th International AFIR Colloquium*, Zurich. (Available at http://www.afir2005.ch)

[15] Hull, J., 2003, Options, Futures and Other Derivatives: Fifth Edition, *Prentice Hall*, New Jersey.

[16] JP Morgan, 2007, LifeMetrics: A Toolkit for Measuring and Managing Longevity and Mortality Risks. (Available at http://www.jpmorgan.com/).

[17] Lane, M.N., 2000, Pricing Risk Transfer Transactions, *ASTIN Bulletin*, 30, 259-293.

[18] Lane, M.N., and Beckwith, R., 2005, The 2005 Review of the Insurance Securitization Market: Game On!, Lane Financial L.L.C. (Available at http://lanefinancialllc.com/).

[19] Lane, M.N., and Beckwith, R., 2006, How High is Up? The 2006 Review of the Insurance Securitization Market, Lane Financial L.L.C. (Available at http://lanefinancialllc.com/).

[20] Lane, M.N., and Beckwith, R., 2007, That was the Year that was! The 2007 Review of the Insurance Securitization Market, Lane Financial L.L.C. (Available at http://lanefinancialllc.com/).

[21] Liao, H.H., Yang, S.S., and Huang, I.H., 2007, The Design of Securitization for Longevity Risk: Pricing Under Stochastic Mortality Model with Tranche Technique, *Presented at the Third International Longevity Risk and Capital Market Solutions Symposium*, Taipei.

[22] Lin, Y., and Cox, S.H., 2005, Securitization of Mortality Risks in Life Annuities, *The Journal of Risk and Insurance*, 72(2), 227-252.

[23] Luciano, E., and Vigna, E., 2005, Non Mean Reverting Affline Processes for Stochastic Mortality, In *Proceedings of the 15th International AFIR Colloquium*, Zurich. (Available at http://www.afir2005.ch).

[24] Purcal, S. T., 2006, Supply Challenges to the Provision of Annuities, School of Actuarial Studies, UNSW, Working Paper, Sydney.

[25] Schrager, D.F., 2006, Affine Stochastic Mortality, *Insurance: Mathematics and Economics*, 38, 81-97.

[26] Sherris, M., and Wills, S., 2007, Financial Innovation and the Hedging of Longevity Risk, *Presented at the Third International Longevity Risk and Capital Market Solutions Symposium*, Taipei.

[27] Wadsworth, M., 2005, The Pension Annuity Market: Further Research into Supply and Constraints, The Association of British Insurers, London.

[28] Wang, S.S., 1996, Premium Calculation by Transforming the Layer Premium Density, *ASTIN Bulletin*, 26, 71-92.

[29] Wang, S.S., 2000, A Class of Distortion Operations for Pricing Financial and Insurance Risks, *Journal of Risk and Insurance*, 67, 15-36.

[30] Wang, S.S., 2002, A Universal Framework for Pricing Financial and Insurance Risks, *ASTIN Bulletin*, 32, 213-234.

[31] Wills, S. and Sherris, M., 2008, Integrating Financial and Demographic Longevity Risk Models: An Australian Model for Financial Applications, *Actuarial Studies Working Paper*, The University of New South Wales.