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Optimum strategies for UK state pension deferral with particular reference to partner's right to inherit benefits.

John Dagpunar.

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The Pensions Institute
Cass Business School
City University London
106 Bunhill Row
London EC1Y 8TZ
UNITED KINGDOM

http://www.pensions-institute.org/

Optimum strategies for UK state pension deferral with particular reference to partner's right to inherit benefits

John Dagpunar, Mathematical Sciences, University of Southampton

Correspondence addresses: j.dagpunar@soton.ac.uk jdagpunar@hotmail.com

Abstract:

A person who defers a UK state pension does so with the prospect of taking an extra pension or lump sum on termination of the deferral. A married or civil partner of a deferrer can inherit these benefits on the death of the deferrer. We propose a mathematical model for maximising the expected joint benefit, extending the work of Dagpunar (2015) regarding a deferrer without partner. The model applies to a deferrer who attained state pension age before 6 April 2016, the partners of others having no right to inheritance. Features of the model include different accrual rates for the various components of a state pension. We show that the scheme offers very generous joint benefits compared with no deferral. It provides an interesting example of dynamic decision-making under uncertainty.

Keywords: pensions, decision-making, deferred benefits, risk, probability, life expectancy, income inequality, inter-generational fairness, CPI, Triple Lock

1. Background

State pension is payable in the UK to eligible people once they reach state pension age. Under the Pensions Acts of 1995 (Pensions Act 1995) and 2011 (Pensions Act 2011), men born before 6 December 1953 and women born before 6 April 1950 have a state pension age of 65 and 60 respectively. The two Acts incrementally increase women's pension ages according to date of birth, so that state pension age will be 65 by 6 November 2018. At that point male and female state pension ages will be equalised and it will then increase so that by 6 October 2020 state pension age for both males and females will be 66. The Pensions Act of 2014 (Pensions Act 2014) increases state pension age from 66 to 67 between 2026 and 2028. The Pensions Act of 2007 (Pensions Act 2007) increases state pension age from 67 to 68 between 2044 and 2046. Government has provided both a schedule[1] and a state pension calculator[2] for determining state pension age given date of birth, under current legislation.

Pensions changed on 6 April 2016. Someone reaching state pension age before 6 April 2016 receives a weekly (periodic) state pension under the 'old rules'. This comprises a *basic pension*[3] plus any *additional pension*[4] (state second pension [2002-2016] and state earnings related pension [SERPS, 1978-2002]) plus any graduated retirement benefit, incapacity benefit age addition, invalidity addition, and extra pension inherited from a legally recognised partner. Additional pension is not earned while a member of a contracted-out workplace pension. For many, if not the majority of people, it is the basic pension (before deferral) that accounts for most of their state pension. People reaching state pension age on or after 6 April 2016 receive the *new state pension*[5]. In each case entitlement and amounts depend upon National Insurance record and the rules changed for those reaching state pension age on or after 6 April 2016.

[1] State Pension age timetables. Available from https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/310231/spa-timetable.pdf

[2] Check your State Pension Age. Available from https://www.gov.uk/state-pension-age

[3] The basic State Pension. Available from https://www.gov.uk/state-pension

[4] Additional State Pension. Available from https://www.gov.uk/additional-state-pension

[5] The new State Pension. Available from https://www.gov.uk/new-state-pension

For those under the old rules it is possible to defer[6, 7, 8] taking the periodic state pension and to receive an enhancement of 10.4% per year deferred. We will refer to this as the *accrual rate*. For example, a man with state pension age of 65 might choose to defer to his 67th birthday. By that time his basic state pension (had he taken it) would have increased in each of the two years by the so called *triple lock*, that is the maximum of CPI (consumer price index) growth, earnings growth, and 2.5%. The remaining components of the state pension are currently (2016-17) increasing by the CPI growth rate. The duly indexed (uprated) state pension would determine the periodic pension that he will start to receive on his 67th birthday. We will call this his/her *standard pension*. On top of that he would take his periodic *state extra pension* of 20.8% of his standard pension. In subsequent years, under current legislation, the basic pension would continue to be indexed according to the triple lock while other components of the periodic pension, including the extra pension, would be indexed according to the CPI. As an alternative to taking the periodic extra pension, he/she has the option of taking a lump sum equal to the two years pension he/she has forgone, with interest added during the deferral at the rate of at least 2% per annum above bank base rate. In fact this option is available to anyone who has deferred for at least one year, while those who deferred for less than a year can opt for extra pension or return of pension forgone but without interest. In the example above, it is assumed that he/she survives the two years. If he/she does not and dies suddenly he/she receives neither the periodic extra pension nor the lump sum. However, on his/her death while deferring for at least a year, if he/she is married or in a civil partnership, his/her partner can choose between inheriting the periodic extra pension or the lump sum, but will have to wait until he/she reaches state pension age, and then only if at that point in time he/she has neither remarried nor taken on a new civil partnership. If the deceased had deferred for less than a year then the surviving partner would have to inherit the periodic extra pension. If the deferrer dies after terminating deferral and had opted for the lump sum, a surviving partner obviously inherits neither a lump sum nor a periodic extra pension, while if the deferrer had chosen the option of extra pension, then a surviving partner, subject to the caveats above, would inherit the periodic extra pension. For many deferrers the basic pension will account for much if not all of the standard pension. The inheriting partner can inherit 100% of the extra state pension or lump sum that the deferrer has earned on the basic state pension. The corresponding proportions for other components of the standard pension are 50% for Graduated Retirement, 50% for State Second Pension, between 50% and 100% for SERPS, and 50 % of inherited extra pension from a recognised legal partner.

It should be mentioned that there are many circumstances in which deferral will not lead to the building up of an extra pension. The long list includes people receiving income support, universal benefit, income tax credits, disability benefit, and also those in prison. Some people may receive reduced state benefits once they take the extra pension. For those living abroad, once the extra pension is taken it may not be subsequently indexed (uprated), depending on the country.

We now turn to those who take the new state pension and reach state pension age on or after 6 April 2016. In that case the accrual rate is a far less generous 5.8 % per annum. The stated intention was to

[6] Delay (defer) your state pension. Available from https://www.gov.uk/deferring-state-pension/how-to-claim-a-deferred-state-pension

[7] State pension deferral if you reached state pension age before 6 April 2016. Extra information (May 2016). Available from https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/527015/state-pension-deferral-if-you-reached-state-pension-age-before-6-april-2016-extra_information.pdf

[8] State pension deferral if you reached state pension age before 6 April 2016 and you die. Extra information (May 2016). Available from https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/525363/state-pension-deferral-if-you-reached-state-pension-age-before-6-april-2016-and-you-die-extra-information.pdf

make the scheme 'actuarially fair'. In his report[9] to the Department of Work and Pensions the Government Actuary interpreted this to mean '… the benefits available have broadly the same value in terms of cost to the Exchequer … whether the person chooses to defer or not.' In this case there is no alternative for the deferrer to take a lump sum option; and there is no opportunity for a surviving partner to either inherit or take a periodic extra pension. The eligibility of a partner to inherit a lump sum or extra pension from a deferrer, who is subject to the old rules, does *not* depend upon whether the partner is subject to the old or new rules.

Regarding previous work in this area, Dagpunar (2015) devised a probability model that maximises the expected value of the total (real) pension income received by a single deferring pensioner. His model takes explicit account of the risk associated with dying during the deferral period. He concluded that for a single pensioner it is probably not worth deferring under the new rules 'unless there is good reason to believe that an individual's life expectancy is appreciably larger than the UK national average or there are significant income tax advantages.' However, for a single pensioner attaining state pension age before 6 April 2016 Dagpunar shows that there are significant advantages for a single male to defer and considerable advantages for a female to do so.

Other authors who have written on the subject of a single deferrer, but without explicit quantitative modelling of the survival-risk aspect, include Stubbs and Adetunji (2016), Farrar et al. (2012), and Kanabar and Simmons (2016). The motivation for studying deferral is three-fold. Firstly, it can provide guidance for individual pensioners. Secondly, it allows Government to calculate additional benefits to such pensioners who take advantage of the scheme and therefore provides some objective evidence into discussions around inter-generational fairness as well as the effect of the scheme on income inequality. Thirdly, it provides Government with a methodology for assessing the cost to the taxpayer of the scheme. In respect of the second and third objectives we mention previous work on the costs and sustainability of aspects of the UK state pension scheme in Blake and Mayhew (2006), Hemming and Kay (1982), and Agulnik et al. (2000).

In this paper we briefly review the analysis for a single deferrer and then extend it, to a deferrer with partner, showing that the scheme is considerably more generous than for a single deferrer, in terms of the expected joint benefits the pair receive over their lifetimes. It provides an interesting example of dynamic decision-making under uncertainty and is relevant to those who are currently deferring under the old rules.

2. Model for deferring pensioner without partner

Let us now recap as in Dagpunar (2015) the case of a single pensioner who is neither married nor in a civil partnership and who defers under the old rules. In that case the accrual rate is $\beta = 0.104$. We wish to determine an optimal strategy which will maximise the deferrer's expected real-valued pension receipts over his/her lifetime, present-valued back to the deferrer's state pension age.

Let a denote the deferrer's state pension age. Let $m(x)$ denote the mean residual life (average [or expectation of] life expectancy) for the deferrer at age x. Let $r(x)$ denote the mortality rate at age x and $S(x)$ the probability that the individual survives to at least age x. At state pension age a the conditional probability that the deferrer will survive to at least age $a+x$ is $S(a+x)/S(a)$.

Suppose he/she plans to defer for x years and take his/her periodic pension at age $a+x$. At the moment the basic pension is uprated (indexed) in line with the triple lock whereas other parts (including the extra pension) are uprated in line with the CPI. It is not known for how long this discrimination in

[9] Government Actuary's Department. Report by the Government Actuary on the actuarially fair rate of increments for those reaching State Pension age on or after 6 April 2016 and choosing to defer their State Pension beyond State Pension age. Available from http://data.parliament.uk/DepositedPapers/Files/DEP2014-1112/GAD_State_Pension_Deferral_Report.pdf

rates will persist or of course what CPI and Triple Lock rates will be in future years. The present government intends to retain the triple lock, but there is some pressure to remove or modify it at some point, on the grounds that it is excessively expensive and unfair[10]. In the absence of knowledge as to what will happen in the long term it is not unreasonable to assume that all components of the pension are indexed at the same rate. Appendix 1 addresses this assumption and concludes that it is unlikely to introduce any great degree of sub-optimality, as different uprating scenarios change the optimal deferral period only slightly, and induce only a small penalty with respect to using an 'incorrect model' for uprating. With this assumption the present value of rate at which the pension is paid, referred back to state pension age, will be $1+\beta x$ of the deferrer's standard pension at age a. In this case the discount factor used to obtain the Present Value is based upon the uniform uprating rate.

Let $ASP(y,x)$ denote the expected (average) number of standard pension years the individual will receive after deferring y years so far and planning to defer for a total of $x(>y)$ years. Then

$$ASP(y,x)=\frac{S(a+x)}{S(a+y)}(1+\beta x)m(a+x). \tag{1}$$

Differentiating,

$$\begin{aligned}\frac{\partial ASP}{\partial x}&=\left\{-r(a+x)+\frac{\beta}{1+\beta x}+\frac{m(a+x)r(a+x)-1}{m(a+x)}\right\}ASP\\ &=\frac{S(a+x)}{S(a+y)}\{-(1+\beta x)+\beta m(a+x)\}.\end{aligned} \tag{2}$$

Now the mean residual life is decreasing in x and therefore the optimal strategy is to continue to defer while

$$V(x)=-(1+\beta x)+\beta m(a+x)>0\ . \tag{3}$$

The optimal deferral period x^* is given by

$$m(a+x^*)=x^*+\beta^{-1}\ . \tag{4}$$

From (2) we have

$$ASP(y,x)=ASP(y,y)+\int_y^x\frac{S(a+u)}{S(a+y)}V(u)du\ . \tag{5}$$

From (1)

$$ASP(y,y)=(1+\beta y)m(a+y)\ . \tag{6}$$

We interpret $V(u)du$ as the increase in average standard pension years for delaying termination of deferral from age $a+u$ to $a+u+du$, conditional on having survived to age $a+u$. This is evident by the sacrificed pension $-(1+\beta u)du$ that would have been payable in $(a+u,a+u+du)$ and the increased rate of periodic extra pension βdu earned as a result of the delay.

Substituting (4) into (1) we have for $y<x^*$

$$ASP(y,x^*)=\frac{S(a+x^*)(1+\beta x^*)^2}{\beta S(a+y)}\ . \tag{7}$$

[10] Intergenerational fairness, Third report of session 2016-17, HC59, House of Commons Work and Pensions Committee, pp. 24-33. Available from http://www.publications.parliament.uk/pa/cm201617/cmselect/cmworpen/59/5906.htm

Finally, in this section we must mention how an individual will respond to a sudden downgrading in mean residual life, perhaps because of a new medical condition. In that case the optimal decision would be to take the lump sum if the accrued lump sum plus the expected value of the periodic standard pension over the remainder of his/her life exceeds the expected value of the standard *plus* extra pension, that is if

$$x + m(a+x) > (1+\beta x)m(a+x) \tag{8}$$

that is if

$$m(a+x) < \beta^{-1}. \tag{9}$$

Note that if the interest rate of (at least) 2% on top of bank base rate was exactly equal to the assumed common uprating (indexation) rate, then the value of the lump sum, present-valued back to age a, would equate to the x standard pension years that we have used in (8). We have assumed this to be the case, appealing again (following Appendix 1) to the robustness of the optimal deferral period and associated penalty with respect to changes in scenario.

Summarising, the optimal decision rule for a single pensioner can be stated as follows: If $m(a+x) > x + \beta^{-1}$ then continue deferring, else if $m(a+x) > \beta^{-1}$ stop deferral immediately and take the periodic extra plus standard pension, else take the lump sum plus periodic standard pension.

3. Numerical Example for deferrer without partner

Let us consider the case of a woman born between 6 March 1953 and 5 April 1953, the last period of birth allowing a pension under the old rules. She attains state pension age of 63 on 6 March 2016. She chose to defer. From ONS (period) life tables (2013-15)[11], $m(63) = 22.57$, $m(70) = 16.78$, $m(71) = 15.99$. Now, $\beta = 0.104$, so

$$V(7) = -(1+7\beta) + \beta m(63+7) = 0.01712 \tag{10}$$

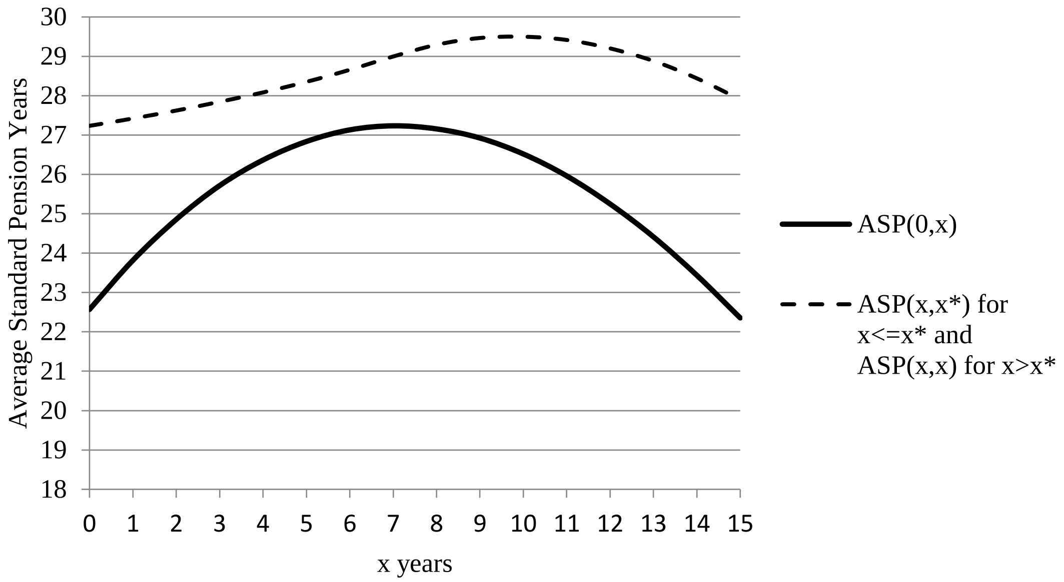
and

$$V(8) = -(1+8\beta) + \beta m(63+8) = \text{-}0.16904\ . \tag{11}$$

Interpolating, $x^* = 7.1$ and $\beta^{-1}(1+\beta x^*)^2 = 29.03$. The conditional probability of surviving to age 70.1 is 0.938 so $ASP(0, x^*) = 0.938 \times 29.03 = 27.23$ which represents a 21% uplift on the 22.57 years she can expect with no deferral. Now suppose that she has survived to age x where $x \le x^*$. Then she should plan to terminate deferral after x^* years with expected future benefits of $ASP(x, x^*)$. However, if $x > x^*$ she would take the extra pension immediately and the expected future benefit is $ASP(x, x)$. Figure 1 shows the expected future benefit $ASP(0, x)$ at age 63 for any planned deferral period x, and also the expected future benefit under the optimal policy at age 63+x having survived to that age.

[11] https://www.ons.gov.uk/file?uri=/peoplepopulationandcommunity/birthsdeathsandmarriages/lifeexpectancies/datasets/nationallifetablesunitedkingdomreferencetables/current/nltuk1315reg.xls

Figure 1: Average standard pension years for female with state pension age of 63 (old rules)



The conclusion is that if such a woman is in a financial position to defer her pension it is worth doing. Dagpunar (2015) has produced graphs showing the uplift for men and women of various state pension ages. As in that paper, the analysis presented here takes no account of possible different taxation due to a delay in taking pension benefits. In that paper he also examines the effect of deviating from the ONS tables, where an individual may feel his/her average life expectancy will be greater than that given in the ONS tables.

4. The case of deferrer with married or civil partner

We will consider a deferrer (A) who has a state pension age of a and his/her partner (B) having age b at the time at which A reaches age a. We will consider the case where b is at least equal to B's state pension age. This means that if A dies first then B will immediately be eligible for inheriting benefits as described in section 1, unless A had previously taken the lump sum. We will see that this leads to an attractive rule of thumb for determining optimal deferral period. The case where b is less than B's state pension age is perfectly amenable to numerical calculation, but introduces extra mathematical complexity and no correspondingly simple rule of thumb.

Let $m_A(x)$ and $m_B(x)$ denote their respective mean residual lives at ages x and let $M(x_A, x_B)$ denote the expected time to the second death of A and B given that they have respective ages x_A and x_B. Let $S_A(x)$ and $S_B(x)$ be the respective survivor functions for A and B and $r_A(x)$ and $r_B(x)$ be their respective mortality rates.

Let γ denote that proportion of the standard pension that can earn an extra pension or lump sum, 100% of which is inheritable, and let $1-\gamma$ denote the remaining proportion that can earn an extra pension or lump sum, 100θ % of which is inheritable. Let $\beta = 0.104$ denote the basic rate of accrual of extra pension and let

$$\beta_1 = \{\gamma + (1-\gamma)\theta\}\beta \tag{12}$$

denote the rate of accrual of extra pension for an inheriting partner.

It follows that if the deferrer has just stopped deferral after *x* years, where both deferrer and survivor are still alive, the expected joint income until second death is

$$\beta_1 xM(a+x,b+x)+(\beta-\beta_1)xm_A(a+x)+m_A(a+x) \quad . \tag{13}$$

expressed as before in standard pension years.

It is possible that either A or B dies before the end of the planned deferral period x is reached. If A predeceases B at age $a+u$ (where $u<x$), then B can choose either to inherit a lump sum of $\beta_1 u/\beta$ standard pension years or a periodic extra pension of $\beta_1 u$ per year. If B predeceases A at age $b+u$ (where $u<x$), then A can choose, either to take a lump sum of u standard pension years plus the periodic standard pension, or take both periodic extra pension of βu per year and standard pension, or (if $m_A(a+w)=w+\beta^{-1}$ where $w>u$) continue to defer, planning to take the extra plus standard pension when A reaches age $a+w$.

Bringing these together, the joint average future standard pension years after *y* years of deferral and planning to defer $x(>y)$ years is

$$\begin{aligned}
ASP(y,x) &= \frac{S_A(a+x)S_B(b+x)}{S_A(a+y)S_B(b+y)}\{\beta_1 xM(a+x,b+x)+(\beta-\beta_1)xm_A(a+x)+m_A(a+x)\} \\
&+\int_y^x r_A(a+u)\frac{S_A(a+u)S_B(b+u)}{S_A(a+y)S_B(b+y)}\frac{\beta_1}{\beta}\max[u,\beta u m_B(b+u)]du \\
&+\int_y^x r_B(b+u)\frac{S_A(a+u)S_B(b+u)}{S_A(a+y)S_B(b+y)}\max\begin{bmatrix} u+m_A(a+u),\{1-H(m_A(a+u)-u-\beta^{-1})\}(1+\beta u)m_A(a+u)+ \\ H(m_A(a+u)-u-\beta^{-1})(1+\beta w)m_A(a+w)S_A(a+w)/S_A(a+u)\end{bmatrix}du
\end{aligned} \tag{14}$$

where $H(.)=1$ for positive arguments, otherwise equals zero, and where

$$m_A(a+w)=w+\beta^{-1} \quad . \tag{15}$$

Note that the last term of (14) relies upon the single deferrer strategy derived in section 2, where the optimal deferral period is *w*. In appendix 2 it is shown that

$$M(x_A,x_B)=m_A(x_A)+m_B(x_B)-\int_0^\infty \frac{S_A(x_A+u)S_B(x_B+u)}{S_A(x_A)S_B(x_B)}du \tag{16}$$

and in appendix 3 that

$$\frac{\partial ASP(y,x)}{\partial x}=\frac{S_A(a+x)S_B(b+x)}{S_A(a+y)S_B(b+y)}V(x) \tag{17}$$

where

$$\begin{aligned}
V(x) &= -(1+\beta x)+\beta_1 M(a+x,b+x)+(\beta-\beta_1)m_A(a+x)+\frac{\beta_1}{\beta}r_A(a+x)\max[x-\beta x m_B(b+x),0] \\
&+r_B(b+x)\max\begin{bmatrix} x-\beta x m_A(a+x), \\ H(m_A(a+x)-x-\beta^{-1})\{(1+\beta w)m_A(a+w)S_A(a+w)/S_A(a+x)-(1+\beta x)m_A(a+x)\}\end{bmatrix} .
\end{aligned} \tag{18}$$

$V(x)dx$ is the conditional increase in average (expected) joint standard pension years for delaying termination of deferral by a further increment dx. With hindsight, this result is evident from a marginality argument, analogous but more elaborate to that used for the single deferrer.

It follows from (17) that A should continue to defer while $V(x) > 0$. This will allow identification of the first local optimum. Numerical experience suggests that this will be a global optimum, and indeed it is difficult to imagine a pathological case where that would not be so.

We may write

$$ASP(y,x) = ASP(y,y) + \int_y^x \frac{S_A(a+u)S_B(b+u)}{S_A(a+y)S_B(b+y)} V(u)du \tag{19}$$

where from (14)

$$ASP(y,y) = \beta_1 yM(a+y, b+y) + (\beta - \beta_1)ym_A(a+y) + m_A(a+y)\ . \tag{20}$$

$ASP(y,y)$ is the average (expected) joint future standard pension years to be received after A terminates deferral at age $a+y$, conditional upon both A and B being alive. Putting $y=0$ into (19) we have

$$ASP(0,x) = m_A(a) + \int_0^x \frac{S_A(a+u)S_B(b+u)}{S_A(a)S_B(b)} V(u)du \quad . \tag{21}$$

All this assumes that while A is deferring and both A and B are alive, that as time passes the life expectancies follow the national average and decrease continuously. However, it may be that one or other of A or B experiences a discontinuous reduction in life expectancy which might trigger immediate termination of deferral with the lump sum option. The condition for this is that the lump sum exceeds the expected extra pension that would be enjoyed by the pair until the second death. The condition for this is

$$\beta_1 xM(a+x, b+x) + (\beta - \beta_1)xm_A(a+x) < x \tag{22}$$

that is

$$\frac{\beta_1}{\beta} M(a+x, b+x) + \left(1 - \frac{\beta_1}{\beta}\right) m_A(a+x) < \frac{1}{\beta} \quad . \tag{23}$$

If the lump sum is taken in this way, then obviously B will not inherit.

Assuming condition (23) is not previously triggered, A will continue to defer while $V(x) > 0$, terminating deferral at x^* where $V(x^*) = 0$. A will then take a periodic extra pension of βx^* per year knowing that if A subsequently dies first then B will inherit a periodic extra pension of $\beta_1 x^*$ per year.

In (18) the r_A and r_B terms are small for relevant values of x (with the r_A term being zero when $m_B(b+x) > \beta^{-1}$ and the r_B term being zero when $x + \beta^{-1} > m_A(a+x) > \beta^{-1}$) and a good approximation to $V(x^*) = 0$ is

$$\frac{\beta_1}{\beta} M(a+x^*, b+x^*) + (1 - \frac{\beta_1}{\beta}) m_A(a+x^*) = \frac{1}{\beta} + x^* \tag{24}$$

which is reminiscent of the optimal rule (4) for the single deferrer. Evidently, if both $m_B(b+x^*) > \beta^{-1}$ and $x^* + \beta^{-1} > m_A(a+x^*) > \beta^{-1}$ then the approximation is exact.

Summarising, the strategy is that while both A and B are alive, A should take the extra pension at the smallest x such that $V(x) \le 0$, unless condition (23) is previously triggered in which case A should take the lump sum. If the approximation (24) is used the strategy may be succinctly expressed as:

While A and B are alive, if $\frac{\beta_1}{\beta} M(a+x,b+x)+(1-\frac{\beta_1}{\beta})m_A(a+x) > \frac{1}{\beta}+x$ then A should continue to defer, else if $\frac{\beta_1}{\beta} M(a+x,b+x)+(1-\frac{\beta_1}{\beta})m_A(a+x) > \frac{1}{\beta}$ then A should take the extra periodic pension, else A should take the lump sum.

If A dies while deferring, B should take the periodic extra pension if $m_B(b+x) > \beta^{-1}$, else B should take the lump sum. If B dies during deferral and $m_A(a+x) > x+\beta^{-1}$ then A should plan to take the extra pension at age $a+w$ where $m_A(a+w) = w+\beta^{-1}$, else if $m_A(a+x) > \beta^{-1}$ then A should take the extra and pension immediately, else A should take the lump sum immediately.

5. Numerical Example for deferrer A and partner B

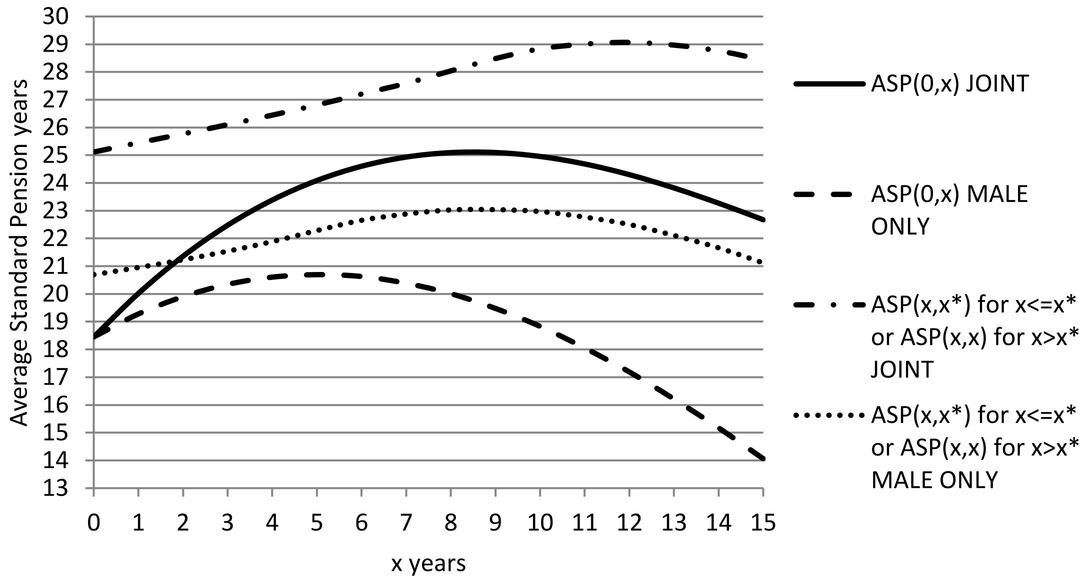
By way of an example let us consider the case of a male (A) who has just achieved state pension age at 65 years, whose entire pension is made up of basic state pension, and therefore $\gamma = 1, \beta = \beta_1 = 1$, and his wife (B) who is 63 years old and is already receiving her own state pension. From ONS period life tables (2012-15), if A decides to take his standard pension immediately, he can expect $m_A(65)$ =18.45 standard pension years on average.

Table 1 shows the mortality rates and mean residual lives extracted from the life tables (2013-15) together with computed $V(x)$ and $M(65+x, 63+x)$. The optimal deferral period is $x^* = 8.5$ years leading to an average of 25.10 joint standard pension years, which is a 36% uplift on a strategy of no deferral. Figure 2 shows average standard pension years both for the joint deferrer and partner and also for a single deferrer. The most obvious feature is the huge extra benefit that accrues to the deferrer and partner compared with that accruing to a single deferrer.

Table 1: $M(65+x,63+x)$ and $V(x)$ for male deferrer (a=65) and female partner (b=63)

x	$r_A(65+x)$	$r_B(63+x)$	$m_A(65+x)$	$m_B(63+x)$	$M(65+x,63+x)$	$V(x)$
0	0.012331	0.006801	18.45	22.57	25.66	1.6843
1	0.013051	0.007308	17.68	21.72	24.75	1.4810
2	0.014401	0.007988	16.90	20.88	23.84	1.2782
3	0.015802	0.008481	16.14	20.04	22.94	1.0765
4	0.017813	0.009481	15.39	19.21	22.04	0.8772
5	0.019495	0.010672	14.66	18.39	21.15	0.6802
6	0.02156	0.011647	13.95	17.59	20.29	0.4864
7	0.024752	0.013015	13.24	16.79	19.42	0.2913
8	0.026989	0.014339	12.57	16	18.56	0.0985
8.5	-	-	-	-	-	0
9	0.030317	0.016116	11.90	15.23	17.72	-0.0935
10	0.033059	0.018277	11.26	14.48	16.90	-0.2827
11	0.036408	0.020312	10.62	13.74	16.07	-0.4725
12	0.040036	0.022436	10.01	13.01	15.27	-0.6599
13	0.044791	0.025071	9.40	12.3	14.47	-0.8394
14	0.049452	0.027839	8.82	11.6	13.70	-0.9992
15	0.056456	0.030916	8.25	10.92	12.94	-1.1487

Figure 2: Comparison of male deferrer (a=65) with female partner (b=63) versus single male deferrer (a=65)



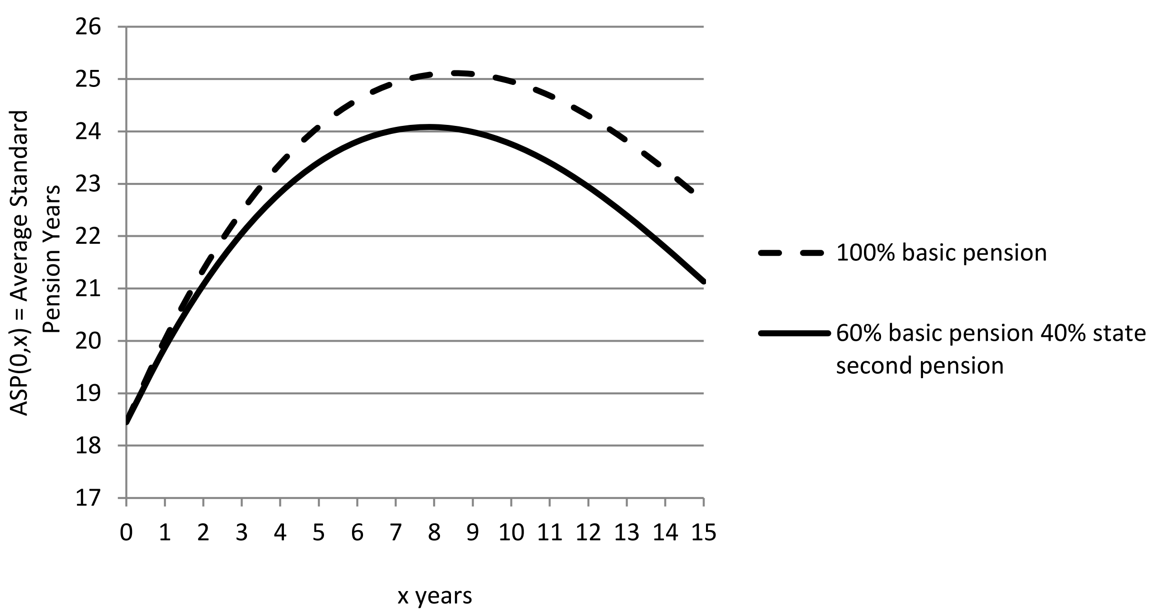
Assuming he chooses the optimum, if A predeceases B after $x\,(<8.5)$ years of deferral then B should inherit the periodic extra pension (rather than take the lump sum) as $m_B(63+x) > \beta^{-1} = 9.62$. If B predeceases A after $x\,(<8.5)$ years of deferral then A should take the periodic extra pension and standard pensions immediately while $m_A(65+x) < x + \beta^{-1}$, that is while $x > 5.0$ years, otherwise he should continue deferring, planning to take the periodic extra pension along with his standard pension at age 65+5.0=70. Of course, if subsequently his life expectancy is downgraded at age $a+v$, where $x < v < 5.0$ and where $m_A(a+v) < \beta^{-1}$ then he should instead take the lump sum plus periodic standard pension.

Applying the approximation (24), we see that it is in fact exact since both r_A and r_B terms in (18) are zero as $m_B(b+x^*) > \beta^{-1}$ and $x^* + \beta^{-1} > m_A(a+x^*) > \beta^{-1}$.

We observe empirically from the $V(x)$ column in Table 1 that it is a decreasing function and consequently that $x^* = 8.5$ is not merely a local maximum but a global maximiser of (14).

Now consider the same couple but this time where A's standard pension comprises 60% basic pension, and 40% of state second pension. Thus $\gamma = 0.6, \theta = 0.5, \beta_1 = (0.6 + 0.4 \times 0.5)\beta = 0.8\beta$. Figure 3 shows $ASP(0,x)$ for this scenario compared with the original scenario. Note that there is little difference in the optimal deferral period and that under the respective optimal deferral periods there is a reduction of about one standard pension year, due to the large extra state second pension component, of which B can only inherit 50%.

Figure 3: Male deferrer (a=65) with female partner (b=63) - two scenarios



6. Summary and Conclusion

The mathematical model developed here provides a useful decision support system for deferral strategies in the case of married and civil partners. A suitable approximation is to continue deferring while both are alive and an appropriately weighted sum of deferrer's and partner's mean residual lives exceeds the reciprocal of the accrual rate plus the time deferred to date. Decision rules are given in the event of the death of either the deferrer or partner during deferral. The numerical example indicates the great advantage offered to a couple over that offered to a single pensioner.

Directions for future work are to investigate: the effect of taxation, cases where a partner has not yet reached state pension age, and the treatment of demographic groups whose life expectancy differs from the UK overall distribution.

Appendix 1: The assumption of equal indexing for different components of the pension

We recall that under current legislation the basic pension is indexed according to the triple lock while other components of the standard pension and also the extra pension are indexed in line with the CPI. Let us suppose that at state pension age the ratio of basic to other pension is $\gamma : 1-\gamma$. Let α and κ denote the index rates for the triple lock and CPI respectively. It follows that after deferring for x years the standard pension will be paid at a rate of $\gamma \exp(\alpha x) + (1-\gamma)\exp(\kappa x)$ of that payable at state pension age. The extra pension at that time will be a multiple βx of this. Therefore at age $a+x+u$ the rate of pension (expressed again as a multiple of the rate at state pension age) will be

$$\gamma \exp(\alpha(x+u)) + (1-\gamma)\exp(\kappa(x+u)) + \beta x\{\gamma \exp(\alpha x) + (1-\gamma)\exp(\kappa x)\}\exp(\kappa u) \ . \qquad \text{(A1.1)}$$

The present value of this will depend upon the choice of a meaningful rate to be used in the discount factor. Arguably, this could be the CPI growth rate since this measures the real value in terms of buying power. Assuming this to be the case, the present value at age a of the pension rate at $a+x+u$ is

$$\begin{aligned}&\{\gamma \exp(\alpha(x+u)) + (1-\gamma)\exp(\kappa(x+u)) + \beta x\{\gamma \exp(\alpha x) + (1-\gamma)\exp(\kappa x)\}\exp(\kappa u)\}\exp(-\kappa(x+u)) \\ &= 1 + \beta x + \gamma\beta x\{\exp((\alpha-\kappa)x) - 1\} + \gamma\{\exp((\alpha-\kappa)(x+u)) - 1\}\end{aligned} \tag{A1.2}$$

and so the expected Net Present Value of that income stream is

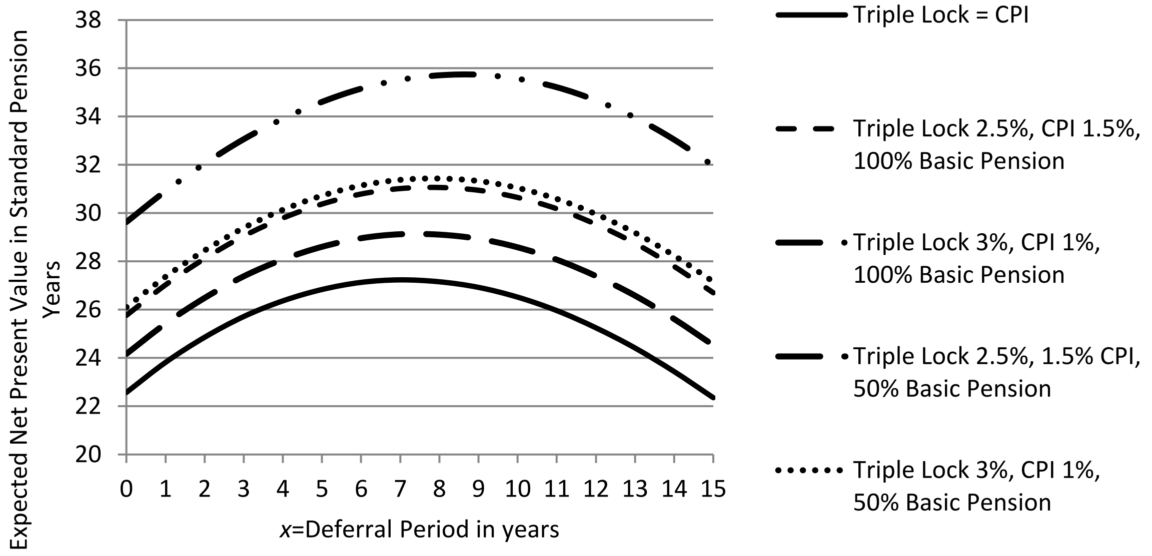
$$\begin{aligned}BSP(0,x) &= \int_0^\infty \frac{S(a+x+u)}{S(a)}(1+\beta x+\gamma\beta x\{\exp((\alpha-\kappa)x)-1\}+\gamma\{\exp((\alpha-\kappa)(x+u))-1\})du \\ &= \frac{S(a+x)}{S(a)}(1+\beta x)m(a+x) + \frac{S(a+x)}{S(a)}\gamma\beta x\{\exp((\alpha-\kappa)x)-1\}m(a+x) \\ &+ \gamma\int_0^\infty \frac{S(a+x+u)}{S(a)}\{\exp((\alpha-\kappa)(x+u))-1\}du \\ &= ASP(0,x) + \frac{S(a+x)}{S(a)}\gamma\beta x\{\exp((\alpha-\kappa)x)-1\}m(a+x) \\ &+ \gamma\int_0^\infty \frac{S(a+x+u)}{S(a)}\{\exp((\alpha-\kappa)(x+u))-1\}du\end{aligned} \tag{A1.3}$$

where $ASP(0,x)$ denotes the probabilistic expectation of the number of standard pension years the individual receives when all components are indexed identically.

We propose that for the purpose of finding an optimal deferral period it will be adequate to take just the first term, $ASP(0,x)$. To test this we calculated (A1.3) for different scenarios for the parameters $\gamma, \alpha,$ and κ. The results are shown in Figure 4. Selecting the maximising value of *x* for any envisaged scenario and then transferring that value to the true (perhaps unknown) scenario results in a very small penalty. One can be reasonably confident therefore in using the *x* that maximises expected benefit under an assumption of equal indexation. Of course, the vertical axis values are different for the various scenarios and it is revealing to note, for example, that even with no deferral the average number of standard pension years under a Triple Lock of 2.5%, a CPI of 1.5% , with 100% Basic Pension, is about 25.9, whereas with equal indexing it is 22.6 yields. This shows the real cost of triple lock if it were to be used in perpetuity and perhaps its implications for inter-generational fairness[12]. The current situation is that Government has warranted the triple lock only until the end of the present Parliament.

[12] Intergenerational fairness, Third report of session 2016-17, HC59, House of Commons Work and Pensions Committee, pp. 24-33. Available from http://www.publications.parliament.uk/pa/cm201617/cmselect/cmworpen/59/5906.htm

Figure 4: Robustness of optimal deferral period with respect to indexation and basic pension component for a single female, State Pension age 63, accrual rate 10.4%



Appendix 2: Expectation of time to second death

Let $M(x_A, x_B)$ be the expected time until the second death of A and B given their current ages are x_A, x_B respectively. The conditional probability that at least one of them will survive a further period u is

$$\begin{aligned}&1-\left(1-\frac{S_A(x_A+u)}{S_A(x_A)}\right)\left(1-\frac{S_B(x_B+u)}{S_B(x_B)}\right)\\&=\frac{S_A(x_A+u)}{S_A(x_A)}+\frac{S_B(x_B+u)}{S_B(x_B)}-\frac{S_A(x_A+u)}{S_A(x_A)}\frac{S_B(x_B+u)}{S_B(x_B)}\end{aligned} \tag{A2.1}$$

and therefore

$$\begin{aligned}M(x_A,x_B)&=\int_0^\infty\left\{\frac{S_A(x_A+u)}{S_A(x_A)}+\frac{S_B(x_B+u)}{S_B(x_B)}-\frac{S_A(x_A+u)}{S_A(x_A)}\frac{S_B(x_B+u)}{S_B(x_B)}\right\}du\\&=m_A(x_A)+m_B(x_B)-\int_0^\infty\frac{S_A(x_A+u)}{S_A(x_A)}\frac{S_B(x_B+u)}{S_B(x_B)}du\end{aligned} \tag{A2.2}$$

Appendix 3: Derivation of $\dfrac{\partial ASP(y,x)}{\partial x}=\dfrac{S_A(a+x)S_B(b+x)}{S_A(a+y)S_B(b+y)}V(x)$

By considering the conditional probabilities of death of A in $(a+x, a+x+dx)$ and B in $(b+x, b+x+dx)$ we have

$$\begin{aligned}\frac{dM(a+x,b+x)}{dx}&=r_A(a+x)\{M(a+x,b+x)-m_B(b+x)\}\\&\quad+r_B(b+x)\{M(a+x,b+x)-m_A(a+x)\}-1\end{aligned} \tag{A3.1}$$

We write (15) as

$$ASP(y,x) = \phi_1(y,x) + \phi_2(y,x) + \phi_3(y,x)\ . \tag{A3.2}$$

Now

$$\frac{\partial \phi_1}{\partial x} = \frac{S_A(a+x)S_B(b+x)}{S_A(a+y)S_B(b+y)} \left\{ \begin{array}{l} -\left[r_A(a+x)+r_B(b+x)\right]\left\{\beta_1 x M(a+x,b+x)+m_A(a+x)\left[1+(\beta-\beta_1)x\right]\right\} \\ +(\beta-\beta_1)m_A(a+x)+\beta_1 x\left[\begin{array}{l} r_A(a+x)\left[M(a+x,b+x)-m_B(b+x)\right] \\ +r_B(b+x)\left[M(a+x,b+x)-m_A(a+x)\right]-1 \end{array}\right] \\ +\beta_1 M(a+x,b+x)+\left[m_A(a+x)r_A(a+x)-1\right]\left[1+(\beta-\beta_1)x\right] \end{array} \right\} \tag{A3.3}$$

$$\frac{\partial \phi_2}{\partial x} = \frac{S_A(a+x)S_B(b+x)}{S_A(a+y)S_B(b+y)} \frac{\beta_1}{\beta} r_A(a+x)\max\{x, \beta x m_B(b+x)\} \tag{A3.4}$$

$$\frac{\partial \phi_3}{\partial x} = \frac{S_A(a+x)S_B(b+x)}{S_A(a+y)S_B(b+y)} r_B(b+x)\max\left[\begin{array}{l} x+m_A(a+x), \left\{1-H\left[m_A(a+x)-x-\beta^{-1}\right]\right\}(1+\beta x)m_A(a+x)+ \\ H\left[m_A(a+x)-x-\beta^{-1}\right]\dfrac{S_A(a+w)(1+\beta w)m_A(a+w)}{S_A(a+x)} \end{array}\right] \tag{A3.5}$$

where $H(.)=1$ for positive arguments, else equals zero. Adding these

$$\frac{\partial ASP}{\partial x} = \frac{S_A(a+x)S_B(b+x)}{S_A(a+y)S_B(b+y)} \times$$

$$\left\{ \begin{array}{l} -(1+\beta x)+\beta_1 M(a+x,b+x)+(\beta-\beta_1)m_A(a+x)+\dfrac{\beta_1}{\beta} r_A(a+x)\max\left[x-\beta x m_B(b+x),0\right] \\ +r_B(b+x)\max\left[\begin{array}{l} x-\beta x m_A(a+x), \\ H(m_A(a+x)-x-\beta^{-1})\left\{(1+\beta w)m_A(a+w)S_A(a+w)/S_A(a+x)-(1+\beta x)m_A(a+x)\right\} \end{array}\right] \end{array} \right\}$$

(A3.6)

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