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# Managing Financially Distressed Pension Plans in the Interest of Beneficiaries<sup>\*</sup>

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**Abstract.** The beneficiaries of a corporate defined benefit pension plan in financial distress care about the security of their promised pensions. We propose to value the pension obligations of a corporate defined benefit plan using a discount rate which reflects the funding ability of the pension plan and its sponsoring company, and therefore depends, in part, on the chosen asset allocation. An optimal valuation is determined by a strategic asset allocation of the pension plan assets which optimizes the funding risk exposure of a representative pension plan member. We provide an empirical application using the General Motors pension plan.

*Key words:* strategic asset allocation, pension plan, default risk, liability, discount rate

*JEL Classification:* G11, G23

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## 1. Introduction

Assume you are an employee and member of the defined benefit pension plan of General Motors at the beginning of 2003. 2002 was a year of falling stock markets and falling interest rates.<sup>1</sup> As a consequence, you notice from the latest financial statements of your employer that pension assets decreased to 60.9 bn USD, while the projected benefit pension obligation (PBO) increased to 80.1 bn USD, resulting in a funding ratio of just 76%.<sup>2</sup> You also notice that the total net worth of General Motors is 6.8 bn USD and therefore insufficient to close the funding gap. Assume you also understand from the extensive press coverage of the General Motors pension plan that the PBO is calculated using a discount rate which exceeds the yield on riskless long-maturity U.S. Treasury bonds by 1.74% which means that the present value of the promised pension payments is considerably larger than the PBO. To make matters worse, you realize that another large U.S. company with severely underfunded pension obligations, United Airlines, filed for bankruptcy in December 2002. Would you not become just a little worried about the security of your promised future pension? Is the value of the pension promise little different to that of a junk bond?

The question we are going to answer in this paper is this: how should we value the pension obligation of a corporate defined benefit pension plan in financial distress?<sup>3</sup> We propose a valuation framework which borrows from the literature of corporate bond pricing to derive a

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<sup>1</sup> A market situation which has become known as the “perfect storm” for defined benefit plans because of its dramatic impact on the plans’ funding ratios of pension assets to liabilities.

<sup>2</sup> The median funding ratio in DB plans sponsored by S&P500 companies was 84% at the end of 2002. The funding situation is slightly worse today with a median funding ratio of 82% at the end of 2010 (Goldman Sachs, 2010).

<sup>3</sup> Although corporate DB pension plans are increasingly being replaced by defined contribution plans, DB plan assets are still substantial and amounted to \$6.6 trillion in the U.S. alone at the end of 2010 (Towers Watson, 2011).

discount rate for the valuation of the pension obligation which reflects the risk that the pension plan and its sponsor default on the pension obligation. This default risk incorporates both the insolvency risk of the plan sponsor and market risk. Both types of risk affect future underfunding probabilities. We denote the default risk as funding risk and the default spread as funding spread in our model of pension assets and liabilities. As in the corporate bond pricing literature,<sup>4</sup> we express this discount rate in terms of a spread over the yield on a riskless government bond of appropriate maturity, which equals the duration of the pension liability in our case. However, we go one step further than this literature and derive the valuation of the pension obligation within a model of the optimal strategic asset allocation of the pension plan. What we are proposing is nothing less than a fully integrated asset-liability management solution for pension plans. The rationale for this is straightforward: the ability of the pension plan and its sponsoring company to fund the future pension payments promised to the beneficiaries depends on the future values of pension plan assets which themselves depend on the current strategic asset allocation policy of the pension plan. Thus, funding spreads which appropriately reflect funding risk depend on the chosen asset allocation. We cannot value the pension obligation without knowing the strategic asset allocation policy of the pension plan.

To find an optimal valuation of the pension obligation, we optimize the strategic asset allocation of the pension plan. Unlike the existing literature on asset-liability management for pension plans (e.g. van Binsbergen and Brandt (2009), Hoevenaars et al. (2008), and Sundaresan and Zapatero (1997), described below), we propose an asset-liability management approach which is directly targeted to address the interests of the pension plan members as

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<sup>4</sup> Das and Sundaram (2000) is an example of this literature which shares a discrete time pricing framework with our paper.

ultimate beneficiaries of the plan. We optimize the funding spread used for the valuation of the pension obligation by means of a strategic asset allocation which is optimal given the preferences of a representative pension plan member with respect to funding risk exposure. Using the optimal funding spread, the present discounted value of expected future pension payments optimally approaches the present discounted value of promised future pension payments. We call the first value the *benefit* of the pension plan beneficiaries and the second value the *liability* of the pension plan and its sponsoring company. The liability should be discounted using the yield on a riskless government bond with a maturity equal to the duration of the pension obligation. The benefit is calculated using a discount rate which exceeds this yield by the funding spread. We describe the preferences of the representative pension plan member using a utility function in the benefit of the pension plan members at some future investment horizon, which naturally equals the duration of the pension obligation. Maximizing the conditional expectation of this utility function leads to an optimal asset allocation and simultaneously to an optimal current value for the pension benefit.

We take the beneficiaries' perspective in deriving the value of pension liabilities and the optimal asset allocation of the plan assets as required by U.S. law. The Board of Trustees of a corporate defined benefit pension plan is responsible for the asset-liability management of the plan. Paragraph 404(a)(1) of the Employee Retirement Income Security Act (ERISA) regulates the role of trustees in the investment process. Fiduciaries are required to act "solely in the interest of the participants and beneficiaries and for the exclusive purpose of providing benefits to participants and their beneficiaries." The objective function we formalize in this paper is wholly consistent with this requirement. Schneider and Pinheiro (2008) point out that a breach of this exclusive purpose rule may result in the loss of a plan's tax-qualified status.

Hence, the trustees have a very strong incentive to act in the best interests of the beneficiaries, indeed to act as if the beneficiaries choose the strategic asset allocation.<sup>5</sup>

Our proposed asset-liability management framework differs from current practices in pension liability valuation and asset allocation in two major aspects. First, corporate pension liabilities are usually valued using actuarial methods. The strategic asset allocation is then determined using an asset-liability model which takes the value of these liabilities as an input. By contrast, our model determines simultaneously the optimal asset allocation and benefit valuation. Second, pension plan sponsors are currently obliged to publish a liability value (the PBO) which is calculated using a discount rate determined by accounting standards. In practice, sponsoring companies often use the average yield to maturity on long-term corporate bonds with a Moody's AA rating (Coronado and Sharpe, 2003) for this purpose. We propose to publish two values: the liability value which is the current value of the pension promise and the benefit value which is the funding-risk-adjusted value of the pension promise. By comparing these values, plan participants and all other stakeholders in the pension plan – shareholders of the sponsoring company, the sponsor, and the pension regulator – will have a realistic assessment of the plan's true funding situation and are able to adjust their behavior accordingly. Yields on corporate bonds are specific to the rating status of the issuer, yet the current practice in pension liability valuation is to treat all pension plans identically and ignore the different funding abilities of different plan sponsors. In other words, discount rates do not reflect the true risk underlying the promised future pension payments, a violation of the basic principles of financial economics. This is what we attempt to address in this paper.

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<sup>5</sup> This does not rule out the possibility that agency problems might arise in later stages of the investment process. In particular, the pension plan might decide to hire investment managers to manage particular asset classes consistent with the strategic asset allocation set by the Board of Trustees. These investment managers might instead pursue their own objectives. Van Binsbergen and Brandt (2009) discuss such agency problems in the context of delegated portfolio management.

We apply our model to the U.S. pension plan of General Motors in December 2002. To better understand our contribution, it will be helpful to preview some of our results. On the basis of the yield on a government bond with maturity equal to the duration of the pension liability, General Motors' pension liability was about 100 bn USD in December 2002. Thus, the funding ratio was about 60%. The net worth of General Motors was about 7% of the liability value. The outcome of our asset-liability modeling exercise yields a funding-risk-adjusted benefit value of around 80-85% of the liability value, depending on the relative risk aversion of the representative pension plan member. This means, given the funding ability of General Motors in December 2002, the members of the defined benefit pension plan of the company should have expected a reduction in their pension wealth by around 15-20%.

One could argue that the Pension Benefit Guaranty Corporation (PBGC) was introduced to provide a (partial) hedge against defaults on pension obligations. There are two reasons why we believe that the valuation of the corporate defined benefit obligation should not include the recovery option provided by the PBGC.<sup>6</sup> First, unless the government fully underwrites any funding gap, the PBGC is itself subject to default risk, as a result of both systematic risk – all corporate pension funds are affected by falling stock markets and interest rates at the same time – and moral hazard in the form of increased risk taking behavior by companies whose liabilities are covered by a guaranty fund like the PBGC (as shown by McCarthy and Neuberger, 2005). Second, we are evaluating the obligations of the plan sponsor from the viewpoint of the plan members. They certainly want to avoid a situation where the PBGC has to step in to secure pension payments, since this is also likely to be associated with unemployment and a reduction in pension benefits for many members. Correspondingly, we propose to minimize the funding risk in the absence of any intervention by the PBGC.

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<sup>6</sup> Pennacchi and Lewis (1994) derive the value of PBGC insurance using contingent claim techniques.

Petersen (1996) and Ippolito (2002) have already proposed that the discount rate for the valuation of the pension liability should reflect funding risk. However, they do not formalize an approach to obtain the funding spreads and, hence, funding-risk-adjusted discount rates. This is what we deliver in this paper. We believe that our approach brings significant advantages to all stakeholders of the pension plan compared with current practice. Most importantly, it provides plan members with a realistic value of their pension wealth and so allows them to adjust their life-cycle consumption and savings trajectories if the benefit falls short of the liability. In particular, they may choose to compensate a reduction in expected pension payments with increased private savings.

The pension regulator should also be interested in our approach, since it limits the discretion of firms in setting the discount rates for the valuation of the pension obligation. The outcome of our asset-liability model is a unique funding-risk-adjusted discount rate for each pension plan. Bergstresser et al. (2006) and Addoum et al. (2010) provide evidence that the expected return on plan assets, another assumption required by U.S. pension accounting standards, tends to be used to manipulate earnings. Similarly, Cocco and Volpin (2007) show that insider trustees, who are also executive directors of the sponsoring company, tend to act in favor of the shareholders of the sponsor, rather than in the interests of the pension plan members. The discount rate is likely to be used in a similar strategic way. We provide an example later in the paper.

The shareholders of the pension plan sponsor are another group of stakeholders that profit from our valuation approach. Coronado and Sharpe (2003) and Franzoni and Marín (2006) find that the market does not correctly value firms with a defined benefit pension plan. Coronado and Sharpe report that all companies with a defined benefit plan are overvalued, while Franzoni and Marín show that the market only overvalues companies with underfunded

pension liabilities. The benefit value we compute provides a plan-specific evaluation of the sponsor's ability to fund the pension promise which is likely to increase transparency for the shareholders of the sponsoring company and ameliorate the overvaluation problem.

The sponsoring company itself will benefit from the information on future underfunding probabilities that are calculated for the determination of the funding spread. Rauh (2006) shows that, for companies facing financial constraints, capital expenditures decline by the amount of mandatory contributions to their defined benefit pension plans. Again, our approach immediately highlights possible future financial constraints arising from the current decisions of the plan sponsor. Henceforth, the plan sponsor will no longer be surprised by the need to make future mandatory contributions to the plan, nor by the consequential requirement to curtail corporate investment or dividends.

Van Binsbergen and Brandt (2009) derive the strategic asset allocation for a corporate defined benefit pension plan which explicitly accounts for such mandatory contributions. They formulate the objective function of a fund manager who manages the assets of the pension plan on behalf of the plan sponsor and is concerned about reputation loss emerging from the need of the sponsor to pay mandatory contributions. By contrast, our objective function is formulated to represent the interests of the plan beneficiaries. While we do not explicitly describe a contribution policy for the plan, we treat the plan as an integral part of the company and effectively merge the net worth of the sponsor with the pension plan assets. We do this in recognition that the sponsor is ultimately liable for closing any funding gap. In our model, contributions are merely a shift of assets from the company's balance sheet to the pension plan and do not affect funding risk. Van Binsbergen and Brandt (2009) do not consider funding risk and instead value liabilities using the yield on a government bond or a weighted average of past yields. However, like van Binsbergen and Brandt, we adopt the

Brandt et al. (2005) simulation methodology to solve the dynamic asset allocation problem.

Hoevenaars et al. (2008) propose an asset-liability model to determine the asset allocation of a defined benefit plan. They assume the perspective of the plan sponsor, but do not integrate the plan and the sponsor. By contrast, we assume the perspective of the plan members and view the pension plan as an integrated part of the sponsoring company. Moreover, we derive an optimal strategic asset allocation under the assumption that rebalancing occurs optimally in future to adjust to time-varying investment opportunities, while Hoevenaars et al. consider suboptimal rebalancing to a portfolio which is the same at all rebalancing times. Unlike Hoevenaars et al., we are not only interested in the optimal asset allocation, but also in the optimal liability valuation and we explicitly address funding risk.

Sundaresan and Zapatero (1997) relate the valuation of pension liabilities and the allocation of pension plan assets to the lifetime marginal utility of the worker. They show that the worker will retire when the ratio of pension benefits to current wages reaches a critical value which depends on the discount rate. The discount rate and asset allocation strategy is chosen from the perspective of the pension plan sponsor, while we assume the perspective of the pension plan member. The worker in the Sundaresan and Zapatero model behaves as if there is no funding risk, while the pension plan beneficiaries in our model are concerned about this risk. Finally, in contrast with Sundaresan and Zapatero (1997), we consider time variation in investment opportunities.

The remainder of the paper is structured as follows. Section 2 describes the asset-liability model. The output from this model is an optimal strategic asset allocation and a valuation of the pension benefit which is consistent with this allocation. In Section 3, we apply our model to a pension plan in financial distress: the General Motors U.S. pension plan in December 2002. Section 4 concludes.

## 2. Asset-Liability Management

We propose a new approach to the management of the assets and liabilities of a corporate defined benefit pension plan which reflects the interests of the plan beneficiaries.

### 2.1 PENSION LIABILITY VALUATION

We investigate a stylized corporate defined benefit pension plan which promises its beneficiaries a constant nominal pension payment of magnitude  $E$  at intervals of length  $h$  up to some date of maturity  $Mh$ . The present discounted value of the *promised* future pension payments

$$L_t = \sum_{m=1}^M \frac{E}{(1 + Y_t^{mh})^{mh}} \quad (1)$$

defines the *liability* value,  $L_t$ , of the pension plan.  $Y_t^{mh}$  denotes the current yield-to-maturity on a riskless zero-coupon bond with maturity  $mh$ . Discounting using a riskless rate is appropriate because we are valuing promised pension payments and can therefore treat them as though they are guaranteed.

Equation (1) is valid if we assume that the plan is sufficiently large that longevity risk is diversified away. This is in line with van Binsbergen and Brandt (2009) and Hoevenaars et al. (2008). Like these authors, we also assume that the maturity of the pension liability is constant which holds for a pension plan in a stationary state where the distribution of age cohorts and accrued benefit rights of plan members remains constant over time. Finally, like Hoevenaars et al., we assume that new contributions to the plan exactly offset any increase in accrued pension rights. The overarching purpose of these assumptions is to allow us to focus on the change in liability value arising exclusively from changes in the yield curve.

Plan beneficiaries are concerned about the present discounted value of *expected* future pension payments which we call the *benefit* value ( $B_t$ ) henceforth. The benefit may differ from the liability of the pension fund if there is a probability,  $\pi_t^{mh}$ , that the pension plan and its sponsoring company default on their pension obligation at any time  $t + mh$ ,  $m = 1, \dots, M$ . Recognizing that the sponsoring company is ultimately liable for the obligation of the pension plan, default occurs if the assets of the pension plan and the net worth of the sponsoring company are insufficient to fund the liability in period  $t + mh$ . Based on these considerations, we will refer to the risk of default as funding risk and to the default probability,  $\pi_t^{mh}$ , as the underfunding probability. In this case, we can apply the fundamental equation of asset pricing (see Cochrane, 2001) to obtain the current value of the future pension payment as

$$\begin{aligned} P_t^{mh} &= E_t[M_{t+mh}\text{Payoff}_{t+mh}] \\ &= E_t[M_{t+mh}]E_t[\text{Payoff}_{t+mh}] + \text{cov}_t(M_{t+mh}, \text{Payoff}_{t+mh}) \end{aligned} \quad (2)$$

with components

$$E_t[M_{t+mh}] = (1 + Y_t^{mh})^{-mh} \quad (3)$$

$$E_t[\text{Payoff}_{t+mh}] = (1 - \pi_t^{mh})E + \pi_t^{mh}\lambda_t^{mh}E. \quad (4)$$

$M_{t+mh}$  denotes the  $mh$ -period stochastic discount factor with conditional expectation (3). The expected pension payoff is derived in (4) as the probability-weighted sum of the pension payoffs in the states of over- and underfunding. In the case of underfunding, only a recovery fraction,  $\lambda_t^{mh}$ , of the promised pension payment,  $E$ , will be paid off.

The covariance term in (2) is a risk correction term (Cochrane, 2001). For our purposes, it is convenient to replace this additive term with a multiplicative term,  $(1 + \theta_t^{mh})^{-mh}$ , where  $\theta_t^{mh}$  defines the funding-risk premium. Then we can rewrite (2) as

$$P_t^{mh} = \frac{(1 - \pi_t^{mh})E + \pi_t^{mh}\lambda_t^{mh}E}{(1 + Y_t^{mh})^{mh}(1 + \theta_t^{mh})^{mh}} = \frac{E}{(1 + Y_t^{mh})^{mh}(1 + \delta_t^{mh})^{mh}} \quad (5)$$

with components

$$(1 + \theta_t^{mh})^{-mh} = 1 + cov_t(M_{t+mh}, \text{Payoff}_{t+mh}) / (E_t[M_{t+mh}]E_t[\text{Payoff}_{t+mh}]) \quad (6)$$

$$(1 + \delta_t^{mh})^{-mh} = (1 + \theta_t^{mh})^{-mh}(1 - \pi_t^{mh} + \pi_t^{mh}\lambda_t^{mh}). \quad (7)$$

The first equality in (5) follows from replacing (3), (4), and (6) in equation (2). For the second equality in (5), we use the promised pension payment in the numerator which then needs to be discounted by a funding-risk-adjusted discount factor using the funding spread,  $\delta_t^{mh}$ , defined in (7), over the yield of a riskless bond with maturity  $mh$ . By varying  $m$ , (7) defines a term structure of funding spreads. We introduce the funding spreads to emphasize the point that the discount rate for calculating the present value of all future pension payments should reflect the degree of funding risk in the case where future pension payments are defaultable.<sup>7</sup> The funding spread increases with the funding-risk premium. For a given funding-risk premium, the funding spread increases with an increasing underfunding probability and decreases with an increasing recovery fraction. Now we can derive the benefit value as follows

$$B_t = \sum_{m=1}^M \frac{E}{(1 + Y_t^{mh})^{mh}(1 + \delta_t^{mh})^{mh}} \quad (8)$$

The term structure of funding spreads is completely defined by  $\pi_t^{mh}$ ,  $\lambda_t^{mh}$ , and  $\theta_t^{mh}$ . We delay discussion of the funding-risk premium,  $\theta_t^{mh}$ , to later. We define for  $m = 1, \dots, M$

$$\pi_t^{mh} = Prob_t \left( \frac{A_{t+mh} + N_{t+mh}}{L_{t+mh}} < 1 \right) \quad (9)$$

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<sup>7</sup> Our approach to deriving funding spreads resembles the derivation of credit spreads in credit risk models. Das and Sundaram (2000) provide a discrete time reduced-form model which leads to credit spreads of the form (7).

$$\lambda_t^{mh} = E_t \left( \frac{A_{t+mh} + N_{t+mh}}{L_{t+mh}} \mid \frac{A_{t+mh} + N_{t+mh}}{L_{t+mh}} < 1 \right). \quad (10)$$

Here  $A_{t+mh}$  denotes the pension plan assets and  $N_{t+mh}$  the net worth<sup>8</sup> of the sponsoring company in period  $t + mh$ . Funding risk at horizon  $t + mh$  exists if there is a positive conditional probability that the total funding ratio,  $G_{t+mh} = (A_{t+mh} + N_{t+mh})/L_{t+mh}$ , falls below one. We call  $G_{t+mh}$  the total funding ratio because it relates the sum of pension assets and net worth of the sponsoring company to the pension liability. The more familiar funding ratio,  $F_{t+mh} = A_{t+mh}/L_{t+mh}$ , describes the funding position in the pension plan only and ignores the liability of the plan sponsor. The underfunding probability in (9) takes into account both the insolvency risk of the plan sponsor which affects  $N_{t+mh}$  and market risk which affects the future assets,  $A_{t+mh}$ , and liabilities,  $L_{t+mh}$ , of the pension plan. The recovery ratio in (10) is the conditionally expected total funding ratio when the latter falls below unity.

It is very important to remember in what follows that the funding spread and thus the benefit in (8) is a function of the current asset allocation of the pension plan which, in turn, contributes to the determination of future pension asset values and therefore influences underfunding probabilities and recovery ratios.

## 2.2 STRATEGIC ASSET ALLOCATION

We observe a pension plan in period  $t$  with funding ratio  $F_t$  and total funding ratio  $G_t$ .<sup>9</sup> The beneficiaries of the pension plan were promised  $L_t$  but realize that their benefit,  $B_t$ , may fall below  $L_t$  in the presence of funding risk. Let us initially take the extreme case of a typical

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<sup>8</sup> More generally, the net worth variable captures those assets of the corporate sponsor which – depending on the seniority of the pension claims – can be used to cover underfunded pension liabilities in the case of default.

<sup>9</sup> We do not discuss the reasons why this particular funding ratio arises at time  $t$ .

pension plan member who is infinitely risk averse. In representing the interests of this member, the board of trustees of the pension plan will allocate the plan assets in such a way that the benefit approaches the liability as closely as possible.<sup>10</sup> The benefit and liability coincide if the funding spreads at all payment dates are zero as can be seen from comparing (1) and (8). In this case, funding risk is eliminated and the defined benefit pension promised to the member is riskless. Note that minimizing the funding spreads is equivalent to maximizing the benefit since the benefit can never exceed the liability. Now assume the more general case of a representative pension plan member with (constant) relative risk aversion  $\gamma$ . This member might optimally choose to take on some funding risk depending on the curvature of his utility function. This suggests a value function for the asset allocation problem of the form

$$\begin{aligned}
V(Z_t, Jh) &= \max_{\{w_{t+jh}\}_{j=0}^{J-1}} E_t \left[ \frac{B_{t+Jh}^{1-\gamma}}{1-\gamma} \right] = \max_{w_t} E_t \left[ \max_{\{w_{t+jh}\}_{j=1}^{J-1}} E_{t+h} \left[ \frac{B_{t+Jh}^{1-\gamma}}{1-\gamma} \right] \right] \\
&= \max_{w_t} E_t [V(Z_{t+h}, (J-1)h)]
\end{aligned} \tag{11}$$

where  $Z_t$  denotes a vector of state variables at time  $t$ . The trustees will determine a sequence of optimal asset allocations,  $w_{t+jh}$ ,  $j = 0, \dots, J-1$ , which maximizes the conditionally expected utility in the benefit at some investment horizon,  $Jh$ . A natural choice for  $Jh$  is the duration of pension liabilities,  $Dh$ . We assume here for notational simplicity that rebalancing periods of length  $h$  are synchronized with pension payment periods. Equation (11) defines a strategic asset allocation problem (Campbell and Viceira, 2002) in the sense that today's optimal asset allocation decision already reflects future optimal rebalancing of the portfolio in

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<sup>10</sup> Recall from the introduction that this objective function is completely in line with ERISA regulations. In particular, trustees are required by law to act "solely in the interest of the participants and beneficiaries" and to diversify investments "so as to minimize the risk of large losses" (ERISA 404(a)(1)).

response to changes in the investment opportunity set described by the vector of state variables. We assume throughout the paper that the pension fund is not allowed to short-sell.

The output from this optimization program is not only a sequence of optimal asset allocations but also a value for the benefit at time  $t$ ,  $B_t$ . Hence, (11) simultaneously solves the asset allocation and benefit valuation problems of the pension plan resulting in a term structure of funding spreads,  $\delta_t^{mh}$ , evaluated at the optimal asset allocation choice. As a consequence, the asset allocation and benefit valuation exercises are completely interdependent. We argued in the introduction that pension plans should be obliged to publish  $B_t$  along side  $L_t$  to give plan beneficiaries the chance to adjust their savings and consumption patterns if  $B_t$  is below  $L_t$ . Solving (11) determines the optimal discount rates for the calculation of  $B_t$ .

The recommendation to publish the present discounted value of both promised and expected pension payments,  $L_t$  and  $B_t$ , stands in stark contrast to the current publication requirements of corporate defined benefit pension plans. As pointed out in the introduction, pension plans currently publish one liability value, which has the form of the benefit in (8) with the important difference that the funding spread (usually the average spread of AA-rated long-term corporate bonds) is determined exogenously and independent of the actual funding risk associated with the specific pension plan. In our case, the funding spread is fully consistent with the current total funding ratio of the pension plan and its asset allocation.

The benefit at the investment horizon can be rewritten as  $B_{t+Jh} = B_t(1 + R_{t+Jh}^{B(Jh)})$ , where we introduce the convention that subscripts on return variables denote the time the return is realized and superscripts in parentheses denote the holding period. The  $Jh$ -period benefit return follows from (5) and (8) as

$$\begin{aligned}
1 + R_{t+Jh}^{B(Jh)} &= \prod_{j=1}^J (1 + R_{t+jh}^{B(h)}) = \prod_{j=1}^J \frac{\sum_{m=1}^M P_{t+jh}^{mh}}{\sum_{m=1}^M P_{t+(j-1)h}^{mh}} \\
&= \prod_{j=1}^J \sum_{m=1}^M \frac{P_{t+(j-1)h}^{mh}}{\sum_{m=1}^M P_{t+(j-1)h}^{mh}} \frac{P_{t+jh}^{mh}}{P_{t+(j-1)h}^{mh}} = \prod_{j=1}^J \sum_{m=1}^M v_{t+(j-1)h}^{mh} \frac{P_{t+jh}^{mh}}{P_{t+(j-1)h}^{mh}} \\
&= \prod_{j=1}^J \sum_{m=1}^M v_{t+(j-1)h}^{mh} \frac{(1 + Y_{t+jh}^{mh})^{-mh} (1 + \delta_{t+jh}^{mh})^{-mh}}{(1 + Y_{t+(j-1)h}^{mh})^{-mh} (1 + \delta_{t+(j-1)h}^{mh})^{-mh}} \\
&= \prod_{j=1}^J \sum_{m=1}^M v_{t+(j-1)h}^{mh} (1 + S_{t+jh}^{mh(h)}) (1 + \Delta_{t+jh}^{mh(h)})
\end{aligned} \tag{12}$$

where  $S_{t+jh}^{mh(h)}$  is the  $h$ -period net return on a default-free zero-coupon bond with maturity  $mh$ ,  $\Delta_{t+jh}^{mh(h)}$  the  $h$ -period net return on the corresponding funding spread, and  $v_{t+(j-1)h}^{mh}$ ,  $m = 1, \dots, M$ , a number of weights with obvious definition which sum to unity.

Equation (12) involves the term structure of interest rates and funding spreads for maturities ranging from  $h, \dots, Mh$ . In practice, a single discount rate is usually used for the valuation of the pension obligation. This interest rate should be appropriate for the duration of the pension obligation. In this case, (12) can be simplified to

$$1 + R_{t+Jh}^{B(Jh)} = \prod_{j=1}^J (1 + S_{t+jh}^{Dh(h)}) (1 + \Delta_{t+jh}^{Dh(h)}). \tag{13}$$

It is only the second factor,  $1 + \Delta_{t+jh}^{Dh(h)}$ , in this equation that can be influenced by the strategic asset allocation. For funding spreads of size zero at all rebalancing times, the second factor becomes unity,  $\forall j$ , and (13) reduces to the  $Jh$ -period liability holding return, which we denote as  $1 + R_{t+Jh}^{L(Jh)}$ .

The strategic asset allocation influences (13) via the underfunding probabilities,  $\pi_{t+jh}^{Dh}$ , and recovery ratios,  $\lambda_{t+jh}^{Dh}$ , for  $j = 0, \dots, J$ . For example, for the current period,  $j = 0$ , these two

quantities depend on the total funding ratio,  $G_{t+Dh}$ , at the duration of the liabilities, as can be seen from (9) and (10). The components of  $G_{t+Dh}$  can be expressed as  $L_{t+Dh} = L_t(1 + R_{t+Dh}^{L(Dh)})$ ,  $N_{t+Dh} = N_t(1 + R_{t+Dh}^{N(Dh)})$  and  $A_{t+Dh} = A_t(1 + R_{t+Dh}^{A(Dh)})$  with

$$1 + R_{t+Dh}^{A(Dh)} = \prod_{j=1}^D (1 + R_{t+jh}^{A(h)}) = \prod_{j=1}^D (1 + R_{t+jh}^{f(h)} + w'_{t+(j-1)h} R_{t+jh}^{e(h)}) \quad (14)$$

where  $R_{t+jh}^{f(h)}$  denotes the  $h$ -period net return on a riskless asset and  $R_{t+jh}^{e(h)}$  denotes a vector of  $h$ -period excess returns above the riskless return for the number of risky assets under consideration.

By now, it should be clear how the asset allocation influences the benefit in our proposed optimization problem (11): the asset allocation affects the portfolio return on plan assets, (14), and, thus, future asset values in the pension plan. These are important components in the determination of underfunding probabilities, (9), and recovery ratios, (10), which, in turn, affect the funding spreads, (7). The funding spreads directly influence the return on benefits, (12). These interactions are certainly more complex than those between the nearest comparable relationship typically considered in the finance literature, namely that between the asset allocation and the terminal value of assets at an investment horizon. But beneficiaries of a defined benefit pension plan are not primarily concerned about the terminal value of plan assets. They are concerned that the promised benefit will be paid off in full during their retirement and this will only happen with certainty if funding risk is eliminated. In this case, funding spreads become zero and the liability of the pension plan and its sponsor equals the benefit promised to the members. Since it is possible that the current total funding ratio of the pension plan is insufficient to completely eliminate the funding risk, funding risk should be minimized. This is precisely the objective of our optimization problem (11).

### 2.3 DERIVING THE FUNDING RISK PREMIUM

We delayed the derivation of the funding risk premium, (6), to this point, because it does not significantly contribute to the understanding of the value function, (11). We propose a simplified form for the funding risk premium which is easy to calculate for pension plan sponsors and which depends on only one additional parameter. This new parameter needs to be set by an organization independent of the sponsor, such as the pension regulator.

From (7), it is clear that we need the funding risk premium,  $\theta_t^{Dh}$ , in addition to the underfunding probability,  $\pi_t^{Dh}$ , and recovery ratio,  $\lambda_t^{Dh}$ , in order to calculate the current funding spread,  $\delta_t^{Dh}$ . The funding spread is determined by the specification of the stochastic discount factor. Since we only need two states of the world for the derivation of the expected payoff in (4), namely those of overfunding and underfunding, we can simplify the derivation of the funding risk premium by similarly decomposing the expected stochastic discount factor  $E_t[M_{t+Dh}] = (1 - \pi_t^{Dh})M_{t+Dh}^o + \pi_t^{Dh}M_{t+Dh}^u$ , where  $M_{t+Dh}^o$  and  $M_{t+Dh}^u$  are the stochastic discount factors in the states of over- and underfunding, respectively. Equation (2) then becomes  $P_t^{Dh} = E_t[M_{t+Dh} \text{Payoff}_{t+Dh}] = (1 - \pi_t^{Dh})M_{t+Dh}^o E + \pi_t^{Dh}M_{t+Dh}^u \lambda_t^{Dh} E$  for  $m = D$ . By equating this expression to the first expression in (5), we obtain

$$(1 + \theta_t^{Dh})^{-Dh} = \frac{(1 - \pi_t^{Dh})}{[1 - \pi_t^{Dh} + \pi_t^{Dh} \lambda_t^{Dh}]} \frac{M_{t+Dh}^o}{E_t[M_{t+Dh}]} + \frac{\pi_t^{Dh} \lambda_t^{Dh}}{[1 - \pi_t^{Dh} + \pi_t^{Dh} \lambda_t^{Dh}]} \frac{M_{t+Dh}^u}{E_t[M_{t+Dh}]} \quad (15)$$

The funding risk premium is now completely determined by  $\pi_t^{Dh}$ ,  $\lambda_t^{Dh}$  and the ratios of the stochastic discount factors in the states of over- and underfunding to the expected stochastic discount factor. Since we are valuing benefits from the viewpoint of members of a pension plan, a consumption-based asset pricing model (see, e.g. Cochrane, 2001) with power utility seems an appropriate choice. In this case,  $M_{t+Dh}^o$  and  $M_{t+Dh}^u$  can be defined exogenously as

$$M_{t+Dh}^o = \beta^{Dh} \left( \frac{C_{t+Dh}^o}{C_t} \right)^{-\gamma} = \beta^{Dh} (g_{t+Dh}^o)^{-\gamma} \quad (16)$$

$$M_{t+Dh}^u = \beta^{Dh} \left( \frac{C_{t+Dh}^u}{C_t} \right)^{-\gamma} = \beta^{Dh} (g_{t+Dh}^u)^{-\gamma} \quad (17)$$

where  $g_{t+Dh}^o = C_{t+Dh}^o/C_t$  and  $g_{t+Dh}^u = C_{t+Dh}^u/C_t$  measure consumption growth in the states of over- and underfunding (corresponding to states of boom and slump, respectively), and  $\beta$  denotes the subjective time-discount factor of the representative investor. For notational convenience, we assume that the coefficient of relative risk aversion,  $\gamma$ , of the representative investor is the same as the one of the representative pension plan member in (11).

The funding risk premium, (15), is positive if  $M_{t+Dh}^u > M_{t+Dh}^o$  which happens when  $g_{t+Dh}^o > g_{t+Dh}^u$ . This is likely to be the case when overfunding corresponds to a state of high asset values and the representative investor in the economy invests in the same asset classes as the pension plan and increases his consumption when his wealth is high. Given (16) and (17), the remaining components of (15) follow as

$$\frac{M_{t+Dh}^o}{E_t[M_{t+Dh}]} = \frac{(g_{t+Dh}^o)^{-\gamma}}{(1 - \pi_t^{Dh})(g_{t+Dh}^o)^{-\gamma} + \pi_t^{Dh}(g_{t+Dh}^u)^{-\gamma}} = [1 - \pi_t^{Dh} + \pi_t^{Dh}\phi^\gamma]^{-1} \quad (18)$$

$$\frac{M_{t+Dh}^u}{E_t[M_{t+Dh}]} = \frac{(g_{t+Dh}^u)^{-\gamma}}{(1 - \pi_t^{Dh})(g_{t+Dh}^o)^{-\gamma} + \pi_t^{Dh}(g_{t+Dh}^u)^{-\gamma}} = [(1 - \pi_t^{Dh})\phi^{-\gamma} + \pi_t^{Dh}]^{-1} \quad (19)$$

where  $\phi = g_{t+Dh}^o/g_{t+Dh}^u$  denotes the relative consumption growth which we set to a constant larger than unity. This parameter only affects the funding risk premium which itself is a quantitatively unimportant component of the funding spread (7). For practical applications of our model,  $\phi$  could be set by the pension regulator or the financial accounting standards board within the rules governing the valuation of pension obligations.

It is useful to derive some comparative statics results from (15) for  $\theta_t^{Dh}$ . First, we can see that  $\theta_t^{Dh}$  is zero, whenever  $\pi_t^{Dh} = 0$  or  $\pi_t^{Dh} = 1$ . Hence, if one of the two possible states of

the world occurs with certainty, the funding risk premium is zero, whether or not this state is favorable or unfavorable for the pension plan member. For conditional underfunding probabilities between the two extreme outcomes,  $0 < \pi_t^{Dh} < 1$ , we can show (after some straightforward but tedious calculations) that the funding risk premium increases with an increasing underfunding probability (for given maturity,  $Dh$ , and recovery fraction,  $\lambda_t^{Dh}$ ) when  $\phi^{-\gamma} > \lambda_t^{Dh}(\pi_t^{Dh})^2(1 - \pi_t^{Dh})^2$ . The premium decreases with an increasing underfunding probability when this inequality is reversed. In the case of maximum uncertainty about the future state of the world,  $\phi^{-\gamma} = \lambda_t^{Dh}(\pi_t^{Dh})^2(1 - \pi_t^{Dh})^2$ , pension plan members will demand the highest risk premium. The funding risk premium decreases with increasing maturity,  $Dh$ , and recovery fraction,  $\lambda_t^{Dh}$ , and becomes zero for  $\lambda_t^{Dh} = 1$ . All comparative statics results therefore conform with a priori expectations.

## 2.4 SOLVING THE OPTIMIZATION PROBLEM

The asset-liability modeling problem, (11), cannot be solved analytically. We use the simulation method developed by Brandt et al. (2005) for dynamic portfolio choice problems to solve the problem. This requires us to generate a large number of future scenarios for all the state and return variables of interest for which we use a vector autoregressive equation system. The dynamic programming problem is then solved backwards by replacing conditional expectations with estimates obtained from cross-scenario regressions of the variables of interest on lagged state variables. At every rebalancing date, we use a grid search over the possible space of asset allocations (excluding short sales as noted before) with a step size of 5%. The Appendix provides the details of the solution technique.

### **3. Application: General Motors**

We apply our proposed asset-liability framework to the specific example of the General Motors U.S. pension plan in December 2002.

#### 3.1 THE GENERAL MOTORS U.S. PENSION PLAN IN DECEMBER 2002

The proposed asset-liability model will generate the most interesting results for corporate defined benefit plans in financial distress as measured by a low total funding ratio. If the total funding ratio is well above unity, funding spreads reduce to zero and – as we will show below – the optimal asset allocation becomes a liability hedging portfolio.<sup>11</sup>

Correspondingly, we focus on a pension plan in deep financial distress. In 2002, General Motors (GM) had the largest pension plan of any U.S. company (SEC, 2009). Table 1 summarizes key indicators of the GM plan. In December 2002, the PBO amounted to 80.1 bn USD. The pension plan assets were only 60.9 bn USD, which is 76% of the PBO. The net worth of GM was 6.8 bn USD and therefore insufficient to eliminate the funding gap between assets and PBO. Given these statistics, the GM pension plan was clearly in financial distress in December 2002. GM employees had every reason to be concerned that they would not receive their promised pension payments in full when they retired. This concern was reflected in a huge amount of press coverage at the time which focused on the funding gap caused by a combination of falling stock markets and interest rates in 2002. Academics also found interest in the GM case. Viceira (2005) presents a case study about the GM pension plan, while Shivdasani and Stefanescu (2010) use GM to show how corporate leverage rates increase substantially when pension assets and liabilities are incorporated into the capital structure.

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<sup>11</sup> This consists of an investment in assets which provide returns that are as highly as possible correlated with the pension liability return.

The PBO is calculated as the present value of future nominal pension payments evaluated at a discount rate chosen by the sponsor within the limits set by the accounting statement No. 87 concerning “Employers’ Accounting for Pensions” published by the U.S. Financial Accounting Standards Board (FASB). The statement requires that “assumed discount rates shall reflect the rates at which the pension benefits could be effectively settled.” FAS87 allows the company “to look to rates of return of high-quality fixed income investments currently available and expected to be available during the period to maturity of the pension benefits.” In practice, sponsoring companies often use the average yield to maturity on long-term corporate bonds with a Moody’s AA rating (Coronado and Sharpe, 2003).

We are able to gain an unusually deep insight into how the discount rate was set within GM in 2002 from SEC (2009). This document is a litigation report for a complaint issued by the Securities and Exchange Commission about misstatements concerning the disclosure of critical pension accounting estimates in GM’s 2002 10-K filings. In respect of the discount rate, the SEC complained that GM proposed using a term structure of yields on high-quality corporate bonds to value the PBO in a conference call with analysts and the press in August 2002, but instead used a single yield from Moody’s AA-rated index for the 10-K filings submitted in March 2003. The duration-matched discount rate implied by the yield curve was 6.0% at the end of December 2002, while the Moody’s index generated a 6.63% yield which, according to SEC (2009), was adjusted by GM to a 6.75% discount rate based on a survey about pension accounting assumptions among U.S. companies. This is clearly a good example of the discretionary freedom that companies have in the determination of pension liability discount rates, which we criticized in the introduction of this paper.

Moody’s AA index measures the average yield on AA-rated bonds with maturities of 20 years and above. At the end of 2002, the yield on a riskless government bond with a 20-year

maturity was 5.01%. Thus, the spread implied by the discount rate used by GM was 1.74%. For our asset-liability model, we need to compute the liability as the present discounted value of all future promised pension payments, discounted with the yield on a riskless bond of appropriate duration, 5.01% in the present case. While this liability value is not published by GM, we can nevertheless estimate it from the PBO and the average pension payments of GM throughout the years 1997-2002 which equal 5.88 bn USD (calculated from Viceira, 2005). If we assume a constant annual pension payment of this amount, the size of the PBO is consistent with a maturity value of 40 years.<sup>12</sup> This corresponds to a duration of the liability of 13 years. Using the riskless yield of 5.01%, we estimate a liability value of 100.8 bn USD. Hence, the PBO understates the present value of promised pension payments by about 20%. The current funding ratio of GM in 2002 was  $F_t = 60\%$  and the total funding ratio (including the net worth of GM) was  $G_t = 67\%$ . While these figures are adequate for our purposes, GM would have been in a position to calculate them more precisely. We also conduct comparative statics exercises to see how our results change when we alter the observed funding ratio.

Table 1 summarizes the various discount rates and associated liability values. The table also contains information on the asset allocation adopted by the GM pension plan at the end of 2002. The plan had an allocation of 55% in stocks, 34% in bonds and 10% in real estate. We can compare this asset allocation and the selected spread of 1.74% above the yield on a riskless bond with the optimal asset allocation and funding spread derived from our model, in order to see how well the beneficiaries' interests are represented by the actions of GM.

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<sup>12</sup> Only GM know how their pension payments evolve over time to maturity. The maturity of 40 years that we estimate probably understates the true maturity of the GM pension liabilities, because pension payments for the current stock of employees are likely to decrease over time. Our assumption of a constant stream of pension payments, while consistent with the simplifying assumption made in section 2.1 above, should be treated as a first approximation.

### 3.2 DATA AND ESTIMATED RETURN DYNAMICS

Following Campbell et al. (2003), van Binsbergen and Brandt (2009), Hoevenaars et al. (2008) and many others, we model the log return dynamics with a vector autoregressive equation system with one lag. We include the asset classes – cash, bonds and stocks – that are standard in the dynamic portfolio choice literature (e.g. Campbell et al. (2003), and van Binsbergen and Brandt, 2009). Although GM had a small allocation to real estate in 2002, Hoevenaars et al. (2008) show that (listed) real estate plays a negligible role in optimal portfolios derived from a dynamic asset-liability model, so we merge real estate with stocks in what follows. We use the nominal 3-month Treasury bill rate obtained from FRED (Federal Reserve Economic Data, item TBM3) for cash, the return on a portfolio of government bonds with maturities of between 5 and 10 years from CRSP (Center for Research in Security Prices, item le\_120) for bonds, and the value-weighted CRSP stock market index including distributions (CRSP item vwretd) for stocks.

We use the return on equity to proxy the return on the net worth of the sponsoring company. The return on equity refers to the ratio of net income to stockholder's equity for GM. Both variables are obtained from Compustat (items NIQ and SEQQ).

Consistent with GM's own practice, we model liability returns using yields on a 20-year constant maturity bond obtained from FRED (item TCMNOMY20).<sup>13</sup> As in Hoevenaars et al. (2008), we use the log-linear approximation suggested by Campbell et al. (1997) to model the log return on liabilities ( $s_{t+1}$ ) from log yields ( $y_t, y_{t+1}$ ) as  $s_{t+1} = 0.25y_{t+1} - D(y_{t+1} - y_t)$

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<sup>13</sup> This time series has missing observations between January 1987 – September 1993. We replace these missing values with forecasts from a linear regression of 20-year maturity yields on 10- and 30-year maturity yields (FRED items TCMNOMY10 and TCMNOMY30) using monthly data between February 1977 – December 1986 and October 1993 – May 2009.

where  $D$  is the constant duration of liabilities. The factor 0.25 results from using quarterly data but annual yields. We set  $D = 13$  to approximate the duration of GM's pension liability.

Our choice of state variables is guided by the literature on return predictability (e.g. Chen et al., 1986) and dynamic asset allocation (e.g. Campbell et al., 2003). We employ the log dividend-price ratio, the log term spread and the log credit spread as state variables. The log dividend-price ratio is defined as the difference between the log cumulative one-year distributions on the value-weighted CRSP stock market index and the corresponding log price index (CRSP item `vwindx`), the log term spread is the difference between the log yield of a 5-year zero-coupon bond obtained from the CRSP Fama-Bliss dataset (CRSP item `yield5`) and the log nominal T-bill rate, the log credit spread is the difference between the log yields of Moody's BAA- and AAA-rated corporate bonds (FRED items `BAANA` and `AAANA`).

The final sample comprises quarterly data from 1970.II - 2002.IV. 1970.II is the first quarter for which we observe the return on equity for GM, 2002.IV is the period in which we conduct the asset-liability modeling study for GM. Descriptive statistics for all variables are shown in Table 2. Over our sample period, GM achieved an average return on equity of 15.2% per year. The average nominal T-bill rate was 6.4% and the average bond and stock excess returns were 2.7% and 5.7%, respectively. The average liability return was 9.7%. The constructed bond return series is highly correlated with the constructed liability return series with a correlation coefficient of 0.71. Stock returns have a positive correlation (0.31) with liability returns while cash returns have a negative correlation (-0.18).

Table 3 shows the estimation results. Like Campbell et al. (2003), Hoevenaars et al. (2008), and many other studies, we find evidence for mean reversion in log stock excess returns caused by the log dividend-yield.

### 3.3 OPTIMIZATION RESULTS

Table 4 shows the optimization results of the asset-liability management study for GM in December 2002 for plan member coefficients of relative risk aversion,  $\gamma$ , lying between 1 and 9. The first panel of the table summarizes the input parameters. As discussed before, we set the duration of the pension liability,  $D$ , to 13 years. The investment horizon,  $J$ , equals the liability duration. Corresponding to annual rebalancing and quarterly data, we set  $h = 4$ . We assume that relative consumption growth,  $\phi$ , is 1.5.

The second panel of Table 4 contains information on the observed (December 2002) funding situation of the GM pension fund expressed in percent of the liability value,  $L_t$ . We can normalize the liability value to unity, since only the ratio of assets to liabilities is relevant for the outcome of the optimization exercise. The funding ratio is  $F_t = 0.60$  and the total funding ratio (including the net worth of the sponsor)  $G_t = 0.67$ . The third and fourth panels of Table 4 contain the optimization results. Panel 3 summarizes the optimal asset allocation, while panel 4 displays relevant information on the funding spread, its components and the implied pension benefit value.

All optimizations in this and the following tables generate a zero allocation to cash. This is not too surprising, since we are dealing with a long-run asset-liability model. The riskless asset in this context is not cash, but a liability hedging portfolio. The allocation to stocks decreases from 70% for  $\gamma = 1$  to 30% for  $\gamma = 9$ . If we aggregate GM's observed allocation to real estate with stocks, we find an allocation to risky assets of 65%. In our framework, such a portfolio only would be optimal for very low degrees of risk aversion approaching  $\gamma = 1$ .

The underfunding probabilities increase from  $\pi_t = 63\%$  to  $\pi_t = 71\%$  when  $\gamma$  increases from 1 to 9. This is because lower allocations to stocks increase underfunding probabilities, since stocks have higher expected returns than other assets and also exhibit long-run mean reversion. Recovery ratios first increase from  $\lambda_t = 79.5\%$  to  $\lambda_t = 79.8\%$  when  $\gamma$  increases

from 1 to 5 and then decrease to  $\lambda_t = 79.6\%$  for  $\gamma = 9$ . Recall that the recovery ratio is the conditionally expected total funding ratio, given that the latter falls below unity. This conditional expectation is not monotonically increasing in the allocation to stocks. The conditional expectation of the total funding ratio is also shown in panel 4 of Table 4 and decreases from  $E_t[G_{t+Dh}] = 95\%$  to  $E_t[G_{t+Dh}] = 90\%$  when  $\gamma$  increases from 1 to 9. The annualized funding risk premium,  $(1 + \theta_t^{Dh})^h - 1$ , increases from 0.02% to 0.05%, confirming the increase in funding risk as the optimal asset allocation moves from aggressive to conservative.

In summary, these results imply an annualized funding spread,  $(1 + \delta_t^{Dh})^h - 1$ , of between 1.24% for  $\gamma = 1$  and 1.75% for  $\gamma = 9$ . A plan member with a low degree of risk aversion will choose a lower discount rate than a more risk-averse member. This will then be reflected in the benefit value which decreases from  $B_t = 85.2\%$  to  $B_t = 79.8\%$  when  $\gamma$  increases from 1 to 9. Bearing in mind that  $B_t$  is a measure of expected pension wealth, a difference of the order of 5.4 percentage points is economically very significant. This illustrates our main result: given GM's funding ability in December 2002, plan members could reasonably anticipate that the effective value of their pension payments ( $B_t$ ) was only around 80 – 85% of their promised pension payments ( $L_t$ ). It would have been sensible of them to have adjusted their consumption and savings behavior to compensate for this expected loss in retirement benefits.

It is interesting to compare the optimized asset allocation and benefit valuation outcomes with the actual portfolio and discount rate chosen by GM in December 2002. Recall that GM allocated about 65% of the pension assets to risky assets (real estate and stocks) and 35% to bonds. At the same time, the company chose a discount rate which implied a spread of 1.74% over the yield on a government bond. Most interestingly, our asset-liability model generates

almost the same spread (1.75%) when  $\gamma = 9$ . Our model could therefore be used to justify the assumed discount rate for the calculation of the PBO if there is reason to believe that the representative pension plan member is highly risk averse. However, the optimal asset allocation consistent with this funding spread implies an allocation of 70% to bonds and 30% to stocks. With 65% of its pension assets allocated to risky assets, GM chose a strategic asset allocation which was much too risky for the presumed preferences of the representative plan member. Such inconsistencies are avoided in our proposed asset-liability model, because the optimal strategic asset allocation and optimal discount rate are determined simultaneously.

### 3.4 COMPARATIVE STATICS

We will now use  $\gamma = 5$  in Table 4 as a benchmark for a number of comparative statics exercises. We will depart from the parameters and observed funding ratios describing the GM pension plan in December 2002 to obtain further insights into the way our model operates. In Table 5, we vary the funding ratio, in Table 6, we vary the investment horizon, and in Table 7, we vary the liability duration. All other parameters and input values in these tables remain the same as in Table 4. The benchmark case is included in all tables for comparison.

The observed funding ratio is varied between  $F_t = 0.4$  and  $F_t = 1.2$  in Table 5. The results suggest that a pension plan should optimally shift from an aggressive portfolio to a liability hedging portfolio as the funding ratio increases from severe underfunding to overfunding. A funding ratio of unity implies an optimal allocation of 70% to bonds, while  $F_t = 0.4$  implies 30% in bonds and  $F_t = 1.2$  implies 75% in bonds. Annualized funding spreads decrease substantially from 3.08% to 0.09% over the same range of funding ratios. Correspondingly, the benefit increases from  $B_t = 67\%$  to  $B_t = 99\%$ . At a funding ratio of 100%, the benefit still falls short of the liability by about 4.5% because of the remaining funding risk over the duration of the liability. Most pension plans had funding ratios in the range  $F_t \in [0.8, 1.0]$  in the

adverse economic environment of 2002. For these pension plans, we derive optimal funding spreads of between 0.85% and 0.35%. These spreads are much smaller than the yield spread on AA-rated bonds of 1.74%. Thus, the common practice of using a discount rate in line with the average yield on long-term AA-rated bonds leads to PBO values for most pension plans, which are much too small to reflect the value of the benefits that plan members can realistically expect to achieve. As already emphasized in the introduction, it violates a key principle in financial economics if pension plans with different funding abilities use the same discount rate. The discount rate should reflect the plan-specific funding risk.

Figure 1 shows the optimal allocation to stocks and the optimal funding spread for a larger range of observed funding ratios between  $F_t = 0.2$  and  $F_t = 2$ . It is readily apparent that as the funding ratio increases, the funding spread converges to zero and the optimal portfolio becomes the liability hedging portfolio. Nevertheless, the optimal portfolio continues to include an allocation to stocks of about 20%, even for funding ratios above 1.6. Figure 2 shows that underfunding probabilities monotonically switch from unity to zero over the same range of observed funding ratios, while recovery ratios move in the opposite direction from near zero to unity.

The investment horizon is varied between  $J = 1$  and  $J = 17$  years in Table 6. It is important to note that the liability duration is fixed at  $D = 13$  years. Strategic asset allocation models usually generate higher allocations to stocks when the investment horizon increases because stocks become less volatile in the long run due to mean reversion (see, e.g., Campbell et al. 2003). But in our asset-liability model, we do not find this pattern. Instead, we find only a small change in the optimal allocation to stocks from 50% to 40% when we increase the investment horizon from 1 to 17 years. This is because the asset value at the liability duration date is what is most relevant for the determination of the current funding spread and its

associated benefit value. The investment horizon is of second-order importance in our model. It follows from this that a reasonable choice for the investment horizon is the duration of the pension liability as assumed in Table 4.

For Table 7, we assume a constant investment horizon with  $J = 13$  years, while varying the duration of pension liabilities between  $D = 1$  and  $D = 17$  years. We find that the optimal allocation to stocks decreases from 70% for  $D = 1$  to 50% for  $D = 17$ . At the same time, the annualized funding spread decreases from 45% to 1%. Given a total funding ratio of 67%, pension benefits at the one-year horizon are extremely risky and this is reflected in the large funding spread. The conditionally expected total funding ratio at the one-year horizon is only 69%. The objective function (11) is maximized by reducing funding spreads as much as possible for the given degree of relative risk aversion. This is achieved by increasing the allocation to stocks. As the liability duration increases, both the conditionally expected total funding ratio and the net worth of the sponsor increase, and hence the need to decrease funding spreads further through additional allocations to stocks diminishes. Figure 3 shows a more complete picture of this relationship for an extended range of liability durations,  $D \in [1, 19]$ . It turns out that the allocation to stocks starts increasing for very long liability durations exceeding  $D = 15$  years. At these durations, annualized funding spreads are 1% and less. The asset allocation strategy starts to exploit the mean reversion in stock returns. Funding spreads continue to decrease as the figure shows. The funding spread shown in Figure 3 can also be interpreted as a term structure of funding spreads. In the presence of severe underfunding, funding spreads are very large at the short horizon but converge towards zero for long maturities. Figure 4 shows the term structure of the corresponding underfunding probabilities and recovery ratios. Underfunding probabilities monotonically decrease with

increasing liability duration, while recovery ratios increase as long as the allocation to stocks decreases and decrease afterwards.

## **4. Conclusions**

In this paper, we propose a new approach to the valuation of the pension obligation of a corporate defined benefit pension plan. We borrow from the literature on corporate bond pricing and derive a discount rate which consists of the yield on a riskless government bond plus a funding spread which depends on the ability of the plan sponsor to fund promised future pension payments. The funding spread converges to zero with decreasing underfunding probabilities and increasing recovery ratios. We view the pension fund as an integral part of the company and merge the pension plan assets with the net worth of the sponsoring company for the purpose of computing total funding ratios.

We demonstrate that the proposed discount rate depends on the chosen asset allocation which influences future funding abilities. To find an optimal discount rate, we propose an optimal strategic asset allocation which optimizes funding spreads. Our aim is to reflect the interests of the pension plan beneficiaries who care about the security of their future pensions. The resulting asset-liability model leads simultaneously to an optimal strategic asset allocation and an optimal valuation of the pension obligation which is consistent with the asset allocation. This stands in marked contrast with current practice, in which the valuation of liabilities and the allocation of pension assets are treated as separate tasks.

We argue that our approach has important advantages for all stakeholders of the corporate pension plan. In particular, plan beneficiaries get a clearer picture of the current funding-risk-adjusted value of their pension promise. If this turns out to be too low, they can optimally adjust their savings behavior. Our approach also removes the discretion of the plan sponsor to

choose the discount rate in a strategic way, for example, to manipulate reported earnings. Finally, our approach increases transparency for the sponsoring company and, especially, its shareholders who are now better able to plan for future contributions into the pension plan and to value the sponsoring company more accurately.

We applied our asset-liability model to the U.S. pension plan of General Motors in December 2002. With a reported funding ratio of pension assets to projected benefit obligations of 76%, the GM pension plan was in financial distress after a period of both falling stock markets and interest rates. Our model implies that the beneficiaries of the GM plan should have expected a reduction in their pension wealth of the order of 15-20%, depending on the relative risk aversion of the representative pension plan member. We found that the discount rate chosen by GM in December 2002 could be justified in our model if the representative pension plan member is very risk averse. However, our model also implies a much more conservative asset allocation than the one chosen by GM.

Pension plans which are sufficiently well funded to reduce funding risk to a negligible amount optimally choose an asset allocation in our framework which is close to a liability hedging portfolio but which additionally includes a small allocation to stocks to help keep future underfunding probabilities low as a result of the higher expected mean-reverting return to stocks.

Finally, we note that the optimal discount rates implied by our model tend to be smaller than those used by companies with moderate underfunding for calculating their projected benefit obligations. The current practice of using the same discount rate for all pension plans regardless of their individual funding abilities is misleading and carries no justification in financial economics. A revision to the accounting standards that report the valuation of corporate defined benefit obligations is a clear policy implication from our analysis.

## Appendix: Simulation Approach

We use the simulation approach suggested by Brandt et al. (2005) to solve the dynamic programming problem (11). We first generate a large number (5,000) of  $(J + D)h$ -period forecasts for all the variables of the vector autoregressive equation system presented in Table 3. We then solve the dynamic programming problem recursively for every rebalancing time,  $t + jh$  for  $j = J - 1, J - 2, \dots, 0$ . At every rebalancing time, we evaluate the objective function in every scenario for a grid of 61 possible funding ratios,  $F_{t+jh}$ , and 231 possible portfolios,  $w_{t+jh}$ , comprising all possible combinations of allocation to cash, bonds, and stocks in 5% steps. From  $F_{t+jh}$  we can calculate  $G_{t+jh}$  and  $g_{t+jh} = \log G_{t+jh}$  in every scenario using the return on the company's net worth. For the calculation of the funding spread,  $\delta_{t+jh}^{Dh}$ , we need the future total funding ratio  $G_{t+(j+D)h}$  which is obtained for every  $F_{t+jh}$  and  $w_{t+jh}$  by projecting the returns on liabilities, the net worth and the respective portfolio in every scenario. In line with the requirements for dynamically consistent behavior emphasized by Cuoco et al. (2008), the risk measure – value at risk in their case, the funding spread in our case – is dynamically reevaluated at every rebalancing time,  $t + jh$ , under the assumption that the asset allocation is kept unchanged over the evaluation interval,  $Dh$ . Let  $1 + R_{t+(j+D)h}^{G(Dh)} = G_{t+(j+D)h}/G_{t+jh}$  denote the gross return on the total funding ratio over the liability duration and  $r_{t+(j+D)h}^{G(Dh)} = \log \left( 1 + R_{t+(j+D)h}^{G(Dh)} \right)$ , the corresponding log return. Then we can compute the components of the funding spread as

$$\pi_{t+jh}^{Dh} = Prob_{t+jh} \left( g_{t+jh} + r_{t+(j+D)h}^{G(Dh)} < 0 \right) = \Phi \left( \frac{-g_{t+jh} - E_{t+jh} \left[ r_{t+(j+D)h}^{G(Dh)} \right]}{V_{t+(j+D)h}^{0.5} \left[ r_{t+(j+D)h}^{G(Dh)} \right]} \right) \quad (A1)$$

$$\lambda_{t+jh}^{Dh} = \frac{1}{\pi_{t+jh}^{Dh}} \exp \left( g_{t+jh} + E_{t+jh} \left[ r_{t+(j+D)h}^{G(Dh)} \right] + 0.5 V_{t+(j+D)h}^{0.5} \left[ r_{t+(j+D)h}^{G(Dh)} \right] \right).$$

$$\Phi \left( \frac{-g_{t+jh} - E_{t+jh} \left[ r_{t+(j+D)h}^{G(Dh)} \right] - V_{t+(j+D)h} \left[ r_{t+(j+D)h}^{G(Dh)} \right]}{V_{t+(j+D)h}^{0.5} \left[ r_{t+(j+D)h}^{G(Dh)} \right]} \right) \quad (\text{A2})$$

using the properties of the truncated lognormal distribution (see Lien, 1985). The conditional expectations are obtained by regressing  $r_{t+(j+D)h}^{G(Dh)}$  on a polynomial in the state variables at time  $t + jh$  across all scenarios as suggested by Brandt et al. (2005). Once the funding spreads are obtained, we can obtain  $B_{t+jh}$  in every scenario.

Assume now we are at rebalancing time  $j = J - 1$ . We evaluate the objective function in  $B_{t+Jh} = B_{t+(J-1)h} (1 + R_{t+Jh}^{B(h)})$  in every scenario for every possible portfolio and funding ratio grid value. For a given funding ratio grid value, the particular portfolio which maximizes the conditionally expected utility in  $B_{t+Jh}$  is optimal. The conditional expectation is again computed by a cross-scenario regression on a polynomial in the state variables at the rebalancing time. Once we have obtained an optimal portfolio for every funding ratio grid value, we regress the optimal  $(1 + R_{t+Jh}^{B(h)})$  on  $B_{t+(J-1)h}$  for every funding ratio grid value in every scenario. We use this auxiliary regression later to match benefit values. At  $j = J - 2$ , we need to evaluate the utility function in  $B_{t+Jh} = B_{t+(J-2)h} (1 + R_{t+(J-1)h}^{B(h)}) (1 + R_{t+Jh}^{B(h)})$ . We obtain the first factor,  $B_{t+(J-1)h} = B_{t+(J-2)h} (1 + R_{t+(J-1)h}^{B(h)})$ , as before for every possible portfolio and funding ratio grid value and use this value as an explanatory variable in the aforementioned auxiliary regression to obtain the second factor  $(1 + R_{t+Jh}^{B(h)})$ . The auxiliary regression provides a match between the optimized end of period  $t + (J - 1)h$  benefit and the optimal benefit return in period  $t + Jh$  resulting from an initial benefit  $B_{t+(J-1)h}$  and replaces alternative interpolation methods (e.g., van Binsbergen and Brandt, 2009).

We continue the process until we reach the current period with  $j = 0$ . In the current period there is no need to estimate conditional expectations with regressions on state variables as the current state variables are known. Hence, sample averages replace the conditional expectations. For  $j = 0$ , we obtain the current value of the benefit,  $B_t$ , which is now a function of the current optimal portfolio,  $w_t$ , and all future optimal portfolios derived from the dynamic programming exercise.

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## Tables

Table 1: General Motors U.S. pension plan in December 2002

General Motors pension plan	Dec. 2002	Unit	Source
Net worth of the plan sponsor	6.81	bn USD	Viceira (2005)
Pension assets	60.90	bn USD	Viceira (2005)
Projected benefit obligation (PBO)	80.10	bn USD	Viceira (2005)
Pension assets as percent of PBO	76.03	percent	own calculation
Assumed discount rate	6.75	percent	SEC (2009)
Yield of Moody's $\geq$ 20 yrs. AA bond	6.63	percent	SEC (2009)
Yield of a 20 yrs. government bond	5.01	percent	FRED
Spread over riskless yield	1.74	percent	own calculation
Average pension payment 1997-2002	5.88	bn USD	Viceira (2005)
Maturity of pension liability	40	years	own calculation
Duration of pension liability	13	years	own calculation
Pension liability	100.76	bn USD	own calculation
Net worth as percent of liability ( $N_t$ )	6.76	percent	own calculation
Pension assets as percent of liability ( $A_t$ )	60.44	percent	own calculation
Pension liability as percent of liability ( $L_t$ )	100.00	percent	own calculation
Funding ratio ( $F_t$ )	60.44	percent	own calculation
Total funding ratio ( $G_t$ )	67.20	percent	own calculation
Allocation to others	1.00	percent	Viceira (2005)
Allocation to bonds	34.00	percent	Viceira (2005)
Allocation to stocks	55.00	percent	Viceira (2005)
Allocation to real estate	10.00	percent	Viceira (2005)

Note: The Projected Benefit Obligation (PBO) was calculated by General Motors using the assumed discount rate. For the calculation of the pension maturity and duration, we assume a constant pension payment at annual intervals equal to the average pension payment over the years 1997-2002. The pension liability is calculated by us using the same assumptions, the same maturity and the yield on a riskless government bond with 20 years to maturity as the discount rate. FRED is the Federal Reserve Economic Data collection.

Table 2: Descriptive statistics

	Mean	Standard deviation	Sharpe ratio	Correlation with liability
Nominal T-bill return	0.0635	0.0134	0.0000	-0.1764
Bond excess return	0.0270	0.0793	0.3398	0.7136
Stock excess return	0.0572	0.1970	0.2903	0.3071
Return on equity	0.1515	0.1071	0.8224	-0.1462
Liability return	0.0972	0.1331	0.2533	1.0000
Log dividend-price ratio	-3.5247	0.4281		
Log term spread	0.0351	0.0314		
Log credit spread	0.0300	0.0112		

Note: The moments of the returns are annualized and are generated by appropriately adjusting the moments for the underlying log returns. The Sharpe ratio is the mean excess return over the nominal T-bill return divided by the standard deviation of the return. The statistics are calculated from quarterly U.S. data from 1970.II-2002.IV. 1970.II is the first quarter for which we observe the return on equity for General Motors. 2002.IV is the period in which we conduct the asset-liability modeling study for General Motors.

Table 3: Vector autoregression estimation results

	Tbill	BXR	SXR	RoE	Liab	DP	Term	Def	Cons	R <sup>2</sup>
Tbill	0.95	-0.01	0.01	0.01	0.00	0.00	0.00	-0.02	0.00	0.91
t-val.	18.3	-2.35	2.58	1.49	-0.13	1.54	0.60	-1.15	1.55	(0.00)
BXR	2.67	-0.13	-0.07	-0.12	0.08	-0.02	0.63	-0.42	-0.12	0.17
t-val.	2.73	-1.13	-2.00	-1.64	1.02	-2.08	4.38	-1.04	-2.34	(0.00)
SXR	-2.47	0.23	-0.07	0.02	0.21	0.05	-0.07	0.37	0.21	0.09
t-val.	-1.04	0.81	-0.77	0.09	1.10	1.91	-0.20	0.38	1.71	(0.15)
RoE	-1.55	-0.16	-0.01	0.43	0.11	0.00	-0.05	0.03	0.04	0.24
t-val.	-1.37	-1.21	-0.26	5.09	1.24	0.04	-0.29	0.07	0.77	(0.00)
Liab	4.78	0.78	-0.10	-0.05	0.00	-0.03	0.79	-0.72	-0.15	0.31
t-val.	3.43	4.66	-2.01	-0.50	0.03	-1.67	3.86	-1.25	-2.08	(0.00)
DP	1.10	-0.18	0.08	0.05	-0.24	0.98	0.01	-0.27	-0.09	0.96
t-val.	0.48	-0.65	0.96	0.29	-1.35	39.4	0.02	-0.29	-0.77	(0.00)
Term	-0.62	0.02	-0.02	-0.02	0.03	0.00	0.74	0.48	0.00	0.71
t-val.	-1.32	0.31	-0.95	-0.60	0.69	-0.06	10.8	2.46	0.15	(0.00)
Def	0.17	-0.01	-0.02	-0.01	-0.01	0.00	0.00	0.87	0.00	0.86
t-val.	1.43	-0.90	-3.77	-1.58	-0.85	0.36	-0.22	17.7	0.68	(0.00)
Residual correlation matrix (volatilities in main diagonal)										
Tbill	0.19									
BXR	-0.61	3.51								
SXR	-0.22	0.22	8.53							
RoE	0.09	-0.16	0.18	4.04						
Liab	-0.62	0.82	0.27	-0.09	5.02					
DP	0.21	-0.20	-0.98	-0.15	-0.24	8.21				
Term	-0.71	-0.04	0.09	0.02	-0.02	-0.09	1.68			
Def	-0.32	0.37	0.23	-0.04	0.41	-0.23	0.08	0.42		

Note: The estimation is based on quarterly U.S. data from 1970.II - 2002.IV. 1970.II is the first quarter for which we observe the return on equity for General Motors. 2002.IV is the period in which we conduct the asset-liability modeling study for General Motors. Tbill denotes the nominal log Treasury bill rate, BXR the log bond excess return, SXR the log stock excess return, RoE the return on equity for General Motors, Liab the log liability return, DP the log dividend-price ratio, Term the log term spread, Def the log credit spread and Cons the intercept. Numbers in parentheses are p-values. The maximum eigenvalue of the slope matrix is 0.9781.

Table 4: Optimization results when varying the coefficient of relative risk aversion

1) Input: parameters					
Relative risk aversion ( $\gamma$ )	1	3	5	7	9
Relative consumption growth ( $\phi$ )	1.5	1.5	1.5	1.5	1.5
Investment horizon ( $J$ ) in years	13	13	13	13	13
Liability duration ( $D$ ) in years	13	13	13	13	13
2) Input: observed assets and liabilities					
Net worth of the sponsor ( $N_t$ )	0.07	0.07	0.07	0.07	0.07
Pension assets ( $A_t$ )	0.60	0.60	0.60	0.60	0.60
Pension liability ( $L_t$ )	1.00	1.00	1.00	1.00	1.00
Funding ratio ( $F_t$ )	0.60	0.60	0.60	0.60	0.60
Total funding ratio ( $G_t$ )	0.67	0.67	0.67	0.67	0.67
3) Output: optimal asset allocation					
Allocation to cash ( $1 - w_t'1_2$ )	0.00	0.00	0.00	0.00	0.00
Allocation to bonds ( $w_{t,1}$ )	0.30	0.45	0.55	0.65	0.70
Allocation to stocks ( $w_{t,2}$ )	0.70	0.55	0.45	0.35	0.30
4) Output: optimal benefit valuation					
Exp. total fund. ratio ( $E_t[G_{t+Dh}]$ )	0.9471	0.9305	0.9194	0.9083	0.9028
Underfunding probability ( $\pi_t^{Dh}$ )	0.6326	0.6610	0.6809	0.7010	0.7109
Recovery ratio ( $\lambda_t^{Dh}$ )	0.7953	0.7978	0.7981	0.7971	0.7960
Funding risk premium ( $1 + \theta_t^{Dh})^h$ )	1.0016	1.0038	1.0049	1.0052	1.0053
Funding spread ( $1 + \delta_t^{Dh})^h$ )	1.0124	1.0150	1.0164	1.0171	1.0175
Pension benefit ( $B_t$ )	0.8524	0.8245	0.8098	0.8020	0.7981

Note: Input and output refers to required input to and resulting output from the asset-liability modeling study.

We assume annual pension payment and rebalancing intervals. Accordingly, we set  $h = 4$  with quarterly data.

Table 5: Optimization results when varying the observed funding ratio

1) Input: parameters					
Relative risk aversion ( $\gamma$ )	5	5	5	5	5
Relative consumption growth ( $\phi$ )	1.5	1.5	1.5	1.5	1.5
Investment horizon ( $J$ ) in years	13	13	13	13	13
Liability duration ( $D$ ) in years	13	13	13	13	13
2) Input: observed assets and liabilities					
Net worth of the sponsor ( $N_t$ )	0.07	0.07	0.07	0.07	0.07
Pension assets ( $A_t$ )	0.40	0.60	0.80	1.00	1.20
Pension liability ( $L_t$ )	1.00	1.00	1.00	1.00	1.00
Funding ratio ( $F_t$ )	0.40	0.60	0.80	1.00	1.20
Total funding ratio ( $G_t$ )	0.47	0.67	0.87	1.07	1.27
3) Output: optimal asset allocation					
Allocation to cash ( $1 - w_t'1_2$ )	0.00	0.00	0.00	0.00	0.00
Allocation to bonds ( $w_{t,1}$ )	0.30	0.55	0.65	0.70	0.75
Allocation to stocks ( $w_{t,2}$ )	0.70	0.45	0.35	0.30	0.25
4) Output: optimal benefit valuation					
Exp. total fund. ratio ( $E_t[G_{t+Dh}]$ )	0.7098	0.9194	1.1327	1.3478	1.5592
Underfunding probability ( $\pi_t^{Dh}$ )	0.9154	0.6809	0.3293	0.1037	0.0245
Recovery ratio ( $\lambda_t^{Dh}$ )	0.6698	0.7981	0.8685	0.9048	0.9249
Funding risk premium ( $1 + \theta_t^{Dh}$ ) <sup><math>h</math></sup>	1.0027	1.0049	1.0050	1.0027	1.0008
Funding spread ( $1 + \delta_t^{Dh}$ ) <sup><math>h</math></sup>	1.0308	1.0164	1.0085	1.0035	1.0009
Pension benefit ( $B_t$ )	0.6737	0.8098	0.8963	0.9555	0.9880

Note: Input and output refers to required input to and resulting output from the asset-liability modeling study.

We assume annual pension payment and rebalancing intervals. Accordingly, we set  $h = 4$  with quarterly data.

Table 6: Optimization results when varying the investment horizon

1) Input: parameters					
Relative risk aversion ( $\gamma$ )	5	5	5	5	5
Relative consumption growth ( $\phi$ )	1.5	1.5	1.5	1.5	1.5
Investment horizon ( $J$ ) in years	1	5	9	13	17
Liability duration ( $D$ ) in years	13	13	13	13	13
2) Input: observed assets and liabilities					
Net worth of the sponsor ( $N_t$ )	0.07	0.07	0.07	0.07	0.07
Pension assets ( $A_t$ )	0.60	0.60	0.60	0.60	0.60
Pension liability ( $L_t$ )	1.00	1.00	1.00	1.00	1.00
Funding ratio ( $F_t$ )	0.60	0.60	0.60	0.60	0.60
Total funding ratio ( $G_t$ )	0.67	0.67	0.67	0.67	0.67
3) Output: optimal asset allocation					
Allocation to cash ( $1 - w_t'1_2$ )	0.00	0.00	0.00	0.00	0.00
Allocation to bonds ( $w_{t,1}$ )	0.50	0.50	0.55	0.55	0.60
Allocation to stocks ( $w_{t,2}$ )	0.50	0.50	0.45	0.45	0.40
4) Output: optimal benefit valuation					
Exp. total fund. ratio ( $E_t[G_{t+Dh}]$ )	0.9335	0.9360	0.9173	0.9194	0.9075
Underfunding probability ( $\pi_t^{Dh}$ )	0.6554	0.6522	0.6827	0.6809	0.7000
Recovery ratio ( $\lambda_t^{Dh}$ )	0.7966	0.8001	0.7946	0.7981	0.7933
Funding risk premium ( $1 + \theta_t^{Dh})^h$ )	1.0052	1.0052	1.0049	1.0049	1.0047
Funding spread ( $1 + \delta_t^{Dh})^h$ )	1.0164	1.0160	1.0167	1.0164	1.0169
Pension benefit ( $B_t$ )	0.8097	0.8132	0.8064	0.8098	0.8043

Note: Input and output refers to required input to and resulting output from the asset-liability modeling study.

We assume annual pension payment and rebalancing intervals. Accordingly, we set  $h = 4$  with quarterly data.

Table 7: Optimization results when varying the liability duration

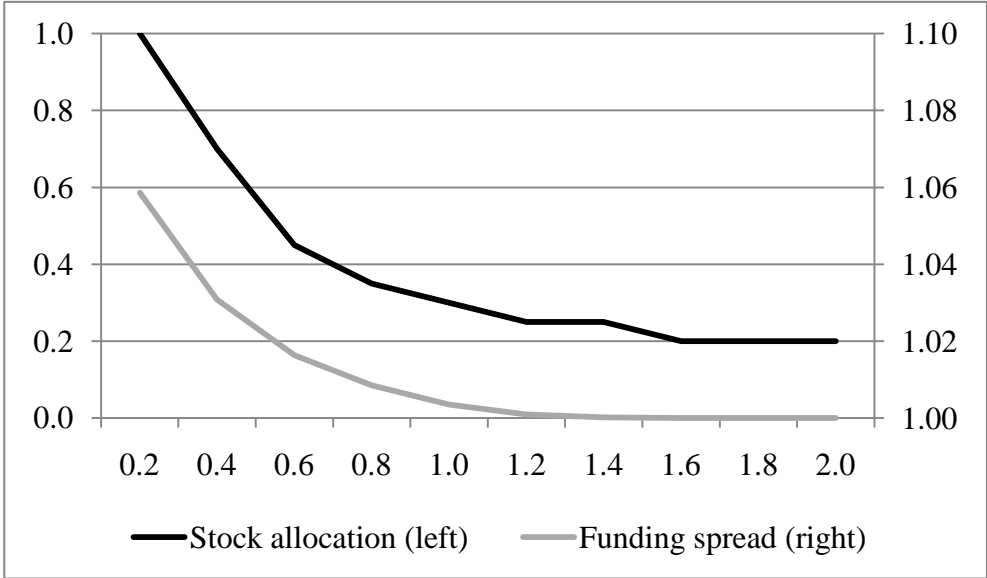
1) Input: parameters					
Relative risk aversion ( $\gamma$ )	5	5	5	5	5
Relative consumption growth ( $\phi$ )	1.5	1.5	1.5	1.5	1.5
Investment horizon ( $J$ ) in years	13	13	13	13	13
Liability duration ( $D$ ) in years	1	5	9	13	17
2) Input: observed assets and liabilities					
Net worth of the sponsor ( $N_t$ )	0.07	0.07	0.07	0.07	0.07
Pension assets ( $A_t$ )	0.60	0.60	0.60	0.60	0.60
Pension liability ( $L_t$ )	1.00	1.00	1.00	1.00	1.00
Funding ratio ( $F_t$ )	0.60	0.60	0.60	0.60	0.60
Total funding ratio ( $G_t$ )	0.67	0.67	0.67	0.67	0.67
3) Output: optimal asset allocation					
Allocation to cash ( $1 - w_t'1_2$ )	0.00	0.00	0.00	0.00	0.00
Allocation to bonds ( $w_{t,1}$ )	0.30	0.35	0.45	0.55	0.50
Allocation to stocks ( $w_{t,2}$ )	0.70	0.65	0.55	0.45	0.50
4) Output: optimal benefit valuation					
Exp. total fund. ratio ( $E_t[G_{t+Dh}]$ )	0.6909	0.7497	0.8190	0.9194	1.1273
Underfunding probability ( $\pi_t^{Dh}$ )	0.9992	0.9584	0.8590	0.6809	0.4147
Recovery ratio ( $\lambda_t^{Dh}$ )	0.6907	0.7356	0.7704	0.7981	0.8056
Funding risk premium ( $1 + \theta_t^{Dh}$ ) <sup><math>h</math></sup>	1.0003	1.0026	1.0039	1.0049	1.0056
Funding spread ( $1 + \delta_t^{Dh}$ ) <sup><math>h</math></sup>	1.4478	1.0629	1.0287	1.0164	1.0106
Pension benefit ( $B_t$ )	0.6907	0.7371	0.7753	0.8098	0.8361

Note: Input and output refers to required input to and resulting output from the asset-liability modeling study.

We assume annual pension payment and rebalancing intervals. Accordingly, we set  $h = 4$  with quarterly data.

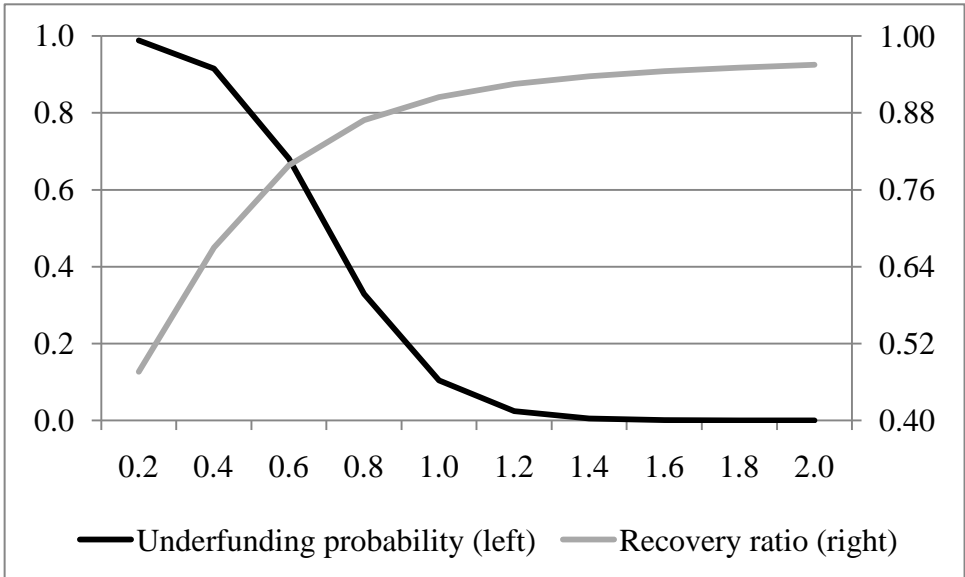
# Figures

Figure 1: Allocation to stocks and funding spreads for different observed funding ratios



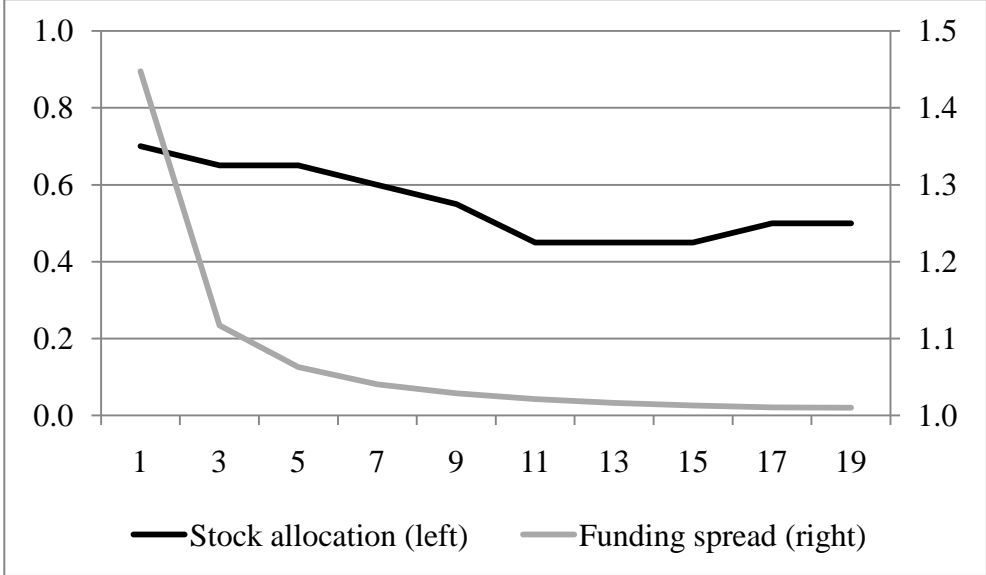
Note: The horizontal axis shows the observed funding ratio. The vertical axes denote the allocation to stocks on the left-hand side and the funding spread on the right-hand side. Based on  $\gamma = 5, \phi = 1.5, J = 13,$  and  $D = 13.$

Figure 2: Underfunding probabilities and recovery ratios for different observed funding ratios



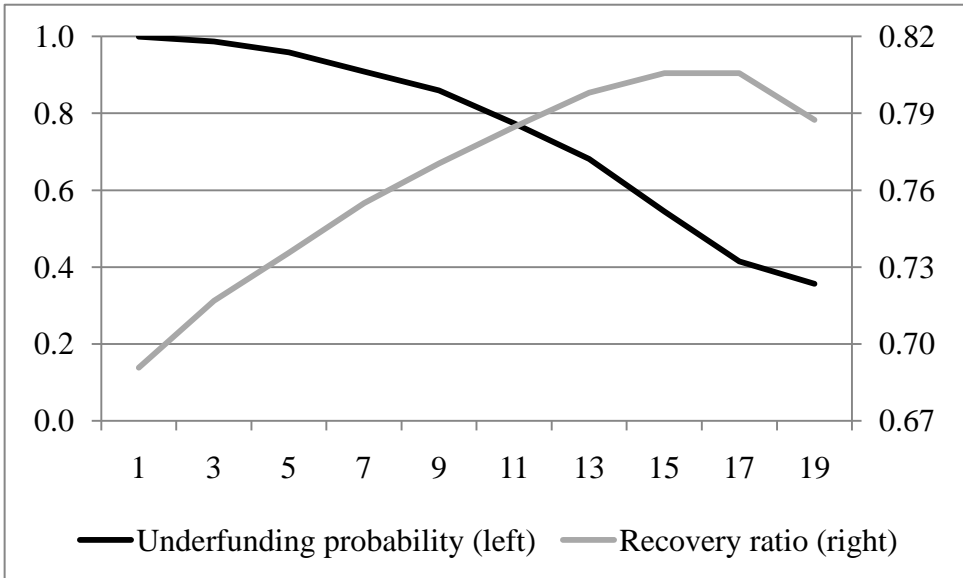
Note: The horizontal axis shows the observed funding ratio. The vertical axes denote the underfunding probability on the left-hand side and the recovery ratio on the right-hand side. Based on  $\gamma = 5, \phi = 1.5, J = 13,$  and  $D = 13.$

Figure 3: Allocation to stocks and funding spreads for varying liability duration



Note: The horizontal axis shows the liability duration. The vertical axes denote the allocation to stocks on the left-hand side and the funding spread on the right-hand side. Based on  $\gamma = 5, \phi = 1.5, J = 13,$  and  $A_t = 0.6.$

Figure 4: Underfunding probabilities and recovery ratios for varying liability duration



Note: The horizontal axis shows the liability duration. The vertical axes denote the underfunding probability on the left-hand side and the recovery ratio on the right-hand side. Based on  $\gamma = 5, \phi = 1.5, J = 13,$  and  $A_t = 0.6.$