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# Facing Up to the Uncertainty of Life: The Longevity Fan Charts

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# Facing Up to Uncertain Life Expectancy: The Longevity Fan Charts

By

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**Abstract:** This article uses longevity fan charts to represent the uncertainty in projections of future life expectancy. These fan charts are based on a mortality model calibrated on mortality data for English and Welsh males. The fan charts indicate strong upward sloping trends in future life expectancy. Their widths indicate the extent of uncertainty in these projections, and this uncertainty increases as the forecast horizon lengthens. Allowing for uncertainty in the parameter values also adds considerably to uncertainty in life expectancy.

**Key words:** life expectancy, longevity risk, fan charts

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## 1. INTRODUCTION

This article examines the phenomenon of longevity risk (or the quantifiable uncertainty associated with expected future human lifetimes) and discusses some of its implications.

It has become increasingly clear over the last few years that life expectancy has not only been rising, but has been rising at a much faster rate than was previously anticipated. Future life expectancies are therefore uncertain, and this uncertainty is no longer seriously disputed.<sup>1</sup>

This uncertainty has major implications for those providing services to the elderly such as health and long-term care or pensions. For example, if people are living longer than previously anticipated, then pension providers will be paying out for longer periods, and someone has to bear the resulting higher costs. Uncertain longevity also exposes pension funds, life companies and, indeed, the state itself, to longevity risk, and their exposure to this risk needs to be managed.

The phenomenon of rising but uncertain longevity also has major public policy implications. As the Governor of the Bank of England stated in a recent lecture to the British Academy, “We cannot avoid taking decisions, so we must accept the need to analyse the uncertainty that inevitably surrounds them” (King 2004: 3). And yet, as Dr. King went on to explain, “policy debates continue to be permeated by [an] ‘illusion of certainty’” that refuses to acknowledge the uncertainty intrinsic to any

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<sup>1</sup> To illustrate, the uncertainty of expected longevity was ‘officially’ acknowledged by the UK Continuous Mortality Investigation (CMI) when it published its proposed new mortality tables (the “00 series” tables) in September 2005 (CMI 2006a): the CMI now advises practising actuaries not to rely on a single mortality projection, but instead to rely on a range of scenarios reflecting the uncertainty in its projections.

forecasts of the future. This refusal to acknowledge risks makes it difficult to manage them, and effectively undermines much public policy debate.

Some indication of the scale of the longevity problem can be seen in the revisions made to expected longevity forecasts over the past 20 years. For instance, in 1980 the UK CMI anticipated that a British man who reached 60 in 1999 could expect to live another 21 years; however, by 1999 that forecast was revised upwards to 26 years (CMI 2006a). Over the course of almost 20 years, the expected remaining lifetime of a 60-year old man had increased by 5 years. There can therefore be little doubt that forecasts of expected longevity are indeed highly uncertain.

## **2. ASSESSING LONGEVITY RISK: ALTERNATIVE APPROACHES**

One approach to this problem is to consider alternative expert views about the impact of potential biomedical factors on future longevity. However, the range of alternative views is vast: on the one hand, ‘pessimists’ led by Jay Olshansky (e.g., Loladze 2002; Mizuno *et alia* 2004; Olshansky *et alia* 1990, 2004) have suggested that future life expectancy might level off or even decline due to factors such as obesity and decreased food-derived health benefits associated with higher levels of atmospheric CO<sub>2</sub>. On the other, ‘optimists’ led by James Vaupel (e.g., Oeppen *et alia* 2002; Tuljapurkar *et alia* 2000; Tuljapurkar 2005) argue that there is no natural upper limit to the length of human life. Moreover, even demographers critical of the extrapolative forecasting approach adopted by Vaupel still accept the possibility that scientific advances and the socio-political responses to them might lead to substantial increases in life expectancy over the next century (e.g., de Grey 2006).

An alternative is to carry out stress tests. For example, the United Kingdom Pensions Commission and the Government Actuary's Department (GAD) recently carried out a stress-test exercise in which they hypothesized that mortality rates might have a 1% error that compounds over time. Using this approach they found that male life expectancy at 65 today could lie anywhere between 17.7 and 20.5 years, with a baseline principal projection estimate of 19 years. By 2040, their approach suggested a range between 17.2 and 26.7 around a base case of 21.3 years (Pensions Commission 2004). Another form of stress testing is to compare actual mortality outcomes and more recent projections of future mortality against earlier forecasts. Using this approach, the Pensions Commission found that the latest available forecast of the mortality rate for 65-year old males in 2004 was some 41% lower than anticipated in GAD forecasts made in 1984 (Pensions Commission 2004). These and other results from subjective stress-testing exercises reported in the Pensions Commission Report further confirm that future longevity is highly uncertain.

Figure 1 brings together some of the above projections of life expectancy for 65-year old English and Welsh males. Whether based on stochastic models, deterministic scenario analysis or expert judgement, the Figure clearly shows that there is no consensus whatever surrounding future projections of life expectancy in the UK.

**INSERT FIGURE 1 HERE**

### 3. INVESTIGATING LONGEVITY RISK WITH FAN CHARTS

A third (and arguably more objective) approach is to quantify this uncertainty using ‘fan charts’: these are charts showing some central projection (such as the median, mode or mean) of the relevant variable – in our case, expected longevity – surrounded by a set of probability bounds, each of which gives the range within which the forecasted variable will fall within a specified probability.

The earliest fan charts appear to be the Bank of England’s inflation fan charts which were first published in the February 1996 issue of the Bank’s *Inflation Report*. The first life expectancy fan chart was published in Dr. King’s lecture and gave prediction intervals for female life expectancy at birth from 2004 out to a little past 2055: this fan chart had life expectancy starting out at 81; its central projection then rose to about 86.5 years, with a 90% prediction interval running from a little less than 84 to almost 90 years. King’s longevity fan charts were essentially illustrative and the underlying statistical model was not disclosed. Other life-expectancy fan charts were produced by Sanderson and Scherbov utilizing data for 14 countries but these were based on a rather simple projection model (Sanderson and Scherbov 2004).

We now produce a series of fully calibrated expected-future-lifetime (EFL) fan charts and use them to illustrate the uncertainty in current and past longevity forecasts. Unlike earlier studies, our fan charts are based on an explicit mortality model that is known to provide a good fit to the data, is estimated over alternative sample periods to illustrate the sensitivity of results to sample observations, takes account of possible uncertainty in our parameter estimates and can scientifically

replicated. Our fan charts are calibrated on mortality rate data for English and Welsh males over the period 1962-2002.<sup>2</sup>

Figure 2 shows the longevity fan chart forecasts for 65-year old males using data up to 1982 for a 20-year forecast horizon extending to 2002. The fan chart on the left-hand side of the Figure assumes estimated parameters to be known, and that on the right makes allowance for possible uncertainty in the parameters of the mortality model. Both fan charts have an initial expected future lifetime of 14 years, which is projected to rise gradually over the horizon period considered. The widths of the fan chart intervals also show that the projections of life expectancy are quite uncertain, and that this uncertainty increases as the forecast horizon rises. A comparison of the two fan charts also shows that the fan chart that takes account of parameter uncertainty is considerably wider than the one that assumes parameters to be known with certainty.<sup>3</sup> If we believe that parameters are uncertain – as we presumably should – then this comparison indicates that fan chart projections that ignore parameter uncertainty tend to considerably under-estimate future longevity uncertainty.

The fan charts also show the estimates of subsequently realized values for expected future lifetimes<sup>4</sup> for 65-year old males over this period, indicated by the ‘dots’ in the chart. These typically lie above the central EFL projections, but are not

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<sup>2</sup> The data are provided by the UK Government Actuary’s Department, and these data are fitted to the Cairns *et alia* (3) stochastic mortality model calibrated over two 21-year periods, i.e., 1962-1982 and 1982-2002. Details of the model are given in the supporting online material

<sup>3</sup> The principal reason for this increased width is uncertainty in the underlying trend rather than in the volatility of mortality rates. As our time horizon increases, uncertainty in the trend dominates all other sources of risk in influencing the width of the right-hand side fan chart.

<sup>4</sup> These are the projected values of life expectancy from the stochastic mortality model estimated with data up to 1982, but using realized values of the stochastic processes driving mortality rates for each year between 1983 and 2002. The calculations of these EFL values are explained more fully in the Appendix.

sufficiently high as to cast serious doubt on the reliability of the forecasting model.<sup>5</sup> This gives us some reason to be confident in the model's forecasting ability.

## **INSERT FIGURE 2 HERE**

Figure 3 gives the fan charts forecasts starting from 2002 and extending out over a 50-year horizon to 2052. The expected future lifetime now starts at 18 and again we find the same pattern of a rising trend and increasing uncertainty as the forecast horizon lengthens. The fan charts that allow for parameter uncertainty are very wide indeed - for example, the parameter uncertainty fan chart has a 90% prediction bound for the expected future lifetime of our illustrative males stretching from 22 to almost 32 years – and are considerably wider than those that do not.

We can also see taking account of parameter uncertainty has an asymmetric effect on the fan charts – it pushes down the lower bounds a little, but pushes up the upper bound a lot. This skewed impact also pushes up central projections in the fan charts and leads to even stronger projected improvements to expected future lifetimes.

## **4. CONCLUSIONS**

We believe that longevity fan charts are a useful way of showing the stochastic nature of improvements in life expectancy over time. Our main findings confirm that expected future lifetimes are projected to strongly increase, but we also find that these

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<sup>5</sup> One might also note that the realized EFL values – the dots – in Figure 2 tend to lie in the more central regions of the parameter-uncertainty fan chart than is the case with the parameter uncertainty fan chart. This again suggests that the former fan charts give better representations of longevity uncertainty.

projections are also highly uncertain and become even more uncertain as the forecast horizon lengthens. These findings are bad news for those with obligations to pay pensions or otherwise provide for the elderly, as it forces them to anticipate large numbers of people living to very old ages whilst also addressing the question of how to provide for very uncertain outcomes. We also found that allowing for parameter uncertainty makes the fan charts even wider, i.e., makes us even more uncertain about expected future longevity. This finding makes the bad news even worse.

We would also stress that although our results are based on a model calibrated on UK male mortality experience, we have every reason to expect that similar findings would be obtained for both males and females for any comparable countries. Thus, our findings have disturbing implications for the health, pensions and life insurance industries in many countries, and for public policy generally.

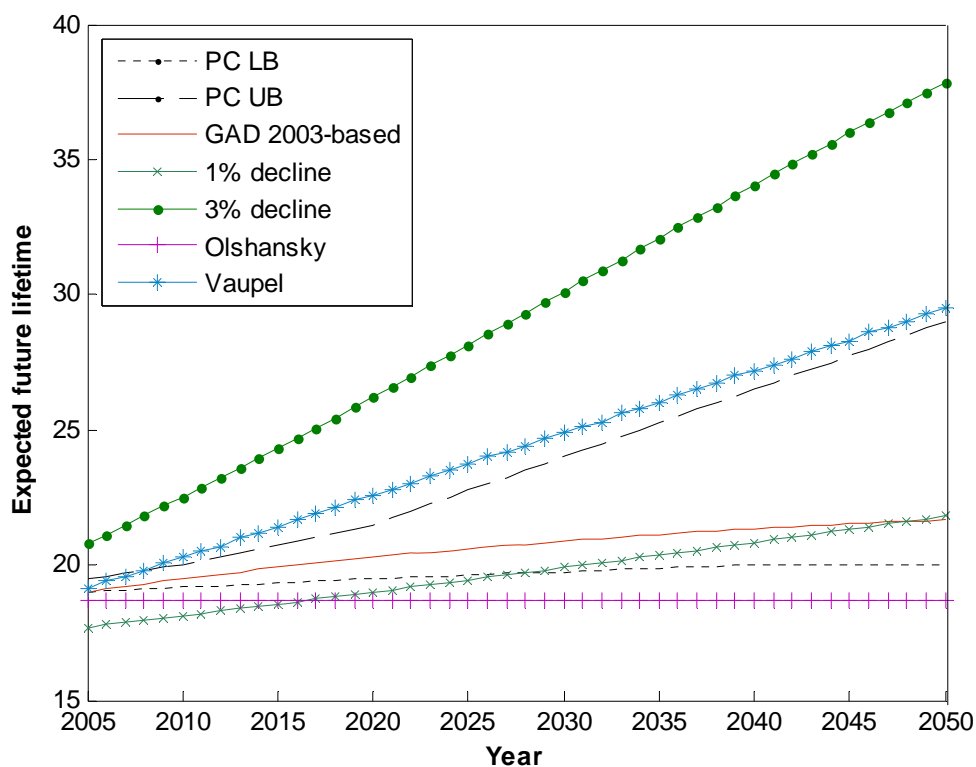
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## FIGURES

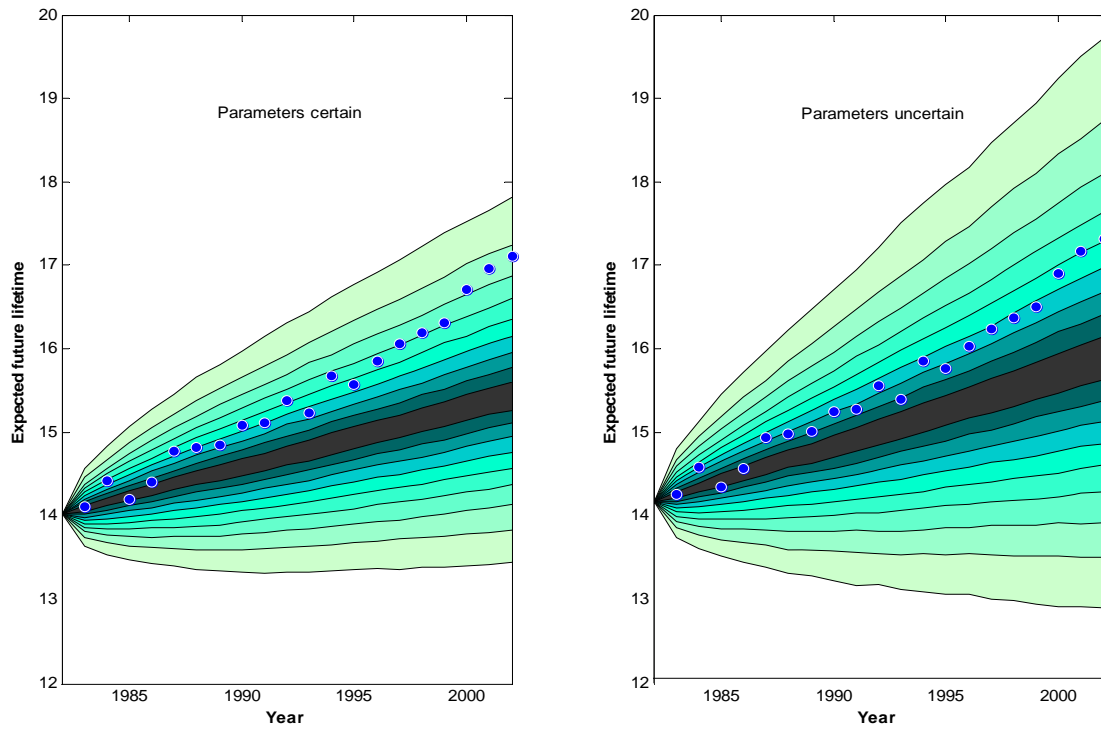
**FIGURE 1: SOME RECENT PROJECTIONS OF LIFE EXPECTANCY FOR  
65-YEAR OLD BRITISH MALES**



Notes: 'PC LB' and 'PC UB' refer to the bounds of the putative 90% prediction interval taken from Fig E.11 of the Pensions Commission Report (14), 'GAD 2000-based' refers to the GAD principal projection based on 2003 data obtained from the same source, '1% decline' and '3% decline' refer to the Pension Commission projections based on 1% and 3% declines in mortality, and is taken from Figure E.4 of the Pensions Commission Report (14), 'Olshansky' and 'Vaupel' refer to the putative Olshansky and Vaupel projections given in the Figure E.5 of the Pensions Commission Report (14).

**FIGURE 2: LONGEVITY FAN CHART FOR 65-YEAR OLD MALES: 1982**

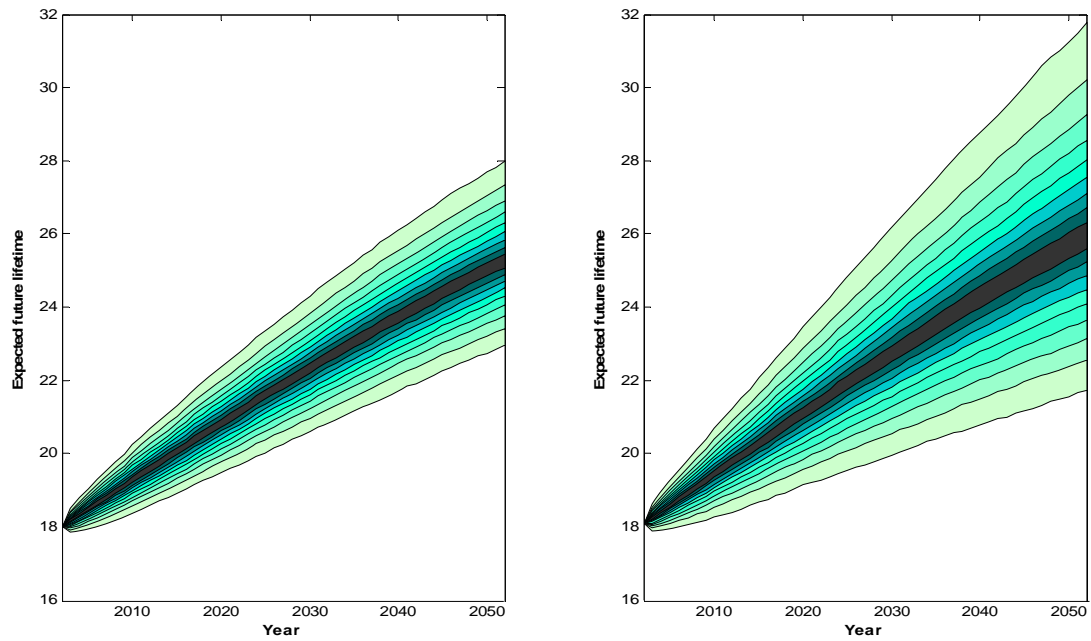
**ONWARDS**



Notes: The chart shows the central 10% prediction interval for expected future lifetime with the heaviest shading, surrounded by the 20%, 30%, ..., 90% prediction intervals with progressively lighter shading. The dots are estimates of subsequently realized expected future lifetimes. Estimates are based on the mortality model of Cairns *et alia* (3) calibrated on GAD data over the period 1962-1982.

**FIGURE 3: LONGEVITY FAN CHART FOR 65-YEAR OLD MALES: 2002**

**ONWARDS**



Notes: The chart shows the central 10% prediction interval for expected future lifetime with the heaviest shading, surrounded by the 20%, 30%, ..., 90% prediction intervals with progressively lighter shading. Estimates are based on the mortality model of Cairns *et alia* (3) calibrated on GAD data over the period 1982-2002.

## APPENDIX: A MODEL OF EXPECTED LONGEVITY

The longevity fan charts are based on an underlying mortality model set out by Cairns *et alia* (2). Let  $q(t, x)$  be the realized mortality rate in year  $t+1$  (that is, from time  $t$  to time  $t+1$ ) of a cohort aged  $x$  at time 0. We assume that  $q(t, x)$  is governed by the following two-factor Perks stochastic process:

$$(1) \quad q(t, x) = \frac{\exp[A_1(t+1) + (t+x)A_2(t+1)]}{1 + \exp[A_1(t+1) + (t+x)A_2(t+1)]}$$

where  $A_1(t+1)$  and  $A_2(t+1)$  are themselves stochastic processes that are measurable at time  $t+1$  ((1), (3)). Now let  $A(t) = (A_1(t), A_2(t))'$  and assume that  $A(t)$  is a random walk with drift:

$$(2) \quad A(t+1) = A(t) + \mu + CZ(t+1)$$

where  $\mu$  is a constant  $2 \times 1$  vector of drift parameters,  $C$  is a constant  $2 \times 2$  lower triangular matrix reflecting volatilities and correlations, and  $Z(t)$  is a  $2 \times 1$  vector of independent standard normal variables. Cairns *et alia* (1) show that this model provides a good fit to UK Government Actuary's Department (GAD) data for English and Welsh males over 1961-2002.

At time  $t$ , we wish to estimate the expected future lifetime for a male aged 65 at that time, which we denote by  $EFL(t)$ .  $EFL(t)$  is a function of  $A_1(t)$  and  $A_2(t)$  with a first approximation given by

$$(3) \quad EFL(t) \approx \frac{1}{2} + \sum_{u=t+1}^{\infty} E[S(u)/S(t)|A_1(t), A_2(t)]$$

where  $E[S(u)/S(t)|A_1(t), A_2(t)]$  is the expected value of  $S(u)/S(t)$  conditional on the values of  $A_1(t)$  and  $A_2(t)$ , and  $S(u)$  is the proportion of males aged 65 at time  $t$  who are still alive at time  $u$ .

For the model defined in equations (1) and (2),  $EFL(t)$  cannot be evaluated analytically. However, a good approximation to  $EFL(t)$  is given by

$$(4) \quad EFL(t) \approx \exp[b_1 + b_2 A_1(t) + b_3 A_2(t) + b_4 A_1(t)^2 + b_5 A_2(t)^2 + b_6 A_1(t)A_2(t)]$$

where the values of the parameters  $b_1, \dots, b_6$  reflect the estimates of  $\mu$  and  $C$  and the inclusion or exclusion of parameter uncertainty in our projections. This approximation was found to work well over a wide range of values for  $A_1(t)$  and  $A_2(t)$  including those that the process is likely to take over the next 50 years.

The parameters of the model were calibrated using estimates obtained for the relevant periods using GAD data for 65-year old English and Welsh males. (The actual parameter values are given below in the Figures.) For each set of parameter values, we simulated 10000 paths of  $A_1(t+1)$  and  $A_2(t+1)$  and used these in (4) to obtain 10000 simulated paths of  $EFL(t)$  over the chosen horizons. For each given  $t$ , the quantiles of  $EFL(t)$  were then obtained from the relevant order statistics of

our ‘sample’ of  $EFL(t)$  values. These quantiles give us the bounds of the fan chart intervals.

Approximation (4) is also used to produce estimates of the realized future life expectancies plotted in Figure 2 as dots. To do so, the parameters  $b_1, \dots, b_6$  are fixed at their 1982 values, and then for each year 1983, ..., 2002 the realized values of  $A_1(t)$  and  $A_2(t)$  are applied to approximation (4). The resulting estimates of EFL can then be regarded ‘as if’ they are the subsequently realized EFL values.

## Figures

The Figures are based on the model calibrated to the following parameter values. Note that the matrix  $C$  is the lower triangular Choleski square root matrix of the matrix  $V$  given below.

*Parameters values for Figure 2 (parameters certain case):*

$$\mu = \begin{bmatrix} -0.0216256340 \\ 0.0001531348 \end{bmatrix}, \quad V = \begin{bmatrix} 0.014677243, -0.000227252 \\ -0.000227252, 0.000003702 \end{bmatrix}, \quad A(0) = \begin{bmatrix} -9.61251 \\ 0.094018 \end{bmatrix},$$

$b_1=0.83697764$ ,  $b_2=-0.87709314$ ,  $b_3=-64.1062208$ ,  $b_4=-0.04702272$ ,  $b_5=-145.6145114$  and  $b_6=-5.56713193$ .

*Parameters values for Figure 2 (parameters uncertain case):*

$$\mu = \begin{bmatrix} -0.0216256340 \\ 0.0001531348 \end{bmatrix}, V = \begin{bmatrix} 0.014677243, -0.000227252 \\ -0.000227252, 0.000003702 \end{bmatrix}, A(0) = \begin{bmatrix} -9.61251 \\ 0.094018 \end{bmatrix},$$

$b_1=0.81096869$ ,  $b_2=-0.88385534$ ,  $b_3=-64.11220593$ ,  $b_4=-0.04698082$ ,  $b_5=-145.61549$  and  $b_6=-5.5321877$ .

*Parameters values for Figure 3 (parameters certain case):*

$$\mu = \begin{bmatrix} -0.0668961480 \\ 0.0005904540 \end{bmatrix}, V = \begin{bmatrix} 0.006114509, -0.0000939164 \\ -0.0000939164, 0.00000150933 \end{bmatrix},$$

$$A(0) = \begin{bmatrix} -10.95043 \\ 0.10582754 \end{bmatrix}, b_1=0.9415389, b_2=-0.8710403, b_3=-64.0888082, b_4=-0.0477268,$$

$b_5=-145.6119686$  and  $b_6=-5.6495324$ .

*Parameters values for Figure 3 (parameters uncertain case):*

$$\mu = \begin{bmatrix} -0.0668961480 \\ 0.0005904540 \end{bmatrix}, V = \begin{bmatrix} 0.006114509, -0.0000939164 \\ -0.0000939164, 0.00000150933 \end{bmatrix},$$

$$A(0) = \begin{bmatrix} 10.95043 \\ 0.10582754 \end{bmatrix}, b_1=0.94212786, b_2=-0.87709228, b_3=-64.65815785, b_4=-0.04832639,$$

$b_5=-149.3423055$  and  $b_6=-5.74534089$ .