To the Future and Back: A Framework for Understanding the Role of Time Preference in the Personal Valuation of Financial Investments

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TO THE FUTURE AND BACK: A FRAMEWORK FOR UNDERSTANDING THE ROLE OF TIME PREFERENCE IN THE PERSONAL VALUATION OF FINANCIAL INVESTMENTS

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ABSTRACT

We use temporal value charts to construct a conceptual framework for understanding how time preference influences the personal valuation of financial investments. In a series of thought investments, an initial value is projected forward to the future and that future value is discounted back to the present. We focus on exponential and hyperbolic valuation. We explain how the personal characteristics and identities of the temporal selves that project and discount can influence time preference and how, in turn, time preference influences personal valuation. Present value equals initial value when there is temporal neutrality (or emmetropia); present value is less than initial value when there is positive time preference (or myopia); and present value exceeds initial value when there is negative time preference (or hyperopia). $1 thought invested today for 30 years might, for example, be worth: $1 today when valued emmetropically; $0.57 when valued myopically; and £1.77 when valued hyperopically.

JEL Classification: G02, D90
PsycINFO classification: 2100

Keywords: Behavioural economics; Cognitive psychology; Time preference; Discount rates; Identity

HIGHLIGHTS

- We explore the psychology underlying the mental simulation of value change over time.
- We focus on exponential and hyperbolic valuation.
- We trace the projection of future value and its discounting back to the present.
- We explain how time preference influences valuation.

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1. INTRODUCTION

This paper was motivated by a discussion at a conference that one of the authors attended some years ago about whether $1 worth of equities is worth more than, less than or the same as $1 worth of bonds. Valuing financial investments, such as equities and bonds, involves mental time travel – to the future and back. In this paper, we examine the psychology underlying the mental simulation of temporal changes in value. We do so by constructing a framework for understanding how time preference influences such personal valuation. The framework comprises an integrated series of “thought investments” in which: today’s value is projected into a future value; and that future value is discounted back to a present value.¹

Chart 1 illustrates two methods by which $1 saved today might be perceived to grow over the next 30 years. In Chart 1, $1 is projected forward 30 years: “exponentially” to $5.74 and “hyperbolically” to $3.32.²

CHART 1: EXPONENTIAL AND HYPERBOLIC GROWTH

![Chart 1: Exponential and Hyperbolic Growth](image)


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¹ Where “thought investment” is used in the same sense as is “thought experiment”.
² We use the compound interest formula \((1+r)^t\) to define the exponential growth function for a \(t\)-year investment horizon. We use the formula \((1+αt)^{γ/α}\), where \(γ\) and \(α\) are constants, to define the hyperbolic growth function (Loewenstein and Prelec, 1992, Laibson, 1998). Chart 1 assumes that \(r = 6\%\) and that \(γ = 1\) and \(α = 4\).
Exponential growth is standard in time value of money calculations with growth rates being postulated not to vary today, tomorrow or, indeed, many years from now. Exponential savers foresee ever-increasing returns, thanks to the benefits of compounding. In contrast, according to Mitchell and Utkus (2004), hyperbolic savers expect $1 saved to grow more rapidly in the short-term than in the long-term. So, they perceive decreasing benefits to long-term savings – rewards are expected to accelerate quickly and then taper off.

The thought investments that follow build on the projections in Chart 1 by discounting these future values back to today's date. As when projecting, economists typically assume that discount functions are exponential (Angeletos et al., 2001), whereas a growing body of experimental evidence suggests that hyperbolic discounting – in which a future value declines at a more rapid rate in the short-term than the long-term – better describes how many of us value delayed rewards (Thaler, 1981, Frederick et al., 2002, Laibson, 2003, Ainslie, 2005 and Berns et al., 2007).

For each of these two methods (exponential and hyperbolic), we consider three expressions of time preference: "emmetropia", in section 2; "myopia", in section 3; and "hyperopia", in section 4.

Emmetropic valuation is temporally neutral – in other words, there is no past, present or future time bias or preference. Temporal neutrality assigns no normative significance per se to the temporal location of benefits and harms within a person's life and demands equal concern for all parts of that life (Brink, 2011). Brink traces the concept of temporal neutrality back to the Epicureans who believed in a symmetry between our pre-natal and post-mortem non-existence – the past time before our birth being a mirror-image of the time to come after our death (Lucretius, cited by Brink, 2011).

We are not always temporally neutral – indifferent about the timing of benefits and harms (Suhler and Callender, 2012). Sometimes we have positive time preference – we prefer present pleasures (and future pain) to future pleasures (and present pain) of a similar magnitude. We are myopic. Sometimes, in contrast, we are hyperopic. We value the future over the present – we have negative time preference.

The Epicureans' presumption of non-existence post-mortem (their belief that the soul dies with the body) imposes a finite time horizon on our existence and, as Olson and Bailey (1981) argue, a finite time horizon is consistent with positive time preference. Negative time preference, on the other hand, has been linked to a desire for improving sequences of outcomes which, in turn, can be attributed partly to anticipation and partly to loss aversion (Loewenstein and Prelec, 1991). Pleasure from anticipation is enhanced if the best comes last, while loss aversion induces a preference for increasing values (gains) over time. Negative time preference is consistent with an extended or infinite time horizon (such as that needed to accommodate a family dynasty or a belief in post-mortem existence).

To explore the psychology underlying these thought investments, we exploit the concept of a life as a series of psychologically connected temporal selves. This is a notion developed by philosophers, such as Parfit (1971, 1982 and 1984), and explored by economists and behavioural scientists, such as

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3 These are the two methods that feature most prominently in the time preference research literature but there are, of course, others – Doyle (2013), for example, identifies over twenty methods.

4 This “symmetry argument” presumes non-existence both before and after life.
Strotz (1955-56) and Frederick (2003 and 2006). The psychological connections at the core of this concept include our memories, personal characteristics and interests all of which contribute to the make-up of our personal identity. In the thought investments discussed below, a Present Self mentally simulates value forward (“prospection”) and a Future Self (or, to be more exact, the Present Self’s projection of its Future Self) mentally simulates value backwards (“retrospection”). So, the direction of travel is forward to the future along the growth curve and back to the present along the discount curve. Prospection requires the Present Self to mentally visit the future; while retrospection requires the Projected Future Self to mentally return to the present (its past). This mental time journey begins and ends at the present time.

Mental time travel allows us to mentally re-live our past or pre-live our future (Suddendorf and Corballis, 2007). Mental time travel theory has traditionally viewed prospection as identical to retrospection, save for the direction of travel (Van Boven et al., 2008). Temporal value research has traditionally taken the same view. Chart 1, for example, is reproduced from Mitchell and Utkus (2004) who entitle their chart “Exponential versus Hyperbolic Discounters: Growth of $1 Over Time”. This heading implies that individuals use the same method to discount as they do to project. Recent research, however, reveals that, for many of us, the ways in which we mentally represent the future and the past are not the same (Kane et al., 2012). Prospection differs, in context and experience, from retrospection (Van Boven et al., 2008). Some researchers acknowledge that projecting and discounting might be psychologically distinct. Doyle (2013), for example, recognises that, while the formulae underlying projecting and discounting might be mathematically equivalent, their underlying mental operations might differ psychologically. He advocates further research to determine whether people adopt a discounting view or a projecting view of intertemporal problems, or indeed, some other view. Our paper offers one such other view – a framework illustrating how projecting and discounting might be combined together to provide personal valuations of financial investments.

As with the chart in Mitchell and Utkus (2004), the specific numbers used in this paper are arbitrary and are chosen to illustrate the different valuation methods – it is not any specific growth rate or discount rate that matters but, rather, the differences between these rates. To keep things simple, we assume a positive expected return – in the form of capital appreciation rather than income – on all thought investments and no inflation over the period. Because it is the differences between temporal value curves, rather than the curves themselves, that matter, we hold the growth curve fixed, for the most part, and allow the discount curve to vary.

2. EMMETROPIC VALUATION

When valuing emmetropically, we use a discount rate that is equal to the growth rate over the valuation period. So, present value equals initial value.

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5 He notes too, analogously, that while $8! = 1·2·3·4·5·6·7·8 = 8·7·6·5·4·3·2·1$, those adopting the former definition (involving an ascending sequence of numbers) tend to give lower estimates of the value of $8!$ than if they are presented with the mathematically equivalent descending sequence definition (Doyle, 2013, citing Tversky and Kahneman, 1974).

6 Fixed, or standard, exponential and hyperbolic growth curves are apposite given that recent research reveals mental time travel to the future to be more prototypical than that back to the past (Kane et al., 2012).
Chart 2 illustrates emmetropic exponential valuation. In this example, $1 is projected forward, at a compound annual growth rate of 6%, to a value of $5.74 and that future value is then exponentially discounted, at an annual rate of 6%, to a present value of $1. So, $1, invested today at 6%, is valued at $5.74 in 30 years' time; and $5.74 in 30 years' time at $1 today. By implication, $1 today is valued at $1.

**CHART 2: EMMETROPIC EXPONENTIAL TEMPORAL VALUE CURVES**

Emmetropic exponential valuation is symmetric. It looks forwards and backwards in time in the same way – discounting is the exact temporal inverse (or reciprocal) of projecting. But, as Chart 3, dealing with the case of emmetropic hyperbolic valuation, shows, a reciprocal discount function is an insufficient condition for symmetry. In the chart, $1 today is projected to grow hyperbolically to $3.32 in 30 years' time; and that future value is then hyperbolically discounted to a present value of $1. So, $1 today is valued at $3.32 in 30 years' time; and $3.32 in 30 years' time at $1 today. Again, by implication, $1 is valued today at $1. Although the function underlying the discount curve in Chart 3 is the reciprocal of that of the growth curve, the discount curve is not a mirror image of the growth curve. Nevertheless, there is what we might call a “distorted” symmetry.

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7 We use the inverted compound interest formula \((1+R)^t\) to define the exponential discount function. Here, \(R\) is the discount rate which, when valuing emmetropically, equals the growth rate \(r\). Chart 2 assumes that \(r=R=6\%\).

8 We use the formula \((1+\alpha t)^{-\gamma/\alpha}\) to define the hyperbolic discount function (Loewenstein and Prelec, 1992, Laibson, 1998).
As Chart 4 illustrates, when valued emmetropically (whether symmetrically or otherwise), the present value of $1 thought invested in, for example, equities for, say, 30 years is the same today as $1 invested in bonds for, say, 20 years, namely $1, even though equities are riskier, offer a higher expected return and are held for a longer period than bonds.
Emmetropic valuation occurs, too, whenever hybrid valuation produces a discount rate equal to the growth rate over the valuation period. Chart 5 provides one such example of emmetropic hybrid valuation: exponential growth combined with hyperbolic discounting. In Chart 5, $1 today is again projected to grow exponentially to $5.74 in 30 years’ time; and that future value is then hyperbolically discounted to a present value of $1. Again, by implication, $1 is valued today at $1. However, unlike the previous examples, present value here is not indifferent to the length of the investment horizon. In this case, assuming no change in the growth rate and discount rate formulae, any reduction in the length of the investment period will result in myopia (present value will be less than initial value), whereas any increase in the investment horizon will cause the temporal value curves to cross over, resulting in hyperopia. We explore myopic valuation and hyperopic valuation in the next two sections.

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9 An alternate form of hybrid valuation would be hyperbolic projection combined with exponential discounting. With this later combination, present value decreases as the length of the valuation period increases.
3. MYOPIC VALUATION

When valuing myopically, we use a discount rate that is higher than the growth rate over the valuation period, i.e., we discount more heavily than when valuing emmetropically. So, present value is less than initial value.

Chart 6 illustrates myopic exponential and hyperbolic valuation. In the first case, $1 is exponentially projected forward at a growth rate of 6% to a value of $5.74 and that future value is then exponentially discounted, at a rate of 8%, to a present value of $0.57. So, $1 today is valued at $5.74 in 30 years’ time; and $5.74 in 30 years’ time at $0.57 today. In the second case, $1 today is projected to grow hyperbolically to $3.32 in 30 years’ time and that future value is then hyperbolically discounted to a present value of $0.57.\(^\text{10}\) So, $1 today is valued at $3.32 in 30 years’ time; and $3.32 in 30 years’ time at $0.57 today.

\(^{10}\) We set \(y = 1\) and \(\alpha = 4\) for the growth function; and \(y = 1\) and \(\alpha = 2.34\) for the discount function.
We consider five possible explanations for myopic valuation. The first is the underestimation of the growth curve. The remaining four relate to discounting.

### 3.1 Short-sightedness

Some of us suffer from inadequacies in our “ability to imagine the future” (Loewenstein, 1992). We are short-sighted (Elster, 1979, cited by Bartels and Urminsky, 2011). It is as if we have a “faulty telescopic faculty” (Pigou, 1920, cited by Frederick et al., 2002). An inability to imagine the future might cause the Present Self to underestimate the growth rate. If the Projected Future Self does not share this faulty faculty, the discount rate might exceed the growth rate.

If the growth rate is not underestimated, however, we must look for other reasons for the mismatch in temporal value curves. If discounting is multi-causal or “multiply-determined” (Bartels and Urminsky, 2011, Berns et al., 2007), we should look for a number of explanations for this asymmetric valuation. We explore four key ones: impatience; existential uncertainty; risk aversion; and loss aversion.
3.2 Impatience

Some of us experience psychological discomfort when asked to defer immediate gratification, even for a short period. We are impatient (Loewenstein, 1992, Olson and Bailey, 1981).

3.3 Existential uncertainty

We noted earlier that a finite time horizon is consistent with positive time preference (Olson and Bailey, 1981). So is uncertainty as to the length of the horizon. A finite time horizon of uncertain length is, of course, a defining feature of human life. The “brevity and uncertainty of human life” (Loewenstein, 1992) means that the Present Self cannot be sure that the Future Self will exist. So, the Projected Future Self cannot be sure that the Future Self exists. If the Projected Future Self believes that the Future Self might not exist to enjoy future values, it might discount them. Indeed, it might discount them hyperbolically – heavily at first and then at a reducing rate as the present approaches and confidence in the continued existence of the self increases.

3.4 Risk aversion

Many of us are risk averse (Andersen et. al., 2008). We discount the expected value of uncertain gains. We prefer a small certain reward to a large uncertain reward of equal expected value (Kahneman and Tversky, 1984). If a risk averse individual perceives equities to be riskier than bonds, $1 worth of equities might be valued at less than $1 worth of bonds.

Risk aversion cannot explain the myopic valuation of riskless investments, unless, of course, even a riskless investment is perceived to have an uncertain value psychologically.¹¹ Myopic valuation of a riskless investment is more likely to be a characteristic of risk-seeking individuals. This explains, too, the use of high discount rates by gamblers and those with risk-seeking behaviours such as alcohol and drug abuse (Chabris et. al., 2008). These are people whose temporal selves are loosely connected – their Present and Future Selves care little for each other.

3.5 Loss aversion

Faced with a choice between a small certain loss and a large uncertain loss of equal expected value, many of us prefer the gamble. Khaneman and Tversky call this tendency, to be risk averse for sure gains and risk seeking to avoid sure losses, the “certainty effect”: we underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty (Kahneman and Tversky, 1979). The certainty effect is encapsulated in “Prospect Theory” (Kahneman and Tversky, 1979) which proposes that value (see Chart 7) is defined by gains and losses in relation to a reference level of wealth and is concave in the domain of gains, due to risk aversion; convex in the domain of losses, as a result of risk seeking; sharply kinked at the reference point; and loss-averse – steeper for losses than for gains by a factor in the range 2-2.5.

¹¹ If the bonds are perceived to be risk-free, they might be valued emmetropically or, even, hyperopically.
How can Prospect Theory explain the use of a discount rate higher than the growth rate? The projection of value (as represented by the growth curve) feels like a loss to the Present Self and a gain for the Future Self. If the growth curve represents a gain for the Future Self, the discount curve must represent a loss of value for it. It feels losses more than gains so, if it suffers from loss aversion, the Projected Future Self will discount at a rate higher than the growth rate.

There are parallels here with the hyperbolic growth curve, in Chart 6, which is concave, like the Prospect Theory gains curve, and with the hyperbolic discount curve, also in Chart 6, which is convex, like the Prospect Theory losses curve. Further, the discount curve is steeper than the growth curve, just as the Prospect Theory losses curve is steeper than its gains curve. These parallels are so compelling that, the behavioral scientist, Caruso and his colleagues recommend that hyperbolic valuation be unified in a three-dimensional value function together with Prospect Theory’s gain-loss asymmetry and what they term “temporal value asymmetry” – the tendency, of many of us, to value future events more highly than equivalent events in the equidistant past (Caruso et al., 2008, D’Argembeau and van der Linden, 2004).

4. HYPEROPIC VALUATION

When we value hyperopically, we use a discount rate lower than the growth rate, i.e., we discount less heavily than when valuing emmetropically. So, present value exceeds initial value.
CHART 8: HYPEROPIC TEMPORAL VALUE CURVES SHOWING BOTH EXPONENTIAL AND HYPERBOLIC GROWTH AND DISCOUNTING

Chart 8 illustrates hyperopic valuation, both exponential and hyperbolic. In the first case, $1 is projected forward exponentially at a growth rate of 6% to a value of $5.74 and then that future value is exponentially discounted, at a rate of 4%, to a present value of $1.77. So, $1 today is valued at $5.74 in 30 years’ time; and $5.74 in 30 years’ time at $1.77 today. In the second case, $1 today is projected to grow hyperbolically to $3.32 in 30 years’ time and that future value is then hyperbolically discounted to a present value of $1.77.\(^\text{12}\) So, $1 today is valued at $3.32 in 30 years’ time; and $3.32 in 30 years’ time at $1.77 today.

What might cause hyperopic valuation? Short-sightedness might cause an individual to overestimate, rather than underestimate, the growth rate. But hyperopia is more typically associated with far-sightedness. First, some of us have a strong desire for accumulation (Loewenstein, 1992). We have self-control and can resist, or abstain from, immediate gratification (Keinan and Kivetz, 2008). Second, some of us obtain pleasure from anticipating the future and its rewards, including a “bequest motive” (Loewenstein, 1992). A spirit of altruism means enjoying giving for its own sake (Cowen and Parfit, 1992). Such altruism can be personally motivated, as by the desire to build a family dynasty, or socially motivated, as by a concern for inter-generational welfare and for the protection of the environment. In short, some of us enjoy wealth for its own sake. Either, or both, of these factors might cause us to discount a future value more lightly than its projected growth rate.

It might be expected that an individual who discounts at a rate lower than the growth rate does so because its temporal selves are strongly connected. Indeed, research confirms that a strong

\(^{12}\) We set \(y = 1\) and \(\alpha = 4\) for the hyperbolic growth function; and \(y = 1\) and \(\alpha = 8.9\) for the hyperbolic discount function.
psychological connection between the temporal selves facilitates saving – individuals who anticipate that their future personal identity will overlap considerably with their current identity tend to accumulate more financial assets than do those who sense little such overlap (Hershfield et al., 2009). If the Projected Future Self cares for the Present Self it might discount lightly the Present Self’s future.

We might expect, too, that an individual who discounts hyperopically would not suffer existential uncertainty, risk aversion or loss aversion but, instead, might be more of a risk seeker. An appetite for risk might cause risky investments to be discounted less heavily than safe ones in which case $1 worth of equities might be valued at more than $1 worth of bonds

Uncertainty about the future can, however, be “double-edged”: it can cause the future to be valued more than the present, as well as less (Olson and Bailey, 1981). Risk aversion, for example, can encourage “saving for a rainy day” (Olson and Bailey, 1981); existential uncertainty can incentivise financial provision for an unexpectedly long life; and, as mentioned earlier, loss aversion is one cause of negative time preference.

6. CONCLUSION

In this paper, we propose an approach to the personal valuation of financial investments based on the concept of mental time travel. We take the exponential and hyperbolic growth curves, in Figure 1 of Mitchell and Utkus (2004), as building blocks with which to construct a framework for understanding how time preference might influence this journey to the future and back. Because prospection and retrospection are psychologically distinct, since our personal characteristics can have different effects on the growth and discount curves, present value in our thought investments is not necessarily the same as initial value. Sometimes it is the same, but sometimes it is higher, sometimes lower.

Empirical time preference studies typically invite subjects to choose between smaller sooner values and larger later values. The researchers can then infer from the choices made personal discount rates and methods (Coller and Williams, 1999, Anderson et. al., 2008, Chabris et. al., 2008). A personal discount rate is deemed to be a measure of an individual’s time preference (Frederick et al., 2002, Chabris, 2008) and the method used an indicator of the consistency of time preference over time. The higher the personal discount rate, the greater is deemed to be the preference for the present. It is implicitly assumed that, when making financial decisions, individuals compare their personal discount rates to a potential investment’s projected rate of return which is determined independently of the discount rate (Doyle, 2013). In contrast, our framework highlights the interdependence of growth and discount rates and illustrates how projecting and discounting might combine in a single personal valuation process.

Emmetropic valuation is temporally neutral. It underlies the financial economics’ principle that, because the expected cashflows from a financial asset discounted at a risk-adjusted rate equals the price of that asset, $1 of cash has the same value as $1 of equities or $1 of bonds (Ralph, 2001). Our myopic and hyperbolic valuation thought experiments illustrate why, in contrast, when offered a choice between a box containing $1 of equities at today’s prices and another containing $1 of bonds, neither to be opened for 30 years, many of us are not indifferent (Jackson, 2006).
We suggest that the attributes which determine positive time preference include short-sightedness, loose psychological connections between the temporal selves, existential uncertainty, risk aversion, and loss aversion, while those that determine negative time preference include self-control, altruism and an appetite for risk. These lists are not exhaustive, nor are these attributes necessarily mutually exclusive. Their interaction determines time preference. For example, an individual who is altruistic might also be risk averse (valuing $1 worth of risky equities at less than $1 worth of risk-free bonds); and an individual with loose psychological connections might also be risk-seeking (valuing equities more than bonds).

Exponential and hyperbolic valuation methods are not mutually exclusive either – they can be combined in a hybrid valuation approach. Our hybrid valuation thought investment (see Chart 5) illustrates how value can depend upon the duration of an investment – value fluctuating from, for example, myopia to hyperopia as the length of the investment period increases. If the financial markets project exponentially, as in Chart 5, individuals who discount hyperbolically will value $1 worth of short-term securities at less than $1 worth of long-term securities. Conversely, if most people demand higher rates of return for short investment periods than for long ones, those who discount the future at a constant rate will value short-term securities over long-term securities (Lowenstein and Prelec, 1992).

Such preferences might be a feature, too, of purely exponential or hyperbolic valuation – if, for example, equities are perceived to be risky in the short-term but safer over the long-term, a risk-averse individual might value them myopically over a short investment period but hyperbolically over a longer horizon. Research by Laury et. al., (2012) suggests, however, that, while subjects will increasingly prefer a larger, later value over a sooner, smaller value as the later value increases, a small percentage of us always prefer the sooner value and a similar proportion always prefer the later value.

The objective of this paper has been to set out a purely conceptual framework explaining how time preference might influence the personal valuation of financial investments. Although our framework is constructed directly from the time preference literature, we have not sought, in this paper, to conduct empirical experiments to validate our thought investments and to explore, in particular, the differences in personal projection and discounting methods and rates identified in the cited literature. We leave that to future work. Nevertheless, we know that in reality personal valuations along the lines discussed in this paper do happen. At the conference one of the authors attended all those years ago, it was quite clear that practising actuaries at that time valued equities hyperbolically: they could quite happily switch a pension fund’s bond holdings, valued at $100 million, into equities and value those equities at $120 million.

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