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Layout & Distribution: Valeria Kozakova

Demographic Issues in Longevity Risk Analysis

Eric Stallard

Objectives

To identify demographic issues in longevity risk analysis relating to measurement and modeling of survival and mortality.

Resolution of such issues is necessary to allow survivor bonds to have coupons that are proportional to the number of survivors at each future date.

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Measurement

Accurate measurement of

- **The inputs to the various models used for forecasting future survival and mortality**

is necessary for ensuring

- **The validity of the outputs of those models**

3

Measurement

Minimal requirements:

- **The initial size and defining characteristics of each cohort**
- **Decrements due to death, emigration, or other censoring events**
- **Increments due to immigration or other forms of cohort recruitment**

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Long-Term vs. Short-Term Trends

Long-term: Mortality declined at about the same rate as disability (~0.6% per year) for older males during 1910-1991 (Costa 2000, 2002)

Long-term: Decline in chronic disease prevalence for older males during 1910-1991 at similar rate (~0.6% per year; Fogel and Costa, 1997)

Short-term: Mortality declined ~1.0% per year slower than disability 1984-1999

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Table 1: Age-Adjusted Central Death Rates at Ages 65 Years and Older (per 100,000)

	Year				
Gender	1900	1910	1990	1999	Annual Rate of Decline
Male	10,612			6,154	0.55%
Female	9,749			4,157	0.86%
Total	10,079			4,898	0.73%
Male		10,444	6,526		0.59%
Female		9,606	4,055		1.07%
Total		9,937	4,986		0.86%

Source: Bell and Miller (2002, Table 1).

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Table 2: Prevalence of Chronic Conditions Among Elderly Male Veterans Aged 65 Years and Older (%)

Condition	Union Army 1910	NHIS 1985-1988	Annual Rate of Decline
Digestive (Hernia/Diarrhea)	84.0	8.0	3.03%
Genito-Urinary	27.3	8.9	1.45%
Circulatory	90.1	40.0	1.06%
CNS, Endocrine, Metabolic, or Blood Disorders	24.2	12.6	0.85%
Musculoskeletal	67.7	42.5	0.61%
Respiratory	42.2	26.5	0.61%
Cancer	2.2	9.2	-1.89%

Source: Fogel and Costa (1997, Table 3).

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Table 3: Prevalence of Chronic Conditions Among Elderly Male Veterans Aged 60-74 Years (%)

Condition	Union Army 1910	NHANES 1985-1988	Annual Rate of Decline
Heart murmur	39.2	3.8	3.00%
Irregular pulse	42.0	8.5	2.07%
Decreased breath or adventitious sounds	37.8	10.8	1.62%
Joint pain/tenderness/swelling	55.0	35.2	0.58%

Source: Costa (2000, Table 1).

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Table 6: Life Expectancy at Ages 65 and 75, United States

Year	Unisex	Males	Females
	At Age 65		
1960	14.3	12.8	15.8
1970	15.2	13.1	17.0
1980	16.4	14.1	18.3
1990	17.2	15.1	18.9
2000	17.9	16.3	19.2
2002	18.2	16.6	19.5
Rate (% per yr.; 42 yr.)	0.58%	0.62%	0.50%
Rate (% per yr.; 22 yr.)	0.47%	0.74%	0.29%
	At Age 75		
1980	10.4	8.8	11.5
1990	10.9	9.4	12.0
2000	11.3	10.1	12.1
2002	11.6	10.4	12.5
Rate (% per yr.; 22 yr.)	0.50%	0.76%	0.38%

Source: NCHS (2003, Table 27); Kochanek et al. (2004, Table 6). 9

Table 7: Unisex Life Expectancy, HIPAA ADL Expectancy, and LTC Institutional Expectancy (in Years at Age 65), United States 1984 and 1999

	Year		
At Age 65	1984	1999	Annual Rate of Change
Life Expectancy	16.64	17.40	0.30%
HIPAA ADL Expectancy	1.86	1.57	-1.13%
LTC Institutional Expectancy	1.11	0.83	-1.94%
LTC Inst/ALF Expectancy	1.11	0.94	-1.09%

Source: Author's calculations based on NLTCS and life tables from Bell and Miller (2002).

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Change in Disabled Life Expectancy Beyond Age *x*

$$e_{Dx,y} - e_{Dx,y_0} = \int_0^\infty \left({}_t p_{x,y}\ \pi_{x+t,y} - {}_t p_{x,y_0}\ \pi_{x+t,y_0} \right) dt$$

$$= \int_0^\infty \left({}_t p_{x,y} - {}_t p_{x,y_0} \right) \pi_{x+t,y_0}\ dt \qquad \text{Survival Increment}$$

$$- \int_0^\infty {}_t p_{x,y} \left(\pi_{x+t,y_0} - \pi_{x+t,y} \right) dt \quad \text{Morbidity Decrement}$$

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Table 8: Components of Change in Unisex Life Expectancy, HIPAA ADL Expectancy, and LTC Institutional Expectancy (in Years at Age 65), United States 1984 and 1999

	Year				
At Age 65	1984	1999	Change	Survival Increment	Morbidity Decrement
Life Expectancy	16.64	17.40	0.76	0.76	-
HIPAA ADL Expectancy	1.86	1.57	-0.29	0.13	-0.42
LTC Institutional Expectancy	1.11	0.83	-0.28	0.09	-0.37
LTC Inst/ALF Expectancy	1.11	0.94	-0.17	0.09	-0.25

Source: Author's calculations based on NLTCS and life tables from Bell and Miller (2002).

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Why is Mortality Declining?

- **Improvements in physical health**
- **Improvements in cognitive health**
- **Improvements in diagnosis and treatment of chronic and disabling illnesses**
- **Innovations in preventive medicine**
- **Pharmaceutical innovation**

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Why is Mortality Declining?

- **Improvements in health-related behaviors**
- **Decreases in hazardous exposures**
- **Improved levels of education**
- **Improved socioeconomic status**

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Factors Affecting Retirement Mortality (FARM)

Age
Alcohol
Education
Gender
Health Behaviors
Income
Marital Status
Obesity
Occupation
Race & Ethnicity
Religion
Smoking

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Quantitative Estimates 1900-1990

- **50% of decline in mortality predicted by changes in height and weight (Fogel and Costa, 1997)**
- **29% of decline in chronic diseases due to occupational shifts (Costa, 2000)**
- **18% of decline in chronic diseases due to declines in infectious diseases (Costa, 2000)**

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Quantitative Estimates 1900-1990

- **37% of decline in functional limitation due to reduced prevalence of chronic diseases (Costa, 2002)**
- **24% of decline in functional limitation due to reduced debilitating effects of chronic diseases (Costa, 2002)**

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Mortality Changes 1960-2000

- **Increases in life expectancy at age 65 due to decreases in death rates from heart and cerebrovascular diseases**
- **Cancer death rates increased, but not enough to offset heart and cerebrovascular decreases**

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Mortality Changes 1960-2000

- **Cancer death rates peaked for males during 1990s (female peak expected 10-20 years after)**
- **Residual causes at age 85+ increased from 27% to 40% of deaths as progress against top 3 causes left "void"**

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Chronic Disease Risk Factor Changes 1960-2000

- **Substantial declines in**
  - **cigarette smoking**
  - **hypertension**
  - **serum cholesterol**
- **Substantial increases in obesity, but not enough to offset favorable declines in other risk factors**

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Is there a Dynamic Equilibrium in Population Health and Survival?

Mortality, morbidity, and disability reflect different aspects of a common, underlying health process; future changes in these three components will be linked.

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Forecasting Models

Several models based on generalizations of the basic life table are considered for use in forecasting future survival and mortality:

- **Lee-Carter Model**
- **Frailty Model**
- **Random Walk Model(s)**

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Lee-Carter Model

$$\ln\left(m_{a,t}\right) = a_a + b_a k_t + e_{a,t},$$

where

$$k_t = k_{t-1} + u + \xi_t$$

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Lee-Carter Model

$$\ln\left(m_{a,t+s}\right) = a_a + b_a\left(k_t + su + \sum \xi_s\right) + e_{a,t}$$

$$\begin{aligned}\operatorname{var}\left(\ln\left(\hat{m}_{a,t+s}\right)\right) = {} & \operatorname{var}(\hat{a}_a) + b_a^2 \operatorname{var}(s\hat{u} + \sum_{r=1}^{s} \xi_r) \\ & + (k_t + su)^2 \operatorname{var}(\hat{b}_a) \\ & + \operatorname{var}(\hat{b}_a) \operatorname{var}(s\hat{u} + \sum_{r=1}^{s} \xi_r) + \operatorname{var}(e_{a,t+s})\end{aligned}$$

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Frailty Model

$$\mu(a \mid X = x) = \mu(x, a) = x \times \mu(1, a)$$

$$l_a / l_0 = S(a) = \exp\left(-\int_0^a \mathrm{E}\left(\mu(X, s) \middle| A > s \right) ds \right)$$

$$\mu(a) = -\frac{d \ln S(a)}{da} = \mathrm{E}(\mu(a \mid X) \mid A > a)$$

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Random Walk Equations

$$d\mathbf{x}_a = u(\mathbf{x}_a, a)dt + d\xi(\mathbf{x}_a, a)$$

$$dS(a \mid \{X_s(w), 0 \le s \le a\}) = -\mu(\mathbf{x}_a, a) S(a \mid \{X_s(w), 0 \le s \le a\})\, da$$

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Fokker-Planck Mortality Model

$$\frac{\partial f_a(\mathbf{x}\,|\mathbf{x}_0)}{\partial a} = -\sum_j \frac{\partial[\, f_a(\mathbf{x}\,|\mathbf{x}_0) u_j(\mathbf{x},a)]}{\partial x_j} + \tfrac{1}{2}\sum_i \sum_j \frac{\partial^2[\, f_a(\mathbf{x}\,|\mathbf{x}_0)\sigma^0_{ij}(\mathbf{x},a)]}{\partial x_i \partial x_j} - \mu(\mathbf{x},a)\, f_a(\mathbf{x}\,|\mathbf{x}_0)$$

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Kolmogorov-Fokker-Planck Mortality Model

$$\frac{\partial f_a(\mathbf{x},d\,|\mathbf{x}_0,d_0)}{\partial a} = -\sum_i \frac{\partial\left[\, f_a(\mathbf{x},d\,|\mathbf{x}_0,d_0) u_{di}(\mathbf{x},a)\right]}{\partial x_i} + \frac{1}{2}\sum_i \sum_j \frac{\partial^2[\, f_a(\mathbf{x},d\,|\mathbf{x}_0,d_0)\sigma_{dij}(\mathbf{x},a)]}{\partial x_i \partial x_j} + \sum_{k\neq d} \lambda_{kd}(\mathbf{x},a)\, f_a(\mathbf{x},k\,|\mathbf{x}_0,d_0) - \left[\sum_{k\neq d} \lambda_{dk}(\mathbf{x},a)\right] f_a(\mathbf{x},d\,|\mathbf{x}_0,d_0)$$

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Linear Dynamics & Quadratic Mortality

$$\mathbf{x}_{i(a+1)} = \mathbf{u}_a + \mathbf{A}\mathbf{x}_{ia} + \mathbf{e}_{i(a+1)}$$

$$\begin{aligned}\mu_a(\mathbf{x}_{ia}) &= \mu_a(\mathbf{0}) + \mathbf{b}'_a\mathbf{x}_{ia} + \tfrac{1}{2}\mathbf{x}'_{ia}\mathbf{B}_a\mathbf{x}_{ia} \\ &= \mathbf{x}'^{*}_{ia}\mathbf{Q}_a\mathbf{x}^{*}_{ia}\end{aligned}$$

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Model Components

The impact of individual-level risk covariates and their change over age, calendar time, and cohort.

The impact of technological innovation.

The impact of limits in the rate of increase in life expectancy and its absolute value.

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Model Components

The changing nature of the mortality process as reflected in –

- **underlying-cause and**
- **multiple-cause of death data and death rates**

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Table 4 -- Ratios of Observed to Expected Age-Standardized Joint Frequencies of Multiple Causes: Unisex Mortality 1998, Age 65+

		Multiple Cause														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		Heart	Malign	CBV	COP	Pneu	Diabt	Suicid	Nephri	ChrLiv	Septic	Alzhm	Athero	Hyper	Aortic	Resid
#	**Multiple Cause**	Dis.	Neopl	Dis.	Dis.	Influ	Mellit		Nephro	Cirrho	-emia	Dis.	-scler.	-tens	Aneur	Dis.
1	Diseases of heart	--	0.50	0.84	1.02	0.75	1.33	0.12	1.12	0.67	0.71	0.68	1.15	1.20	0.91	0.89
2	Malignant neoplasms	0.50	--	0.31	0.69	0.59	0.48	0.17	0.53	0.44	0.58	0.30	0.29	0.58	0.25	0.72
3	Cerebrovascular diseases	0.84	0.31	--	0.54	0.94	1.41	0.06	0.64	0.30	0.76	0.68	1.95	2.13	0.51	0.97
4	ChrObstructivePulDis	1.02	0.69	0.54	--	1.50	0.77	0.19	0.71	0.61	0.66	0.50	0.79	1.00	0.96	1.19
5	Pneumonia and influenza	0.75	0.59	0.94	1.50	--	0.87	0.03	1.08	0.63	2.03	1.41	0.51	0.67	0.35	1.18
6	Diabetes mellitus	1.33	0.48	1.41	0.77	0.87	--	0.12	1.79	1.08	1.21	0.75	1.72	2.57	0.33	0.94
7	Suicide	0.12	0.17	0.06	0.19	0.03	0.12	--	0.05	0.11	0.03	0.12	0.11	0.16	0.05	2.18
8	Nephritis/Nephrosis	1.12	0.53	0.64	0.71	1.08	1.79	0.05	--	1.70	2.20	0.50	1.35	0.28	1.14	1.15
9	ChronLiverDis/Cirrhosis	0.67	0.44	0.30	0.61	0.63	1.08	0.11	1.70	--	1.29	0.18	0.47	0.60	0.39	1.58
10	Septicemia	0.71	0.58	0.76	0.66	2.03	1.21	0.03	2.20	1.29	--	0.83	0.60	0.64	0.64	1.52
11	Alzheimer's disease	0.68	0.30	0.68	0.50	1.41	0.75	0.12	0.50	0.18	0.83	--	0.80	0.82	0.27	0.94
12	Atherosclerosis	1.15	0.29	1.95	0.79	0.51	1.72	0.11	1.35	0.47	0.60	0.80	--	1.62	1.11	0.94
13	Hypertension	1.20	0.58	2.13	1.00	0.67	2.57	0.16	0.28	0.60	0.64	0.82	1.62	--	1.87	1.06
14	Aortic aneurysm	0.91	0.25	0.51	0.96	0.35	0.33	0.05	1.14	0.39	0.64	0.27	1.11	1.87	--	0.93
15	Residual causes	0.89	0.72	0.97	1.19	1.18	0.94	2.18	1.15	1.58	1.52	0.94	0.94	1.06	0.93	--

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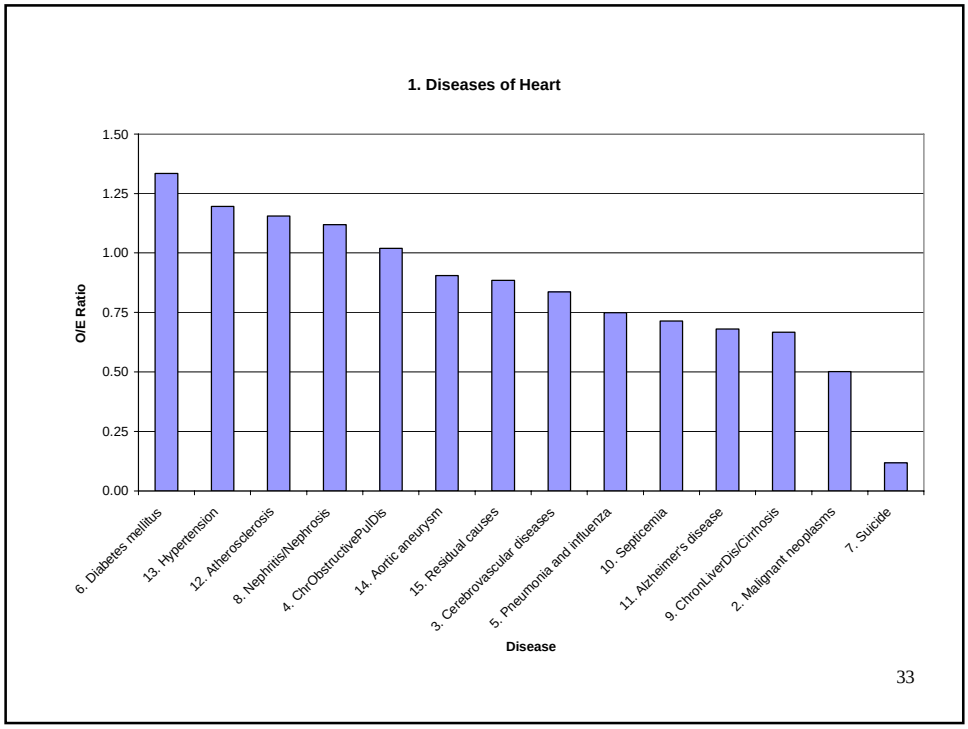
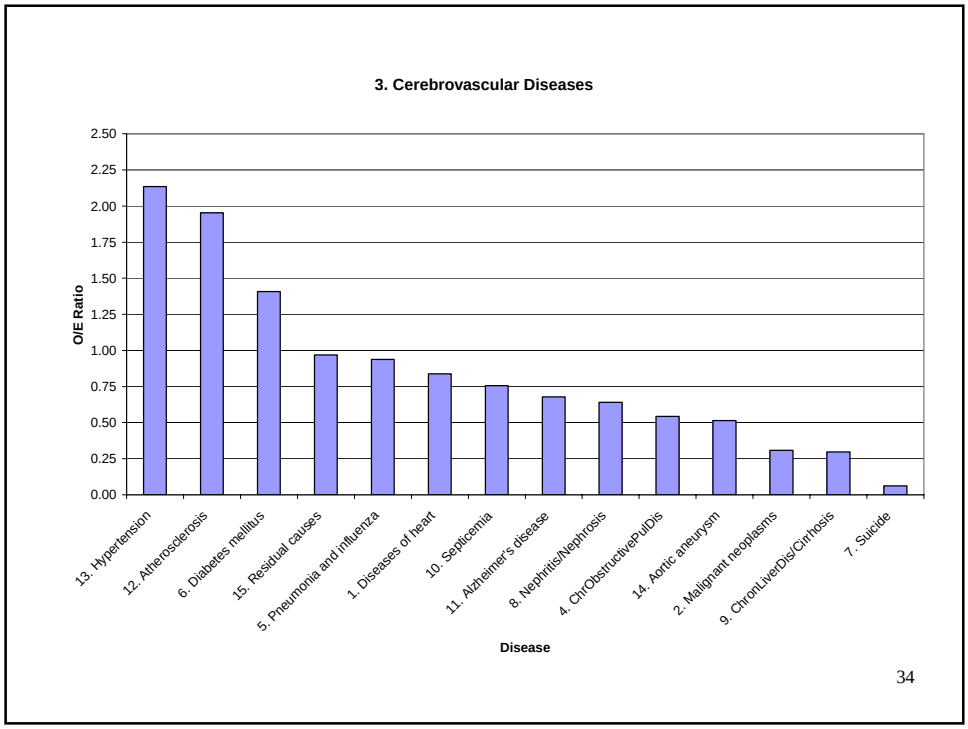
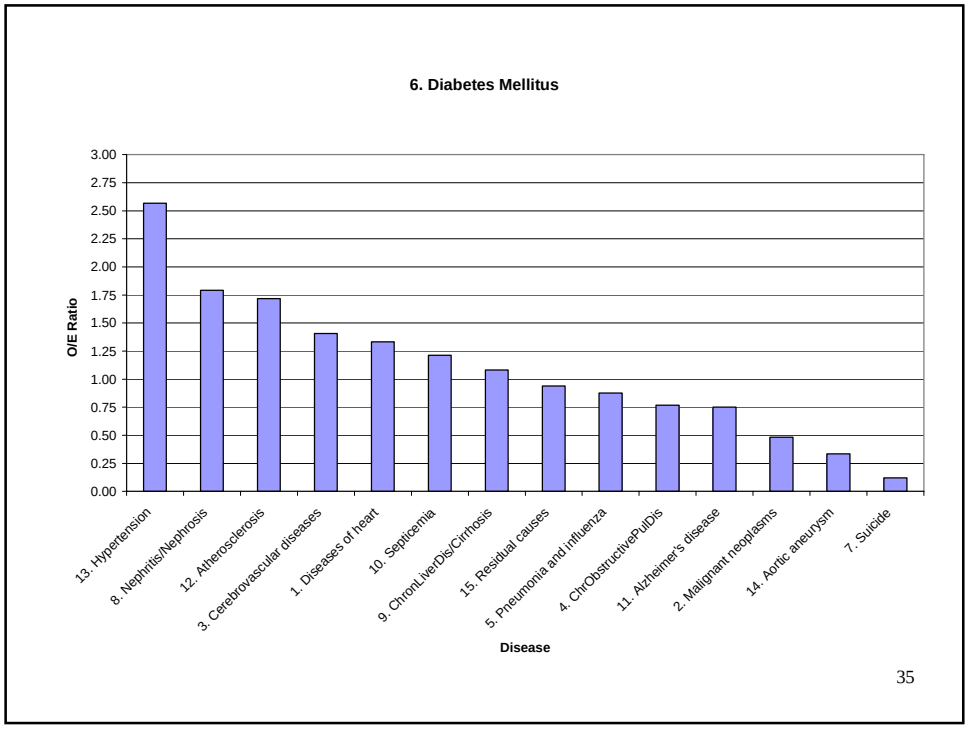
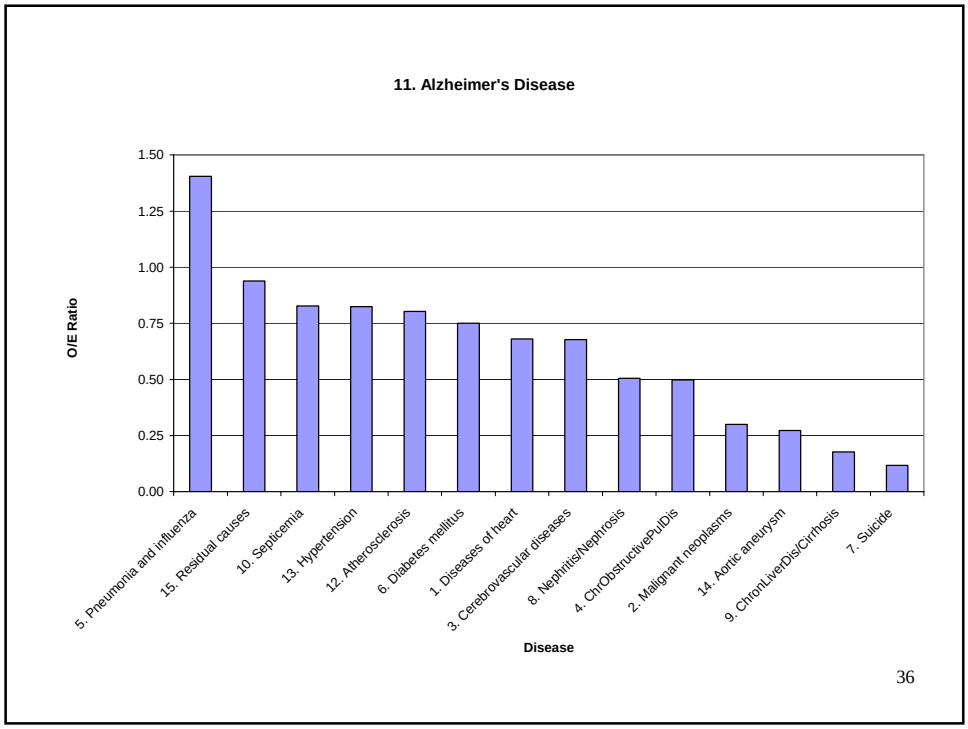
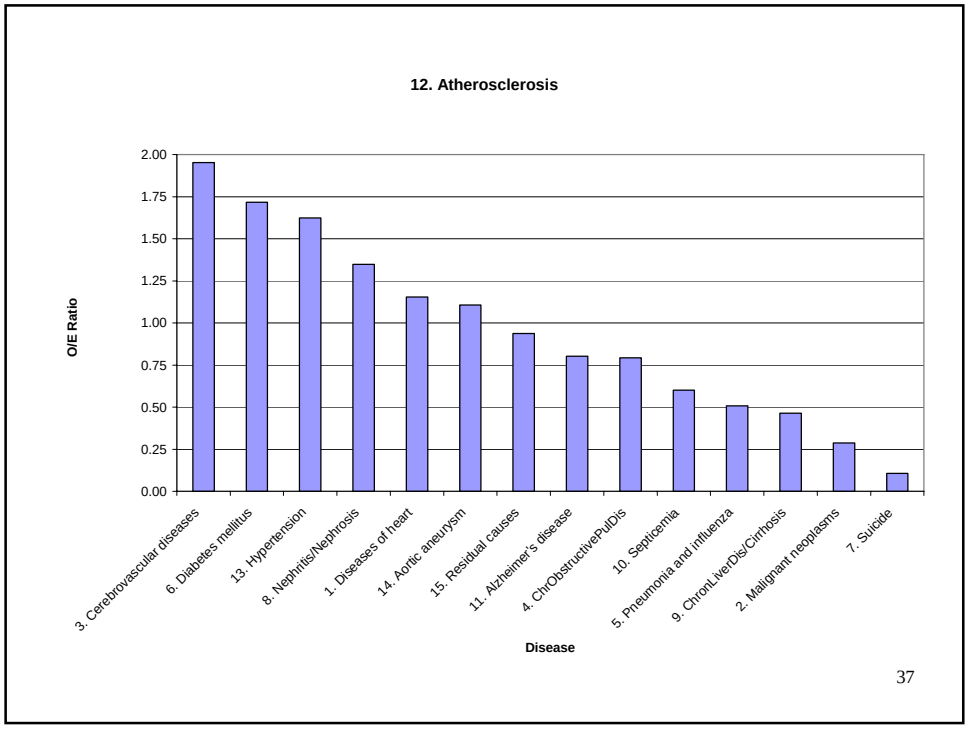

1. Diseases of Heart
O/E Ratio
1.50
1.25
1.00
0.75
0.50
0.25
0.00
6. Diabetes mellitus
13. Hypertension
12. Atherosclerosis
8. Nephritis/Nephrosis
4. ChrObstructivePulDis
14. Aortic aneurysm
15. Residual causes
3. Cerebrovascular diseases
5. Pneumonia and influenza
10. Septicemia
11. Alzheimer's disease
9. ChronLiverDis/Cirrhosis
2. Malignant neoplasms
7. Suicide
Disease
33


3. Cerebrovascular Diseases
O/E Ratio
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
13. Hypertension
12. Atherosclerosis
6. Diabetes mellitus
15. Residual causes
5. Pneumonia and influenza
1. Diseases of heart
10. Septicemia
11. Alzheimer's disease
8. Nephritis/Nephrosis
4. ChrObstructivePulDis
14. Aortic aneurysm
2. Malignant neoplasms
9. ChronLiverDis/Cirrhosis
7. Suicide
Disease
34


6. Diabetes Mellitus
O/E Ratio
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
13. Hypertension
8. Nephritis/Nephrosis
12. Atherosclerosis
3. Cerebrovascular diseases
1. Diseases of heart
10. Septicemia
9. ChronLiverDis/Cirrhosis
15. Residual causes
5. Pneumonia and influenza
4. ChrObstructivePulDis
11. Alzheimer's disease
2. Malignant neoplasms
14. Aortic aneurysm
7. Suicide
Disease
35


11. Alzheimer's Disease
O/E Ratio
1.50
1.25
1.00
0.75
0.50
0.25
0.00
5. Pneumonia and influenza
15. Residual causes
10. Septicemia
13. Hypertension
12. Atherosclerosis
6. Diabetes mellitus
1. Diseases of heart
3. Cerebrovascular diseases
8. Nephritis/Nephrosis
4. ChrObstructivePulDis
2. Malignant neoplasms
14. Aortic aneurysm
9. ChronLiverDis/Cirrhosis
7. Suicide
Disease
36


12. Atherosclerosis
O/E Ratio
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
3. Cerebrovascular diseases
6. Diabetes mellitus
13. Hypertension
8. Nephritis/Nephrosis
1. Diseases of heart
14. Aortic aneurysm
15. Residual causes
11. Alzheimer's disease
4. ChrObstructivePulDis
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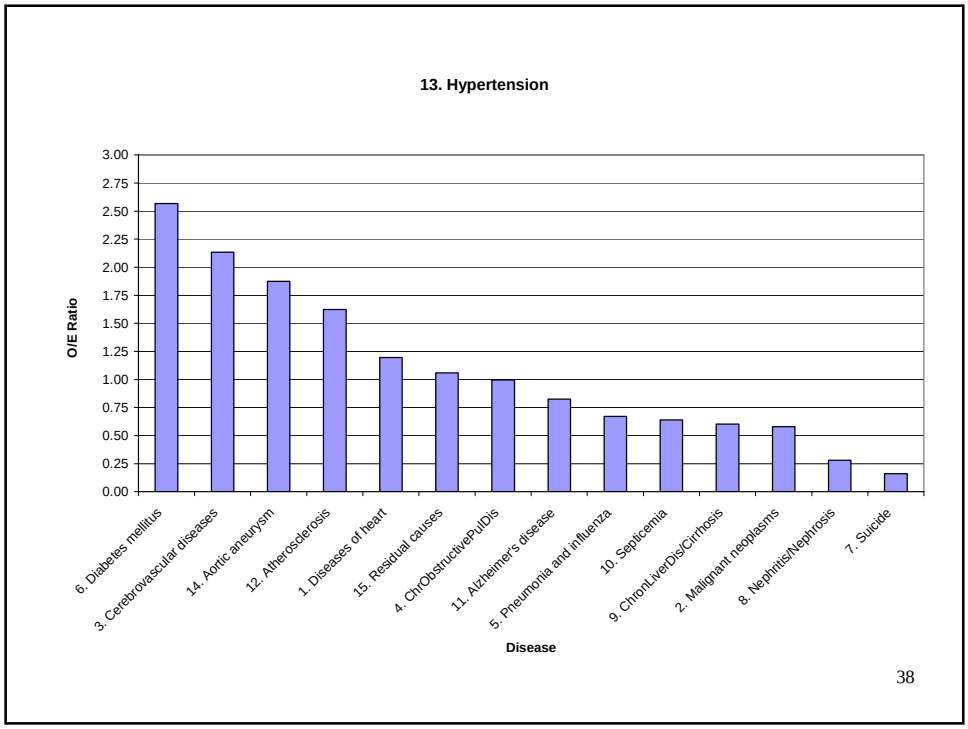
5. Pneumonia and influenza
9. ChronLiverDis/Cirrhosis
2. Malignant neoplasms
7. Suicide
Disease
37


13. Hypertension
O/E Ratio
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
6. Diabetes mellitus
3. Cerebrovascular diseases
14. Aortic aneurysm
12. Atherosclerosis
1. Diseases of heart
15. Residual causes
4. ChrObstructivePulDis
11. Alzheimer's disease
5. Pneumonia and influenza
10. Septicemia
9. ChronLiverDis/Cirrhosis
2. Malignant neoplasms
8. Nephritis/Nephrosis
7. Suicide
Disease
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Stochastic Models

Stochastic life table models with or without risk covariates are needed to reflect the impact of the various factors likely to influence mortality over the forecasting period.

The stochasticity of life table parameters and variation due to model specification errors need to be included in the uncertainty intervals for forecasts.

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Stochastic Models

Stochastic life table models with or without risk covariates can be used:

- **To produce forecasts of the distribution of the proportion of survivors at each future date;**
- **Thereby allowing capital markets to set appropriate rates of investment returns for survivor bonds and similar instruments.**

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