

# Longevity Bonds: Construction, Valuation and Use

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# Introduction

- Longevity risk: risk that members of reference population live on average longer than anticipated
- Longevity risk (LR) is one of the largest sources of risk faced by life companies and pension funds
  - Life expectancy for men aged 60 is 5 years longer in 2005 than anticipated in mortality tables made in 1980s
  - Amounts at risk £2520 bn (about \$4424 bn) (UK Pensions Commission Report) or £40k (\$70k) per person in UK
- This paper looks at Longevity bonds

# Main Features of Swiss Re Bond

- Bond is hedge to issuer; issuer gains if  $S(t)$  very low
  - Hence, bond is hedge against portfolio dominated by life insurance
- Bond is *short-term* bond to protect issuer against *extreme* deterioration in mortality
- $S(t)$  is average of 5 national mortality indices
- Does not involve complicated credit problems
- Bond is coupon-plus-principal bond, but only principal is at risk due to LR
- Spread over LIBOR compensates for principal-at-risk

# EIB/BNP Bond

- Announced in late 2004, to be issued by EIB

# Main Features of EIB/BNP Bond

- Hedge to holder; issuer gains if  $S(t)$  lower than anticipated
- Bond is hedge against portfolio dominated by annuity issues
- Bond is *long-term* bond designed to protect holder against *any* unanticipated improvement in mortality
- $S(t)$  involves a single national survivor index
- Bond involves (quite) complex credit issues
- Bond is annuity-type bond and all coupon payments at risk

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- LBs can vary across many dimensions

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- Collateralized longevity obligations (CLOs) comparable to CDOs
  - Synthetic equivalents, synthetic CLOs etc.

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- Construction using longevity swaps
  - Supplements govt bond with longevity swap

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- Bank puts bond and LS into SPV, SPV provides LB

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- Annuity bond + zero LB for terminal payment produces principal-at-risk LB, etc.

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- 25 years is too short
  - A lot of risk arises from people living well into 90s
- French, UK governments starting to issue ultra-long bonds (50 years)
- If price stability lasts, might expect to see bonds of up to 100 years' maturity

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- Example is EIB/BNP bond

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- Parties involved might also choose further credit enhancement

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- $\Rightarrow$  Need to balance hedging and liquidity


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- Moral hazard and data integrity problems

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- Can measure sensitivities using elasticities

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- Interested in these for illustrative positions for  $T=1$  to 50

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# Illustrative Positions

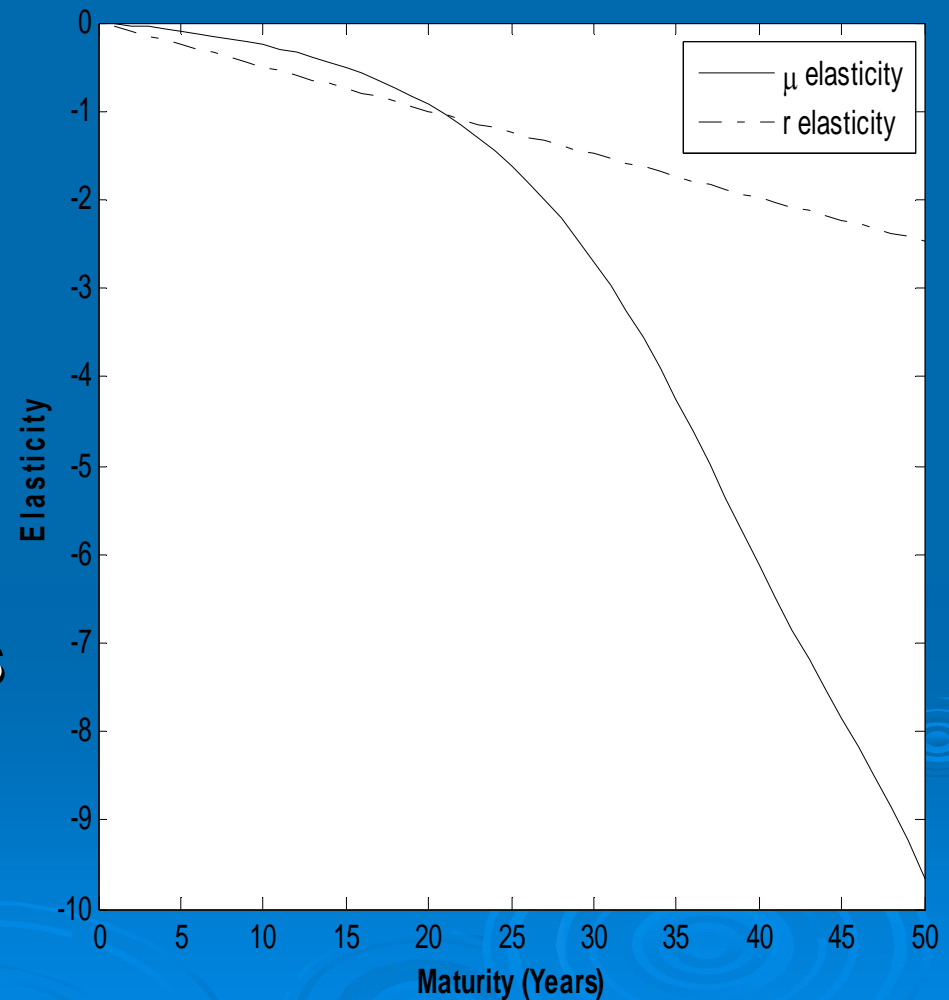
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- Principal-at-risk fixed coupon LBs where principal is  $S(t)$
- + 'inverse' equivalents

# Elasticities for Zero LBs

- Zero LB is long LR and short IR risk
- Implies zero LB is a good hedge for position that is short LR and long IR risk
- Note large elasticities esp. for high  $T$



# Zero LB Example

- Life company might be short LR and long IR risk

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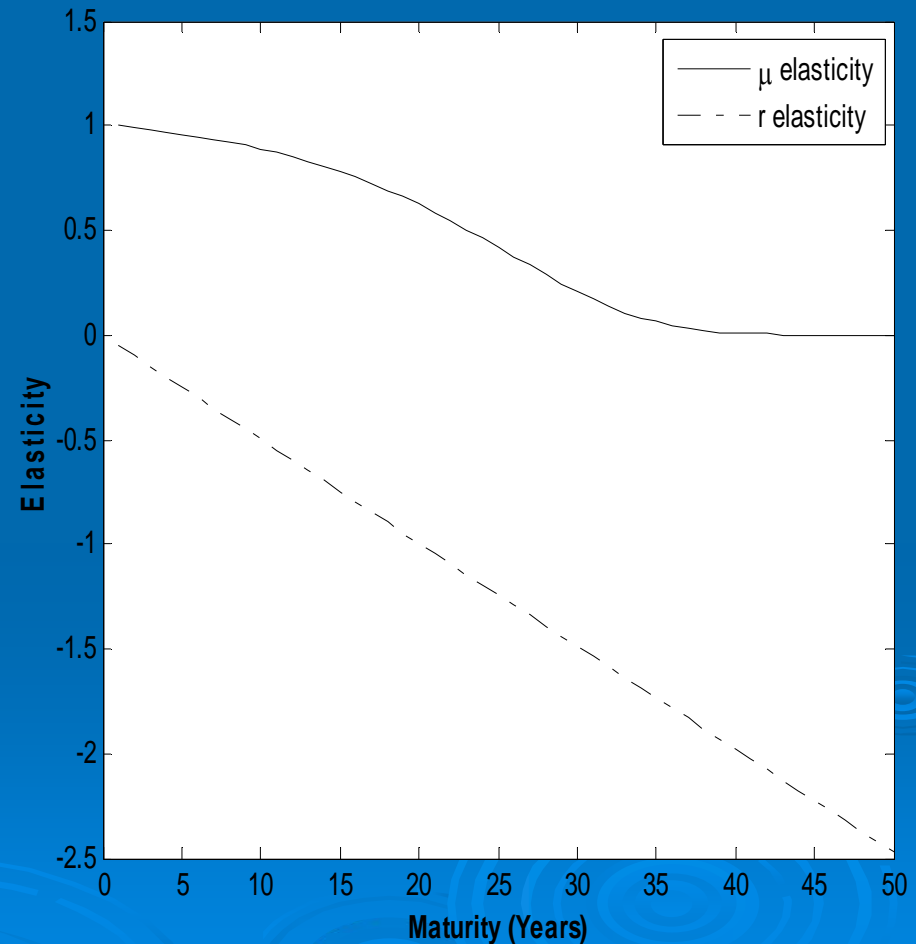
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- Or might use an LB to hedge LR and a conventional hedge for remaining IR risk

# Elasticities for Inverse Zero LBs

- Inverse zero LB is good hedge for position that is long LR and long IR risk
- Would want short to medium term maturity



# Inverse Zero LB Example

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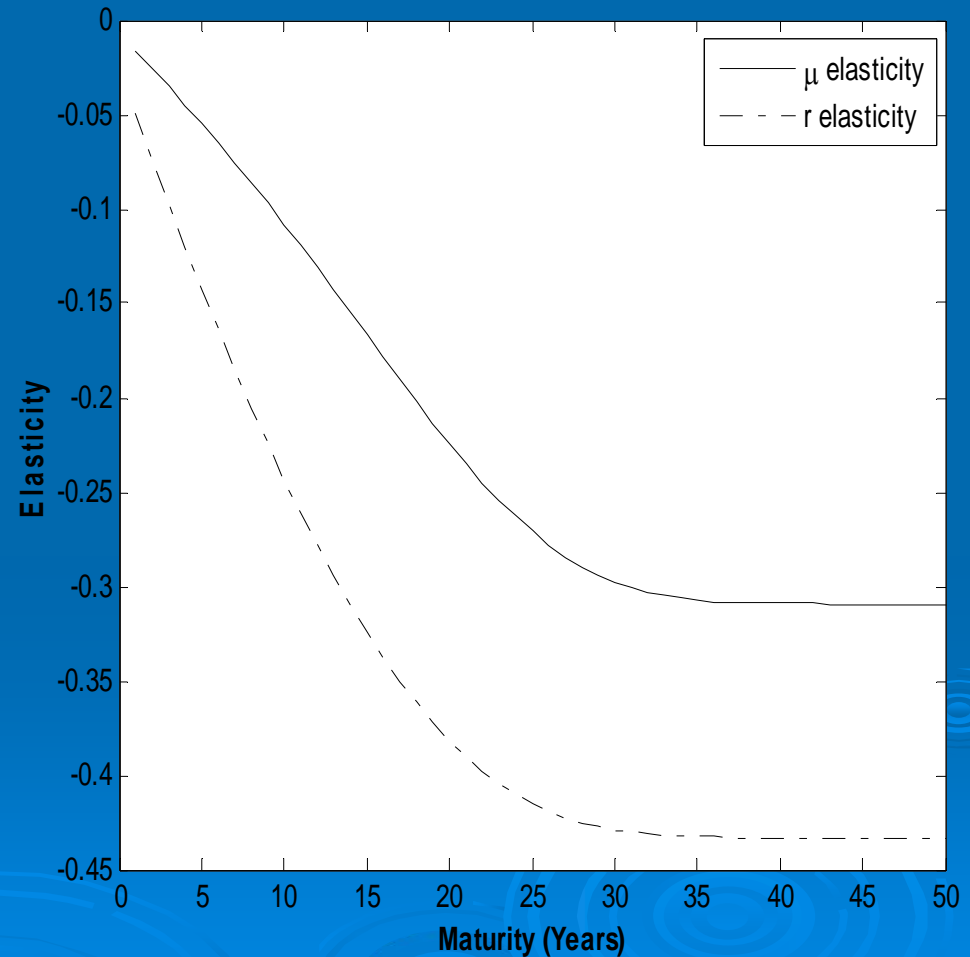
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- The *type* of security we should consider as a hedge depends on nature of LR and (possibly) IR risk exposures
- *Maturity* of hedge instrument depends on the relative sizes of these exposures

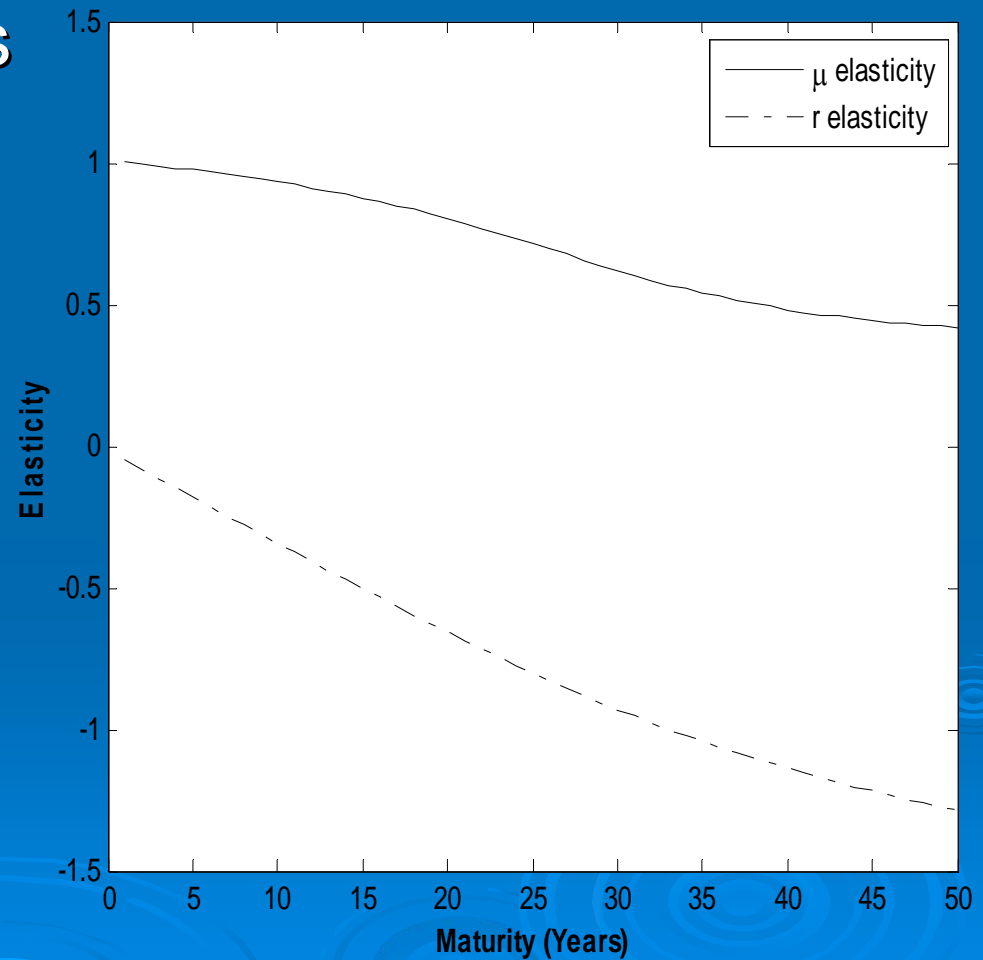
# Elasticities for Classical LBs

- Similar properties as zeros
- Elasticities generally not so large
- Less leverage than zeros



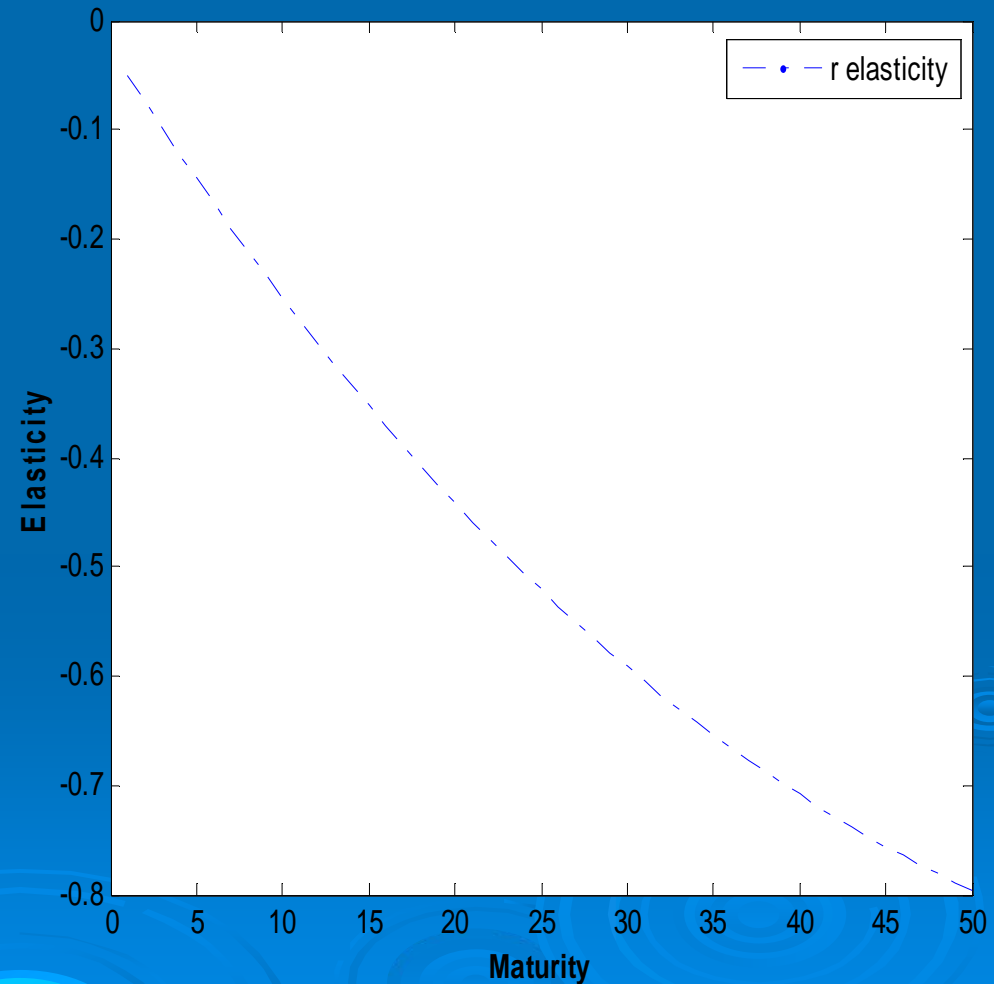
# Elasticities for Classical ILBs

- Similar properties as inverse zeros



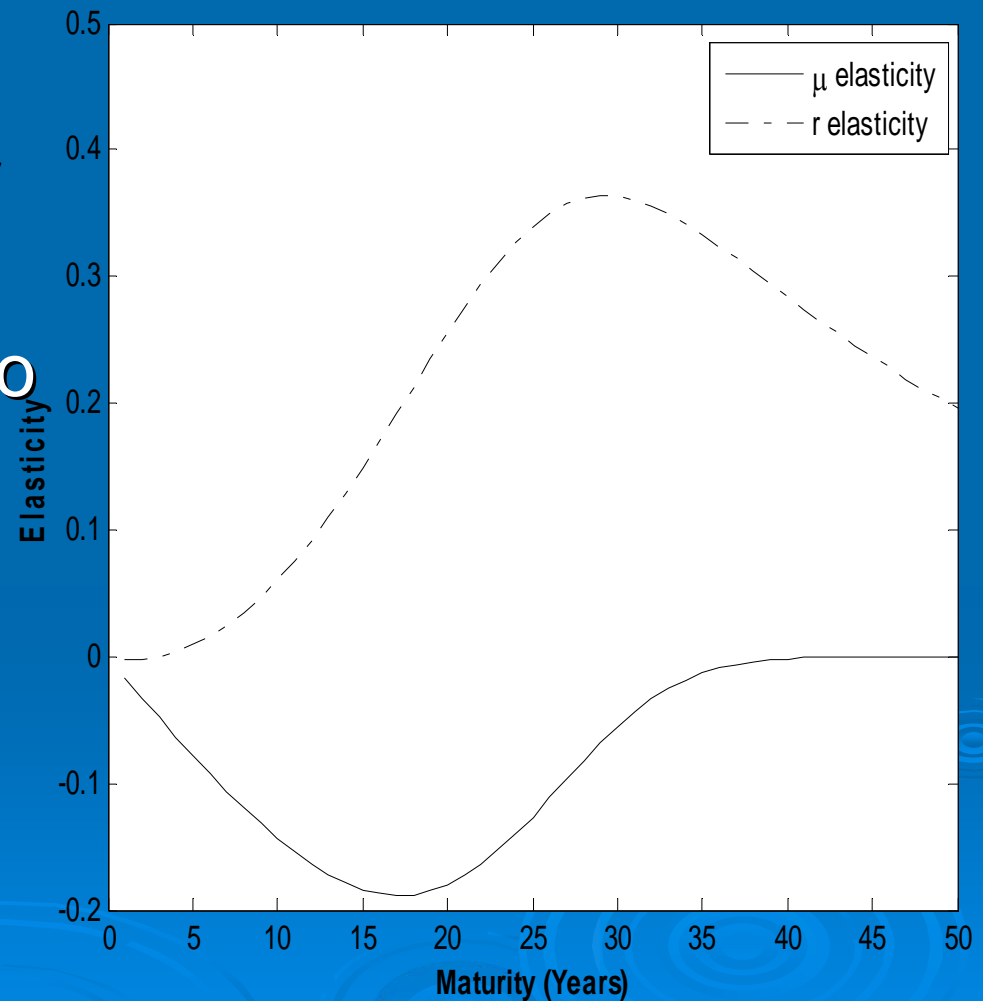
# Elasticities for Conventional AB

- Interesting to note that conventional annuity bonds have much greater IR sensitivity



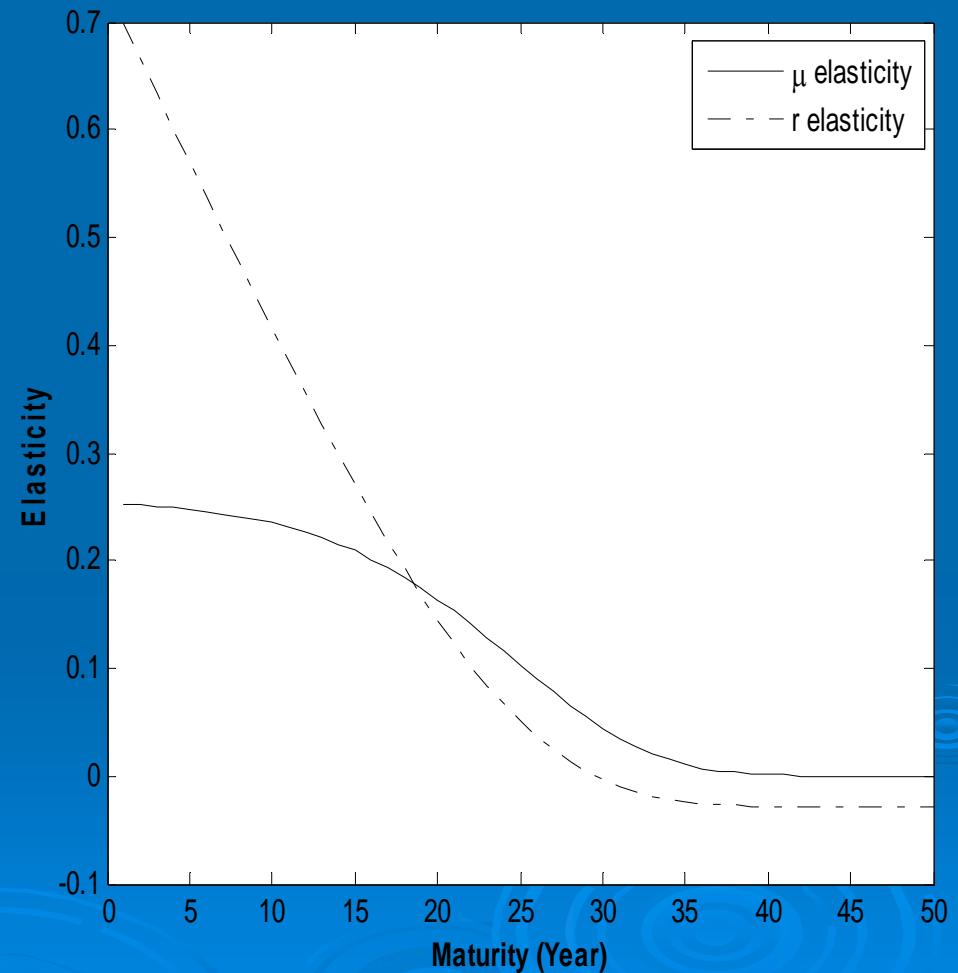
# Floating coupon PAR LBs

- These are very different from earlier ones
- Peaks/troughs due to offsetting effects
- Would hedge a position that is short both risks



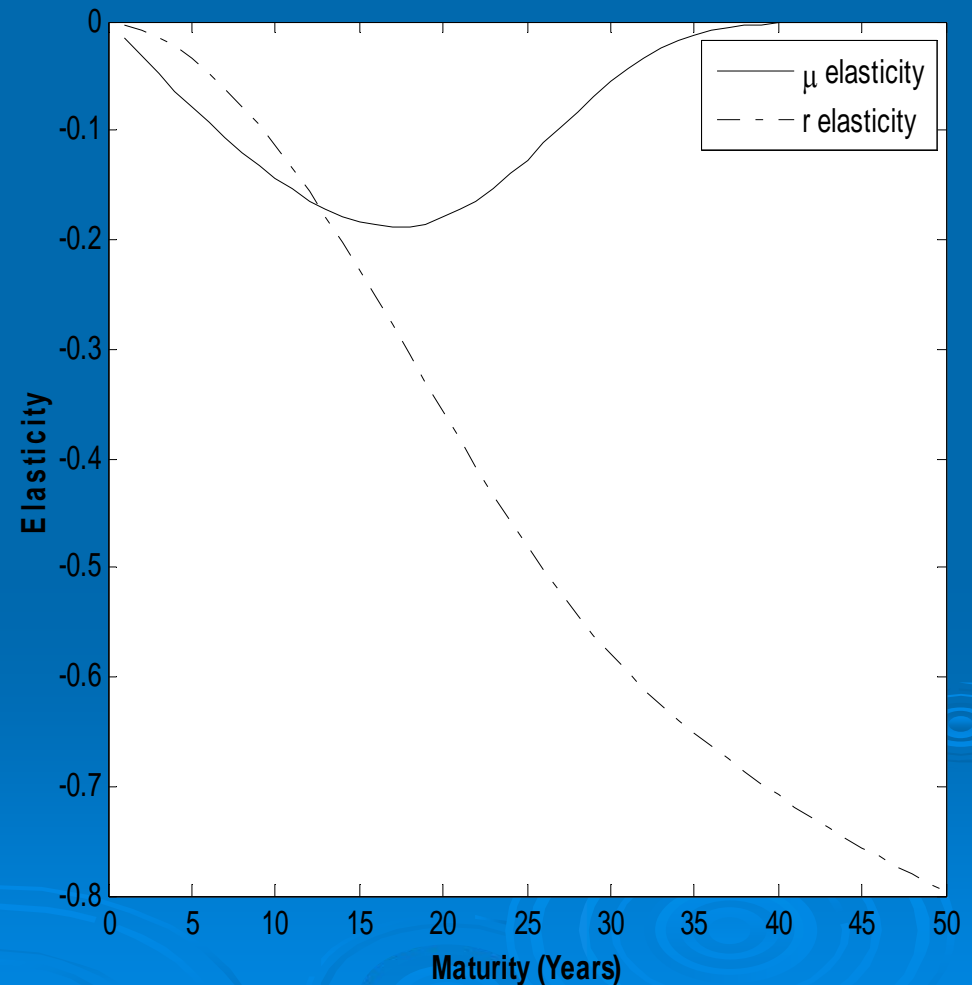
# Floating coupon PAR ILBs

- No peaks/troughs!
- Would hedge a position that is long LR and short IR risk



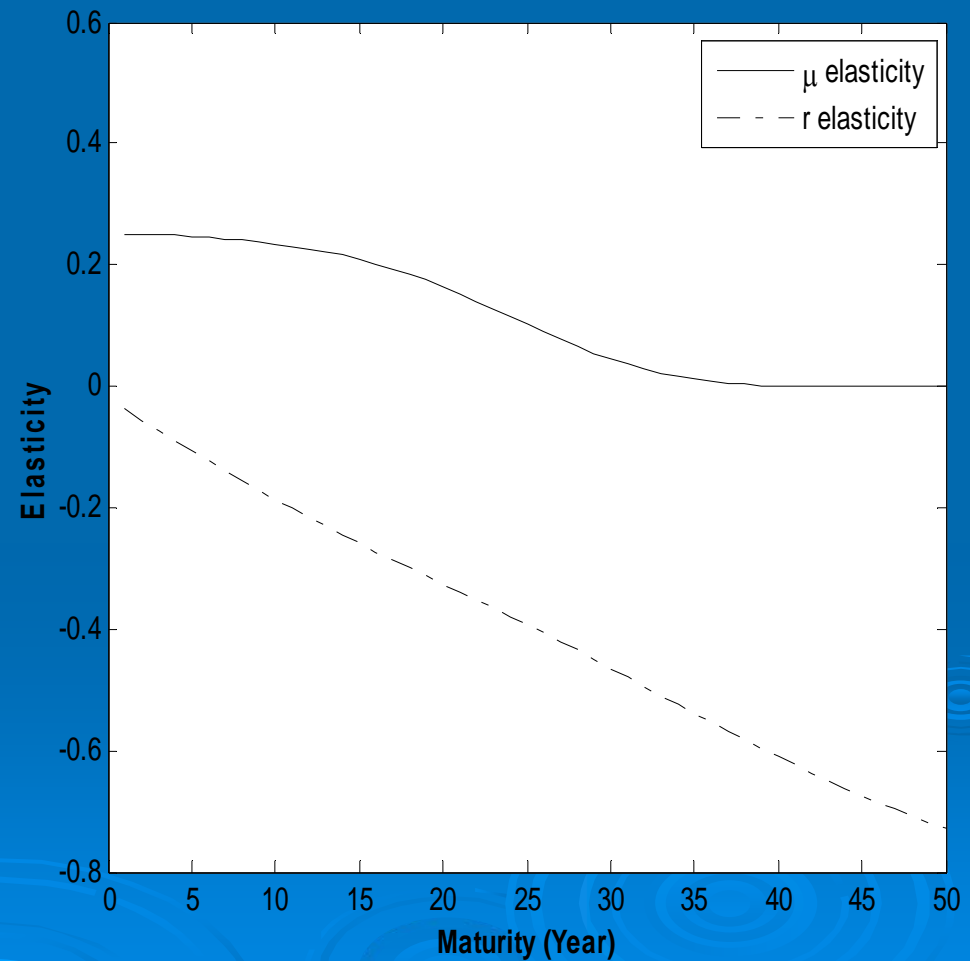
# Fixed coupon PAR LBs

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  - Attractions as low-beta investment opportunities
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